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Research Article

FORMULA TO CALCULATE DISTANCE OF CENTROID FROM ONE OF ITS BASES OF A REGULAR POLYGON WHOSE SIDES ARE ODD NUMBERS

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ABSTRACT

In the study of mechanics, the Centre of gravity/Centroidis the point of an object at which the force of Gravitationcan be considered to be concentrated to act and which undergo no internal motion. The Centroid is an important parameter in Mechanics, Physics and studying Geometric properties especially while evaluating the Moment of inertia. The author has attempted to derive and establish the necessary mathematical formula in order to determine the exact value of distance of Centroid from its base of any Regular polygon, which is having odd number of sides. The formula has been proved with appropriate examples in the chapter 'Result and Discussion'. It is also very useful for Students, Research scholars and persons those who are engaged in Mechanics, Geometry & Physics.

Keywords: *Regular Polygon, Inscribed Circle, Circum Circle, Centroid/Centre of Gravity and Mechanics*

INTRODUCTION

Mechanics (Hibbeler, 2004) is the branch of applied science concerned with the behavior of physical, subjected to forces or displacement and the subsequent effects of the bodies on their environment, which deals with the motion of and forces on objects.In mechanics, the point that may be considered as the center of a one or two-dimensional figure, the sum of the displacements of all points in the figure from such a point being zero is called as *Centroid/ Centre of gravity* (Hibbeler, 2004).

In mathematics and physics, the centroid or geometric center of a two-dimensional region is, informally, the point at which it could be perfectly balanced on the tip of a nail (assuming it has uniform density and a uniform gravitational field). Formally, the centroid of a plane figure or two-dimensional shape is the arithmetic mean (Christopher & Nicholson, 2009) position of all the points in the shape. The definition extends to any object in n-dimensional space: its centroid is the mean position of all the points in all of the coordinate directions.

While in geometry the term *Barycenter*is a synonym for centroid, in physics Barycenter may also mean the physical center of mass or the center of gravity, depending on the context. The center of mass (and center of gravity in a uniform gravitational field) is the arithmetic mean of all points weighted by the local density (Christopher & Nicholson, 2009) or specific weight. If a physical object has uniform density, then its center of mass is the same as the centroid of its shape.

Formula Derivation

The figure 1 showing that *Equilateral triangle* (Weisstein, 2003). ABC be one of the typical regular polygon which is having three sides, 'R' be the *Radius of circum-circle* (Weisstein, 2003), 'r' be the *Radius of in-circle* (Weisstein, 2003) of the polygon, ∠AOC = ∠BOC be θ°, 'O' be the centre of polygon and y' be the distance of Centroidfrom its base.

We already know that the formula for radius of inscribed-circle

$$OC = r = \frac{a}{2}(\cot\theta°)$$

$$\therefore r = y' = \frac{a}{2}\left(\frac{\cos\theta°}{\sin\theta°}\right) ----------------------- [1]$$

Similarly, the formula for radius of circum-circle

$$EC = R = \frac{a}{2}(\mathrm{cosec}^{-1}\,\theta°)$$

$$\therefore R = \frac{a}{2}\left(\frac{1}{\sin\theta°}\right) ----------------------[2]$$

$$\text{Where}, \theta° = \frac{180^0}{n}$$

'n' = Number of sides of the polygon and it is only applicable for 'n' is odd.
Let, h be the Altitude (Weisstein, 2003) of the polygon. In this figure let, EC = h.
Therefore, *h = EC = EO + OC*
Let, OE = R, OC = r
Therefore, h = R + r

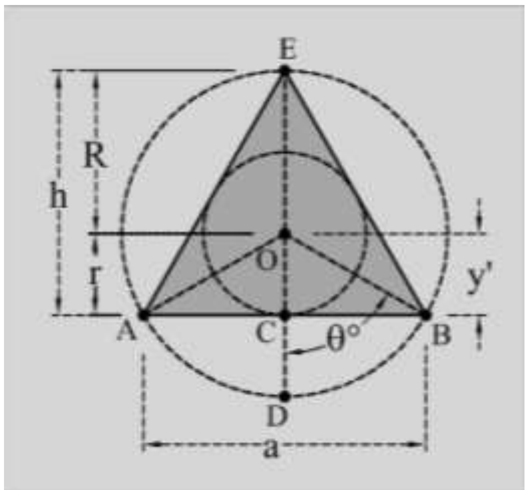

Figure 1: A regular polygon (Equilateral Triangle) with its Inscribed &Circum-Circle

Substituting eqn.1 and 2 in above eqn. we get

$$h = \left(\frac{a}{2\sin\theta^{°}}\right) + \left(\frac{a\cos\theta°}{2\sin\theta°}\right)$$

$$\text{Therefore}, h = \frac{a(1+\cos\theta°)}{2\sin\theta°} ------------------- [3]$$

From eqn. 1 and 3,

$$\frac{y'}{h} = \frac{r}{h} = \frac{a}{2}\left(\frac{\cos\theta°}{\sin\theta°}\right) \div \frac{a}{2}\left(\frac{1+\cos\theta°}{\sin\theta°}\right)$$

$$= \frac{a}{2}\left(\frac{\cos\theta°}{\sin\theta°}\right) \times \frac{2}{a}\left(\frac{\sin\theta°}{1+\cos\theta°}\right)$$

$$\text{Therefore}, \frac{y'}{h} = \frac{\cos\theta°}{1+\cos\theta°}$$

Therefore, $y' = \left(\dfrac{\cos\theta°}{1+\cos\theta°}\right)\text{h} \;-----------------\; [4]$

Where, $\theta° = \dfrac{180^0}{\text{n}}$ and 'n' = No. of sides and 'h' is the altitude of the regular polygon

Eqn.4 is the necessary formula to evaluate the distance of centre of gravity from its base and it is applicable for the polygon which having number of sides is odd.

