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Research Article

NATURE OF ALGEBRAIC STRUCTURE $A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l \;/\; \text{ai} \in F(field)\}$

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ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri Sri Ramakrishana. In the Present work first I proved that (A, +,.) is a non commutative ring. Second I proved $(A, +, *)$ is a commutative ring with unity, where $A = \{a_0 + a_1 i + a_2 j + a_3 k + a4l \;/$ $\text{a}_\text{i} \in F$

Keywords: *Binary Operation, Abelian Group, Semigroup, Ring, Field*

INTRODUCTION
Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that
1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:
1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.
It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.
If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION
Let $A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l \;/\; \text{ai} \in F(field)\}$
And $x = a_0 + a_1 i + a_2 j + a_3 k + a_4 l$, ai $\in F$
$y = b_0 + b_1 i + b_2 j + b_3 k + b_4 l$, bi $\in F$
$z = c_0 + c_1 i + c_2 j + c_3 k + c_4 l$, ci $\in F$

$$-x = (-a_0) + (-a_1)i + (-a_2)j + (-a_3)k + (-a_4)l$$
$$O = 0 + 0i + 0j + 0k + 0l$$
$$1 = 1 + 0i + 0j + 0k + 0l$$

c$x = ca_0 + ca_1 i + ca_2 j + ca_3 k + ca_4 l, c \in F$
Now we define first binary operation + on$A$ as

$$x + y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l\,) + \ (b_0 + b_1 i + b_2 j + b_3 k + b_4 l)$$
$= (a_0 + b_0) + (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k + (a_4 + b_4)l$
………… (1)

$$=> \ x + y = y + x, \forall\, x, y, \in A$$
$$x + (y + z) = (x + y) + z, \forall\, x, y, z \in A$$
$$0 + x = x + 0, \forall\, x \in A$$

$x + (-x) = (-x) + x = 0, \forall\, x \in A$

⇨ (A,+) is an abelian group. ………… (2)

Case 1:

Second binary operation on A as. defined as

$$x.y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l).$$
$$(b_0 + b_1 i + b_2 j + b_3 k + b_4 l)$$
$= (a_0 + a_1 + a_2 + a_3 + a_4)y \forall\, x, y \in A$ ……………(3)

$$=> \ x.(y.z) = (x.y).z, \forall\, x, y, z \in A$$

& $x.y \neq y.x, \forall\, x, y, \in A$

$$(x + y).z = x.z + y.z, \forall\, x, y, z \in A$$
$$x.(y + z) = x.y + x.z, \forall\, x, y, z \in A$$

Hence (A,.) is a non-commutative semi group.

&(A,+,.) is a non-commutative ring……….. (4)

Case 2:

Second binary operation on A as $*$ defined as

$$x * y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l) * (b_0 + b_1 i + b_2 j + b_3 k + b_4 l)$$
$= (a_0 b_0 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1)$
$+ (a_0 b_1 + a_1 b_0 + a_2 b_4 + a_3 b_3 + a_4 b_2)\, i$
$+ (a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3)\, j$
$+ (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_4)\, k$
$+ (a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0)\, l$……….. (5)

⇨ $x * y = y * x, \forall\, x, y \in A$

⇨ $x * (y * z) = (x * y) * z, \forall\, x, y, z \in A$

⇨ $1 * x = x * 1 = x, \forall\, x \in A$

Let $x^{-1} = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l)^{-1}$ be the inverse of x in A
$= (b_0 + b_1 i + b_2 j + b_3 k + b_4 l)$ (say)

By definition

$$(a_0 + a_1 i + a_2 j + a_3 k + a_4 l) * (b_0 + b_1 i + b_2 j + b_3 k + b_4 l)$$
$$= (b_0 + b_1 i + b_2 j + b_3 k + b_4 l) * (a_0 + a_1 i + a_2 j + a_3 k + a_4 l)$$
$= (a_0 b_0 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1)$
$+ (a_0 b_1 + a_1 b_0 + a_2 b_4 + a_3 b_3 + a_4 b_2)\, i$
$+ (a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3)\, j$
$+ (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_4)\, k$
$+ (a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0)\, l$

