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Research Article

NATURE OF ALGEBRAIC STRUCTURE $\boldsymbol{A = \{a_0 + a_1 i + a_2 j + a_3 k \ / \ a_0, a_1, a_2, a_3 \ \in F(Field)\}}$

***Manohar Durge**
ANC, Anandwan Warora
**Author for Correspondence*

ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri SriRamakrishana. In the Present work first I proved that $(A, +, .)$ is a ring. Second I proved that $(A, +, *)$ is a Commutative ring with unity, Where $A = \{a_0 + a_1 i + a_2 j + a_3 k \ / \ a_0, a_1, a_2, a_3 \ \in F\}$.

Keywords: *Binary Operation, Abelian Group, Semigroup, Ring, Field*

INTRODUCTION

Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a \ = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:

1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.

It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.

If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION

Let $A = \{a_0 + a_1 i + a_2 j + a_3 k \ / \ a_0, a_1, a_2, a_3 \ \in F\}$ and
Let $x = a_0 + a_1 i + a_2 j + a_3 k$; $y = b_0 + b_1 i + b_2 j + b_3 k$
$z = c_0 + c_1 i + c_2 j + c_3 k$; $O = 0 + 0i + 0j + 0k$
$1 = 1 + 0i + 0j + 0k$;

$$-x = (-a_0) + (-a_1)i + (-a_2)j + (-a_3)k$$

Here first binary operation on A as + defined as

$x + y = (a_0 + a_1 i + a_2 j + a_3 k) + (b_0 + b_1 i + b_2 j + b_3 k)$
$= (a_0 + b_0) + (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$
………… (1)

$$=> \ x + y = y + x, \forall \ x, y, \in A$$

$$x + (y + z) = (x + y) + z, \forall\, x, y, z \in A$$

$$0 + x = x + 0, \forall\, x \in A$$

$x + (-x) = (-x) + x = 0, \forall\, x \in A$

⇨ $(A,+)$ is an abelian group. ………… (2)

Case 1:

Second binary operation on Aas.defined as

$$x.y = (a_0 + a_1 i + a_2 j + a_3 k).(b_0 + b_1 i + b_2 j + b_3 k)$$

$= (a_0 + a_1 + a_2 + a_3)b_0 + (a_0 + a_1 + a_2 + a_3)b_1 i$

$$+(a_0 + a_1 + a_2 + a_3)b_2 j + (a_0 + a_1 + a_2 + a_3)b_3 k$$

………… (3)

=>

$$x.(y.z) = (x.y).z, \forall\, x, y, z \in A$$

& $x.(y + z) = x.y + x.z, \forall\, x, y, z \in A$

$$(x + y).z = x.z + y.z\ \forall\, x, y, z \in A$$

⇨ (A,.) is a semi group.

⇨ (A,+,.) is a ring.

Case 2:

Second binary operation on A as $*$ defined as

$$x * y = (a_0 + a_1 i + a_2 j + a_3 k) * (b_0 + b_1 i + b_2 j + b_3 k)$$

$= (a_0 b_0 + a_1 b_3 + a_2 b_2 + a_3 b_1) + (a_0 b_1 + a_1 b_0 + a_2 b_3 + a_3 b_2)\, i$

$+ (a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_3)\, j + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)\, k$ …….. (4)

⇨ $x * y = y * x, \forall\, x, y \in A$

⇨ $x * (y * z) = (x * y) * z, \forall\, x, y, z \in A$

⇨ $1 * x = x * 1 = x, \forall\, x \in A$

Let $x * x^{-1} = x^{-1} * x = 1, \forall\, x \in A$

Where $x^{-1} = (a_0 + a_1 i + a_2 j + a_3 k)^{-1}$

$$= (b_0 + b_1 i + b_2 j + b_3 k)$$

Here b_0 is calculated as b_1, b_2, b_3.

$(a_0 b_0 + a_1 b_3 + a_2 b_2 + a_3 b_1) = 1$ …………. (5)

$(a_0 b_1 + a_1 b_0 + a_2 b_3 + a_3 b_2) = 0$ …………. (6)

$(a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_3) = 0$ …………. (7)

$(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) = 0$ …………. (8)

Rewriting eq^n (6),(7) & (8) as

$(a_1 b_0 + a_0 b_1 + a_3 b_2 + a_2 b_3) = 0$

$(a_2 b_0 + a_1 b_1 + a_0 b_2 + a_3 b_3) = 0$

$(a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3) = 0$

⇨ $$\frac{b_0}{\begin{vmatrix} a_0 & a_3 & a_2 \\ a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \end{vmatrix}} = \frac{-b_1}{\begin{vmatrix} a_1 & a_3 & a_2 \\ a_2 & a_0 & a_3 \\ a_3 & a_1 & a_0 \end{vmatrix}} = \frac{b_2}{\begin{vmatrix} a_1 & a_0 & a_2 \\ a_2 & a_1 & a_3 \\ a_3 & a_2 & a_0 \end{vmatrix}} =$$

$$\frac{-b_3}{\begin{vmatrix} a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 \end{vmatrix}} = constant = k$$

$$b_0 = k \begin{vmatrix} a_0 & a_3 & a_2 \\ a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \end{vmatrix}$$

$$b_1 = -k \begin{vmatrix} a_1 & a_3 & a_2 \\ a_2 & a_0 & a_3 \\ a_3 & a_1 & a_0 \end{vmatrix}$$

$$b_2 = k \begin{vmatrix} a_1 & a_0 & a_2 \\ a_2 & a_1 & a_3 \\ a_3 & a_2 & a_0 \end{vmatrix}$$

$$b_3 = -k \begin{vmatrix} a_1 & a_0 & a_3 \\ a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 \end{vmatrix}$$

$$b_0 = k\{a_0^3 - 2a_0a_1a_3 + a_2(a_1^2 + a_3^2) - a_0a_2^2\} \quad \ldots\ldots\ldots\ldots (9)$$

$$b_1 = -k\{a_3^3 - 2a_0a_2a_3 + a_1(a_0^2 + a_2^2) - a_3a_1^2\} \quad \ldots\ldots\ldots\ldots (10)$$

$$b_2 = k\{a_2^3 - 2a_1a_2a_3 + a_0(a_1^2 + a_3^2) - a_2a_0^2\} \quad \ldots\ldots\ldots\ldots (11)$$

$$b_3 = -k\{a_1^3 - 2a_0a_1a_2 + a_3(a_0^2 + a_2^2) - a_1a_3^2\} \quad \ldots\ldots\ldots\ldots (12)$$

From equations (5), (9), (10), (11), (12) we get

$$k = 1 \Big/ \left\{ \begin{matrix} (a_0^4 - a_1^4 + a_2^4 - a_3^4) + 4a_0a_2(a_1^2 + a_3^2) \\ -4(a_1a_3)(a_0^2 + a_2^2) - 2a_0^2a_2^2 + 2a_1^2a_3^2 \end{matrix} \right\} \quad \ldots\ldots\ldots\ldots (13)$$

K = ∞ if $a_0 = a_1$ and $a_2 = a_3$
Hence inverse of each element in A does not exist.
$(A,*)$, is a Commutative monoid.
Hence $(A,+,*)$ Is a Commutative ring with unity.

CONCLUSION

From the above discussion, I come to the following conclusions
first I proved that (A, +,.) is a ring. Second I proved that $(A,+,*)$ Is a Commutative ring with unity. Where A= $\{a_0 + a_1 i + a_2 j + a_3 k \;/\; a_0, a_1, a_2, a_3 \in F\}$

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