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Research Article

GENERALIZED RING OF QUATERNION WITH UNITY

*__Manohar Durge__
ANC, Anandwan Warora
**Author for Correspondence*

ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri SriRamakrishana. In the Present work first I proved that $(A,+,*)$ is a Generalised Ring of Quaternion with unity
Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \;/\; a_i \in F \;\&\; G_i \in C(P)\}$,
And C(P) = Class of $algebraic\ Structure$

Keywords: *Binary Operation, Abelian Group, Ring, Field, Class of Algebraic Structure*

INTRODUCTION

Herstein cotes in 1992
Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that
1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$
Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:
1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.
It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.
If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION

Let $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \;/\; a_i \in F \;\&\; G_i \in C(P)\}$
Where C(P) = Class of $algebraic\ Structure$
Let $x = a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3$;

$$y = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3$$

$z = c_0G_0 + c_1G_1 + c_2G_2 + c_3G_3$;

$$G_0 = 1G_0 + 0G_1 + 0G_2 + 0G_3$$

$$0 = 0G_0 + 0G_1 + 0G_2 + 0G_3$$

$$-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2 + (-a_3)G_3$$

$cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2 + (ca_3)G_3$, c ∈ F.
Here first binary operation + on A defined as

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$$x + y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3) + (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3)$$
$= (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2 + (a_3 + b_3)G_3$
………… (1)

$$\Rightarrow x + y = y + x, \forall x, y, \in A$$
$$x + (y + z) = (x + y) + z, \forall x, y, z \in A$$
$$0 + x = x + 0, \forall x \in A$$

$x + (-x) = (-x) + x = 0, \forall x \in A$
⇨ (A,+) is an abelian group. ………… (2)
Second binary operation * on A defined as

$$x * y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3) * (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3)$$
$$= (a_0b_0 + a_1b_1 + a_2b_2 + a_3b_3)G_0 + (a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2)G_1$$
$$+(a_0b_2 + a_1b_3 + a_2b_0 + a_3b_1)G_2 + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)G_3$$

………… (3)
⇨
$x * y = y * x, \forall x, y, z \in A$

$$x * (y * z) = (x * y) * z, \forall x, y, z \in A$$

$x * G_0 = G_0 * x = x, \forall x, y, z \in A$

Let $x^{-1} = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3$ be the inverse of $a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3$
∴ By definition we get
$(a_0b_0 + a_1b_1 + a_2b_2 + a_3b_3) = 1$ ………. (4)
$(a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2) = 0$ ………. (5)
$(a_0b_2 + a_1b_3 + a_2b_0 + a_3b_1) = 0$ ………. (6)
$(a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0) = 0$ ………. (7)

Rewriting eq^n(5), (6) &(7) as
$a_1b_0 + a_0b_1 + a_3b_2 + a_2b_3 = 0$ ………. (8)
$a_2b_0 + a_3b_1 + a_0b_2 + a_1b_3 = 0$ ………. (9)
$a_3b_0 + a_2b_1 + a_1b_2 + a_0b_3 = 0$ ………. (10)

$\Rightarrow \frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = k\ (constant)$………. (11)

$$F_0 = \begin{vmatrix} a_0 & a_3 & a_2 \\ a_3 & a_0 & a_1 \\ a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_1 = \begin{vmatrix} a_1 & a_3 & a_2 \\ a_2 & a_0 & a_1 \\ a_3 & a_1 & a_0 \end{vmatrix}$$

$$F_2 = \begin{vmatrix} a_1 & a_0 & a_2 \\ a_2 & a_3 & a_1 \\ a_3 & a_2 & a_0 \end{vmatrix}$$

$$F_3 = \begin{vmatrix} a_1 & a_0 & a_3 \\ a_2 & a_3 & a_0 \\ a_3 & a_2 & a_1 \end{vmatrix}$$

$F_0 = \{a_0^3 - a_0(a_1^2 + a_2^2 + a_3^2) + 2a_1a_2a_3\}$
………… (12)

$$F_1 = \{-a_1^3 - 2a_0a_1a_3 + a_1(a_0^2 + a_2^2 + a_2^2)\}$$
………… (13)
$F_2 = \{a_2^3 + 2a_0a_1a_3 - a_2(a_0^2 + a_1^2 + a_3^2)\}$
………… (14)
$F_3 = \{-a_3^3 - 2a_0a_1a_2 + a_3(a_0^2 + a_1^2 + a_2^2)\}$
………… (15)

From eq^n (4), (11), (12), (13), (14) & (15) one obtains

$$k\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3\} = 1$$

$k = {}^1/_{\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3\}} = \infty$, If $a_0 = a_1 = a_2 = a_3$

Hence x^{-1} not exist for each $x \in A$

$(A, +,*)$Is a Generalised Ring of Quaternion with unity

Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \,/\, a_i \in F \,\&\, G_i \in C(P)\}$
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Conclusion

From the above discussion, I come to the following conclusions first I proved that $(A, +,*)$Is a Generalised Ring of Quaternion with unity
Where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \,/\, a_i \in F \,\&\, G_i \in C(P)\}$
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REFERENCES

Herstein IN (1992). *Topics in Algebra* 2nd edition (Wiley Eastern Limited) 26-256. ISBN: 0 85226 354 6.