International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (1) January-March, pp. 149-152/Durge

Research Article

NATURE OF GENERALIZED ALGEBRAIC STRUCTURE $A = \{a_0G_0 + a_1G_1 + a_2G_2 \;/\; a_i \in F \& G_i \in \; C(P) = \text{ class of algebraic structure}\}$

***Manohar Durge**
ANC, Anandwan Warora
**Author for Correspondence*

ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri Sri Ramakrishana. In the Present work first I proved that (A, +,.) is a non abelian ring. Second I proved $(A, +,*)$ is a commutative ring with unity. Third I proved $(A, +, @)$ is a commutative ring with unity, where $A = \{a_0G_0 + a_1G_1 + a_2G_2 \;/\; a_i \in F \;\&\; G_i \;\in\; C(P)\}$ and C(P)=Class of $algebraic\ Structure$, where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \;/\; a_i \in F \;\&\; G_i \;\in\; C(P)\}$, and C(P) = Class of $algebraic\ Structure$.

Keywords: *Binary Operation, Abelian Group, Ring, Field, Class of Algebraic Structure*

INTRODUCTION

Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a \; = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:

1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.

It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.

If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION

Let $A = \{a_0G_0 + a_1G_1 + a_2G_2 \;/\; a_i \in F \& \; G_i \in \; C(P) = \text{ class of algebraic structure}\}$
And x $= a_0G_0 + a_1G_1 + a_2G_2; a_i \in F$
$y = b_0G_0 + b_1G_1 + b_2G_2;\; b_i \in F$
$z = c_0G_0 + c_1G_1 + c_2G_2;\; c_i \in F$
$G_0 = 1G_0 + 0G_1 + 0G_2, 1 \;\in F$
$0 = 0G_0 + 0G_1 + 0G_2, 0 \;\in F$

$$-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2$$
$$x = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2$$
$cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2,\ c\ \in F$

Here first binary operation + on A defined as

$x + y = (a_0G_0 + a_1G_1 + a_2G_2) + (b_0G_0 + b_1G_1 + b_2G_2)$
$= (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2$ …………(1)

$$=> \ x + y = y + x, \forall\ x, y, \in A$$
$$x + (y + z) = (x + y) + z, \forall\ x, y, z \in A$$
$$0 + x = x + 0 = x, \forall\ x\ \in A$$
$$x + (-x) = (-x) + x = 0, \forall\ x \in A$$

$=> (A, +)$is an abelian group.
………… (2)

Case 1:

Second binary operation.onA defined as

$x.y = (a_0G_0 + a_1G_1 + a_2G_2).(b_0G_0 + b_1G_1 + b_2G_2)$
$= (a_0 + a_1 + a_2)y$………… (3)
$x.y \neq y.x, \forall\ x, y, z \in A$

$$x.(y.z) = (x.y).z, \forall\ x, y, z \in A$$

Hence (A,.) is a non-abelian semi group. ………… (4)

Also

$$x.(y + z) = x.y + x.z, \forall\ x, y, z \in A$$

$\&(x + y).z = x.z + y.z, \forall\ x, y, z \in A$ ………… (5)

From (1), (2), (3), (4) & (5) one obtains
(A, +,.) is a non abelian ring.

Case 2:

Second binary operation * on A defined as

$$x * y = (a_0G_0 + a_1G_1 + a_2G_2) * (b_0G_0 + b_1G_1 + b_2G_2)$$

$= (a_0b_0 + a_1b_2 + a_2b_1)G_0 + (a_0b_1 + a_1b_0 + a_2b_2)G_1 + (a_0b_2 + a_1b_1 + a_2b_0)G_2$
………….. (6)

$$=> \ x * y = y * x, \forall\ x, y \in A$$
$$x * (y * z) = (x * y) * z, \forall\ x, y, z \in A$$

$G_0 * x = x * G_0 = G_0, \forall\ x, y \in A$

Let $x^{-1} = \ b_0G_0 + b_1G_1 + b_2G_2$ be the inverse of $x = a_0G_0 + a_1G_1 + a_2G_2$

$=> x * x^{-1} = x^{-1} * x = G_0 = 1G_0 + 0G_1 + 0G_2$

$$(a_0b_0 + a_1b_2 + a_2b_1)G_0 + (a_0b_1 + a_1b_0 + a_2b_2)G_1 + (a_0b_2 + a_1b_1 + a_2b_0)G_2 \ = \ 1G_0 + 0G_1 + 0G_2$$

Hence

$a_0b_0 + a_1b_2 + a_2b_1 = 1$ ………….. (7)
$a_0b_1 + a_1b_0 + a_2b_2 = 0$ ………….. (8)
$a_0b_2 + a_1b_1 + a_2b_0 = 0$ ………….. (9)

Rewriting equation (8) & (9)