Hence we get

$a_0 b_0 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 = 1$ …………(6)
$a_0 b_1 + a_1 b_0 + a_2 b_4 + a_3 b_3 + a_4 b_2 = 0$ ………… (7)
$a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3 = 0$ ………… (8)
$a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_4 = 0$ ………… (9)
$a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0 = 0$ ………… (10)

Rewriting eq^n (7), (8), (9), (10)

$a_1 b_0 + a_0 b_1 + a_4 b_2 + a_3 b_3 + a_2 b_4 = 0$ ………… (11)

$a_2 b_0 + a_1 b_1 + a_0 b_2 + a_4 b_3 + a_3 b_4 = 0$ .... (12)
$a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3 + a_4 b_4 = 0$ .... (13)
$a_4 b_0 + a_3 b_1 + a_2 b_2 + a_1 b_3 + a_0 b_4 = 0$ .... (14)

$$\frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = \frac{b_4}{F_4} = k$$

Where; $F_0 = \begin{vmatrix} a_0 & a_4 & a_3 & a_2 \\ a_1 & a_0 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_4 \\ a_3 & a_2 & a_1 & a_0 \end{vmatrix}$

$F_1 = \begin{vmatrix} a_1 & a_4 & a_3 & a_2 \\ a_2 & a_0 & a_4 & a_3 \\ a_3 & a_1 & a_0 & a_4 \\ a_4 & a_2 & a_1 & a_0 \end{vmatrix}$

$F_2 = \begin{vmatrix} a_1 & a_0 & a_3 & a_2 \\ a_2 & a_1 & a_4 & a_3 \\ a_3 & a_2 & a_0 & a_4 \\ a_4 & a_3 & a_1 & a_0 \end{vmatrix}$

$F_3 = \begin{vmatrix} a_1 & a_0 & a_4 & a_2 \\ a_2 & a_1 & a_0 & a_3 \\ a_3 & a_2 & a_1 & a_4 \\ a_4 & a_3 & a_2 & a_0 \end{vmatrix}$

$F_4 = \begin{vmatrix} a_1 & a_0 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_4 \\ a_3 & a_2 & a_1 & a_0 \\ a_4 & a_3 & a_2 & a_1 \end{vmatrix}$

$=> b_0 = kF_0$ .... (15)
$b_1 = -kF_1$........... (16)
$b_2 = kF_2$ .... (17)
$b_3 = -kF_3$........... (18)
$b_4 = kF_4$........... (19)
From eq^n (6), (15), (16), (17), (18), (19)one obtains
$k\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4\} = 1$
Hence
$k = {}^{1}/_{\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4\}} = \infty$ if$(a_0 = a_1 = a_2 = a_3 = 0 \ \& a_4 \neq 0)$
From the above discussion we get
$x^{-1} = \frac{F_0 - F_1 i + F_2 j - F_3 k + F_4 l}{\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4\}}$not exist for each non zero element in A

$=> x^{-1}$is exist for each non zero element in A.
$Hence$ $(A, +, *)$ Is a Commutative ring with unity.
Where A = $\{a_0 + a_1 i + a_2 j + a_3 k + a_4 l$ / ai $\in F\ (field)\}$

Conclusion

From the above discussion, I come to the following conclusions first I proved that (A, +,.) is a ring. Second I proved that $(A, +, *)$ Is a Commutative ring with unity. Where A $= \{a_0 + a_1 i + a_2 j + a_3 k +$ $a4l$ / $a_i \in F\ (field)$

REFERENCES

Herstein IN (1992). *Topics in Algebra* 2nd edition (Wiley Eastern Limited) 26-256. ISBN: 0 85226 354 6.