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Research Article

NATURE OF GENERALISED ALGEBRAIC STRUCTURE $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \,/\, a_i \in F \,\&G_i \in C(P)$= CLASS OF ALGEBRAIC STRUCTURE$\}$

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ABSTRACT

This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri Sri Ramakrishana. In the Present work first I proved that (A, +,.) is a non abelian ring. Second I proved $(A,+,*)$ is a commutative ring with unity, where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \,/\, a_i \in$ F & $Gi \in C(P)$, and C(P) = Class of *algebraic Structure*.

Keywords: *Binary Operation, Abelian Group, Ring, Field, Class of Algebraic Structure*

INTRODUCTION

Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1 a, b ∈ G implies that a*b ∈ G

2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)

3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G

4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:

1 a+b is in R.

2 a+b = b+a.

3 (a+b)+c = a+(b+c).

4 There is an element 0 in R such that a+0 = a,∀ a ∈ R

5 There exist an element –a in R such that a + (-a) = 0.

6 a*b is in R

7 a*(b*c) = (a*b)*c.

8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.

It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.

If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION

Let $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \,/\, a_i \in F \,\&\, G_i \in C(P)\}$

Where C(P) = Class of *algebraic Structure*

Let $x = a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3$;

$y = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3$;

$z = c_0G_0 + c_1G_1 + c_2G_2 + c_3G_3$;

$$G_0 = 1G_0 + 0G_1 + 0G_2 + 0G_3$$
$$0 = 0G_0 + 0G_1 + 0G_2 + 0G_3$$
$$-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2 + (-a_3)G_3$$
$cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2 + (ca_3)G_3$, $c \in F$

Here first binary operation + on A defined as

$$x + y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3) + (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3)$$
$= (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2 + (a_3 + b_3)G_3$

………… (1)

$$=> \ x + y = y + x, \qquad \forall\, x, y, \in A$$
$$x.(y + z) = (x + y) + z, \qquad \forall\, x, y, z \in A$$
$$0 + x = x + 0, \qquad \forall\, x \in A$$
$x + (-x) = (-x) + x = 0, \forall\, x \in A$

⇨ $(A,+)$ is an abelian group. ………… (2)

Case 1:

Second binary operation. on A defined as

$x.y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3).(b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3)$
$= \ (a_0 + a_1 + a_2 + a_3)\, y, \forall\, x, y \in A$………… (3)
$x.y \neq y.x, \qquad \forall\, x, y, z \in A$

$$x.(y.z) = (x.y).z, \forall\, x, y, z \in A$$

Hence (A,.) is a non abelian semi group ………… (4)

Also

$x.(y + z) = x.y + x.z, \forall\, x, y, z \in A$
$(x + y).z = x.y + y.z, \forall\, x, y, z \in A$

From (1) to (5), one obtains

(A, +,.) is a non abelian ring.

Case 3:

Second binary operation * on A defined as

$$x * y = (a_0b_0 + a_1b_3 + a_2b_2 + a_3b_1)G_0 + (a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2)G_1$$
$$+ (a_0b_2 + a_1b_1 + a_2b_0 + a_3b_3)G_2 \ + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)G_3$$

………… (6)

= >

$x * y = y * x, \forall\, x, y, \in A$
$x * (y * z) = (x * y) * z, \forall\, x, y, z \in A$

$$G_0 * x = x * G_0 = x \forall\, x \in A$$

Let $x^{-1} = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3$ be the inverse of
$x = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3)$ in A.

∴ By definition we get

$$x * x^{-1} = x^{-1} * x = G_0$$

$(a_0b_0 + a_1b_3 + a_2b_2 + a_3b_1) = 1$ …………. (7)
$(a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2) = 0$ …………. (8)
$(a_0b_2 + a_1b_1 + a_2b_0 + a_3b_3) = 0$ …………. (9)
$(a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0) = 0$………. (10)

Rewriting eq^n (8), (9) &(10), one obtains as

$$(a_1b_0 + a_0b_1 + a_3b_2 + a_2b_3) = 0$$
$$(a_2b_0 + a_1b_1 + a_0b_2 + a_3b_3) = 0$$
$$(a_3b_0 + a_2b_1 + a_1b_2 + a_0b_3) = 0$$

$=> \frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = k\ (constant)$……… (11)

$$F_0 = \begin{vmatrix} a_0 & a_3 & a_2 \\ a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_1 = \begin{vmatrix} a_1 & a_3 & a_2 \\ a_2 & a_0 & a_3 \\ a_3 & a_1 & a_0 \end{vmatrix}$$

$$F_2 = \begin{vmatrix} a_1 & a_0 & a_2 \\ a_2 & a_1 & a_3 \\ a_3 & a_2 & a_0 \end{vmatrix}$$

$$F_3 = \begin{vmatrix} a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 \end{vmatrix}$$

$b_0 = kF_0,$
$b_1 = -kF_1,$
$b_2 = kF_2,$
$b_3 = -kF_3$
$b_0 = k\{a_0^3 - 2a_0a_1a_3 + a_2(a_1^2 + a_3^2) - a_0a_2^2\}$
………… (12)
$b_1 = -k\{a_3^3 - 2a_0a_2a_3 + a_1(a_0^2 + a_2^2) - a_3a_1^2\}$
………… (13)
$b_2 = \{a_2^3 - 2a_1a_2a_3 + a_0(a_1^2 + a_3^2) - a_2a_0^2\}$
………… (14)
$b_3 = -k\{a_1^3 - 2a_0a_1a_2 + a_3(a_0^2 + a_2^2) - a_1a_3^2\}$
………… (15)
= >

$$k = 1 \Big/ \left\{ \begin{matrix} (a_0^4 - a_1^4 + a_2^4 - a_3^4) + 4a_0a_2(a_1^2 + a_3^2) - 4a_1a_3(a_0^2 + a_2^2) \\ -2a_0^2a_2^2 + 2a_1^2a_3^2 \end{matrix} \right\}$$

$= \infty \; if \; a_0 = a_1 \& a_2 = a_3$
Hence x^{-1} of each $x \in A$ does not exist.

=> $(A,*)$ is a commutative monoid …………(16)
$also \; x * (y + z) = x * y + x * z, \forall \, x, y, z \in A$
$x * (y + z) = x * z + y * z, \forall \, x, y, z \in A$
………… (17)
$(A, +, *)$Is a commutative ring with unity.
Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \; / \; a_i \in F \; \& \; G_i \in C(P)\}$
And C(P) = Class of $algebraic \; Structure$

Conclusion

From the above discussion, I come to the following conclusions-
first I proved that (A, +,.) is a non abelian ring. Second I proved $(A, +, *)$Is a commutative ring with unity.Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \; / \; a_i \in F \; \& \; G_i \in C(P)\}$ and C(P) = Class of $algebraic \; Structure$.

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