RESULTS AND DISCUSSION

Let, $y' = \text{kh}$, where $\text{k} = \dfrac{\cos\theta°}{1+\cos\theta°}$, $\theta° = \dfrac{180^0}{\text{n}}$ and n is the number of sides of the polygon.

Now by substituting few values of 'n' in the above formula, the corresponding values of 'k' can be calculated.
The calculated values are tabulated below:

Value of 'n'	Name of polygon	θ° (deg.)	k- value
3	Equilateral Triangle	60°	$1/3$
5	Regular Pentagon	36°	≈ 0.4472
7	Regular Heptagon	$\pi/7\,°$	≈ 0.4740
9	Regular Nonagon	20°	≈ 0.4845
15	Regular Penta-decagon	12°	≈ 0.4945
…	…	…	…
…	…	…	…
…	…	…	…
∞	Circle	≈ 0°	$1/2$

Example Problem-1

Find the centre of gravity about its bottom edge of a regular polygon whose sides is 5.8779 units.

Let, $\text{A}_1 = \text{Area of } \Delta\, \text{AOB} = \dfrac{1}{2} \times 5.8779 \times 4.0541$

$\therefore\ \text{A}_1 = 11.8883$ sq. units. Similarly,

$\text{A}_2 = \text{A}_3 = \text{A}_4 = \text{A}_5 = 11.8883$ sq. units.

Total area of the pentagon = 5 × 11.8883 sq. units

∴ Total area of the polygon (A) = 59.4417 sq. units $--------------$[5]

From the drawing (fig.2), the centre of gravity of each triangles are

For Δ AOB, $y' = 0.9549$ units $-----------------------$ [6]

For Δ BOC and Δ AOE, $y_2{}' = y_5{}' = 3.0902$ units $----------------$ [7]

For Δ BOC & Δ AOE, $y_3{}' = y_4{}' = 6.5451$ units $-----------------$[8]

The equilibrium condition for moment of inertia is

$$A \times y' = (\text{A}_1 \times y_1{}') + (\text{A}_2 \times y_2{}') + (\text{A}_3 \times y_3{}') + (\text{A}_4 \times y_4{}') + (\text{A}_5 \times y_5{}')$$

$$\therefore y' = \left[\frac{(\text{A}_1 \times y_1{}') + (\text{A}_2 \times y_2{}') + (\text{A}_3 \times y_3{}') + (\text{A}_4 \times y_4{}') + (\text{A}_5 \times y_5{}')}{\text{A}}\right]$$

Substituting [5], [6], [7] & [8] in above equation, we get

$$y' = \frac{(11.8883 \times 0.9549) + 2(11.8883 \times 3.0902) + 2(11.8883 \times 6.5451)}{59.4417}$$

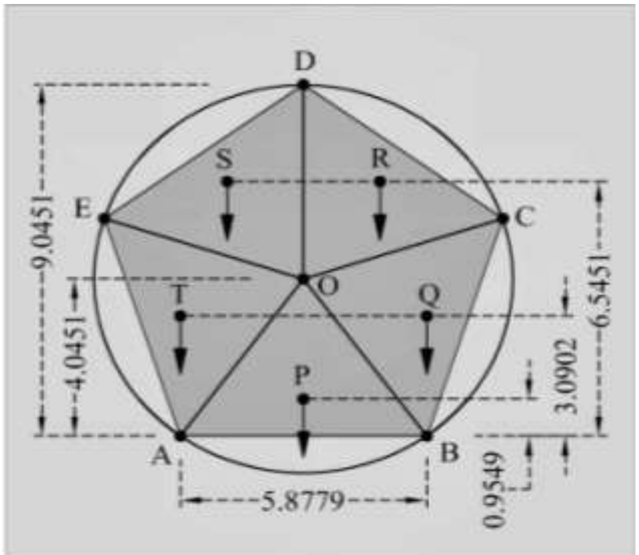

Figure 2: Example of a Regular pentagon with its dimensions

$$\therefore y' = \frac{(11.8883 \times 0.9549) + 2(11.8883 \times 3.0902) + 2(11.8883 \times 6.5451)}{59.4417}$$

$\therefore y' = 4.0451$ units $-----------------------$[9]

The problem figure is a regular pentagon. Therefore, $\theta° = 36°$ and altitude $(h) = 9.0451$
Substituting $\theta° = 36°$ and $h = 9.0451$ in eqn. [4]

$$y' = \left(\frac{\cos\theta°}{1+\cos\theta°}\right) h$$

$$y' = \left[\frac{\cos(36°)}{1+\cos(36°)}\right] \times 9.0451$$

$\therefore y' = 0.4472 \times 9.0451$ units
$\therefore y' = 4.04497$units $-----------------------$ [10]

From [9] & [10], the formula is proved as correct.

Conclusion

The centroid is an important parameter in mechanics and geometric properties especially while evaluating the Moment of Inertia. The author has derived the necessary mathematical formula in order to determine the exact value of centroid of any regular polygon, which is having sides of odd number. The formula has been proved with appropriate examples.

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