$a_1b_0 + a_0b_1 + a_2b_2 = 0$ …………… (10)
$a_2b_0 + a_1b_1 + a_0b_2 = 0$ …………… (11)

$$=> \frac{b_0}{\begin{vmatrix} a_0 & a_2 \\ a_1 & a_0 \end{vmatrix}} = \frac{-b_1}{\begin{vmatrix} a_1 & a_2 \\ a_2 & a_0 \end{vmatrix}} = \frac{b_2}{\begin{vmatrix} a_1 & a_0 \\ a_2 & a_1 \end{vmatrix}} = constant = k$$

⇨ $b_0 = k(a_0^2 - a_1a_2)$ ………… (12)

$b_1 = -k(a_1a_0 - a_2^2)$………… (13)

$b_2 = k(a_1^2 - a_0a_2)$………… (14)
Hence
$k = 1/\{a_0^3 + a_1^3 + a_2^3 - 3a_0a_1a_2\} = \infty$ if $a_0+a_1 + a_2 = 0$
=> x^{-1} does not exist for each of the element in A.
=> $(A,*)$ is a commutative monoid.
=> $(A,+,*)$ is a commutative ring with unity. ………. (15)

Case: 3
Second binary operation @ on A defined as

$$x@y = (a_0G_0 + a_1G_1 + a_2G_2)@(b_0G_0 + b_1G_1 + b_2G_2)$$
$$= (a_0b_0)G_0 + [a_1b_0 + (a_1 + a_2)b_1 + a_1b_2]G_1$$
$$+ [a_2b_0 + a_1b_1 + (a_0 + a_2)b_2]\, G_2 \text{ ………… (16)}$$

$x@y = y@x, \forall\, x, y \in A$

$$x@\,(y@z) = (x@y)@z \forall\, x, y, z \in A$$

$x@G_0 = G_0@x = x, \forall\, x \in A$
Let $x^{-1} = b_0G_0 + b_1G_1 + b_2G_2$ be the inverse of

$x = a_0G_0 + a_1G_1 + a_2G_2$ in A.
∴ By definition

$$x@x^{-1} = x^{-1}@x = G_0 = 1G_0 + 0G_1 + 0G_2$$

$a_0.b_0$=1 ………… (17)
$a_0b_0 + (a_1 + a_2)b_1 + a_1b_2 = 0$ ………… (18)
$a_2b_0 + a_1b_1 + (a_0 + a_2)b_2 = 0$ ………… (19)
Solving eq^n (18) & (19) for b_0, b_1&b_2 one obtains

$$\frac{b_0}{\begin{vmatrix}(a_1 + a_2) & a_1 \\ a_1 & a_0 + a_2\end{vmatrix}} = \frac{-b_1}{\begin{vmatrix}a_1 & a_1 \\ a_2 & (a_0 + a_2)\end{vmatrix}} = \frac{b_2}{\begin{vmatrix}a_1 & (a_1 + a_2) \\ a_2 & a_1\end{vmatrix}} = k$$

$b_0 = k\{(a_1 + a_2)(a_0 + a_2) - a_1^2\}$ ………… (20)
$b_1 = -ka_0a_1$ ………… (21)
$b_2 = k\{a_1^2 - a_2^2 - a_1a_2\}$ ………… (22)
$k = 1/a_0\{a_0a_1 + a_1a_2 + a_0a_2 + a_2^2 - a_1^2\}$ ………… (23)
Substituting the values of k from (23) in equation (21) & (22) one obtains
$b_1 = {-a_1}/\{a_0a_1 + a_1a_2 + a_0a_2 + a_2^2 - a_1^2\}$ ………… (24)

$$b_2 = \{a_1^2 - a_2^2 - a_1a_2\}/a_0\{a_0a_1 + a_1a_2 + a_0a_2 + a_2^2 - a_1^2\}$$

………… (25)
Substituting the values of b_0, b_1&b_2 in LHS of equation (18) & (19) then LHS of equation (18) & (19) does not reduce to zero.
Hence x^{-1} not exist for each $x = a_0G_0 + a_1G_1 + a_2G_2$ in A.
=> $(A, @)$ is a commutative monoid.
Also

$$x@(y + z) = x@y + x@z \forall\, x, y, z \in A$$

$$(x + y)@z = x@z + y@z z\forall\ x, y, z \in A$$

$=>$ $(A, +, @)$is a commutative ring with unity.

Conclusion

From the above discussion, I come to the following conclusions first I proved that(A, +,.) is a non abelian ring. Second I proved $(A, +,*)$Is a commutative ring with unity.Third I proved $(A, +, @)$ is a commutative ring with unity. Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 \ / \ a_i \in F \ \& \ G_i \ \in \ C(P)\}$ and C(P) = Class of $algebraic\ Structure$.

REFERENCES

Herstein IN (1992). *Topics in Algebra* 2nd edition (Wiley Eastern Limited) 26-256. ISBN: 0 85226 354 6.