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Research Article

PROPERTIES OF STRONGLY GP-CONNECTED SPACES

***EL-Maghrabi A.I.[1&2] and AL-Subhi B.J.[2]**
[1]*Department of Mathematics, Kafr EL-Sheikh University, Kafr EL-Sheikh, EGYPT*
[2]*Department of Mathematics, Taibah University, AL-Madinah AL-Munawarh, K.S.A., P.O. Box, 30002*
**Author for Correspondence*

ABSTRACT
The aim of this paper is to introduce and study the concept of strongly generalized precompactness and strongly generalized pre-connectedness which is stronger than the notions of connectedness (Arhangelskii and Wiegandt, 1975; Reilly and Vamanamurthy, 1984) (resp. GO-connectedness (Balachandran *et al.*, 1991)) of topological spaces. Some characterizations of these concepts are discussed.
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INTRODUCTION

In 1970, Levine (1970) introduced the concepts of generalized closed sets of topological spaces as a generalization of closed sets. Mashhour *et al.*, (1982) studied the concept of preopen sets in topological spaces. The purpose of the present paper is to introduce and investigate the concepts of strongly generalized precompactness and strongly generalized preconnectedness which is stronger than the concepts of connectedness (Arhangelskii and Wiegandt, 1975; Reilly and Vamanamurthy, 1984) (resp. GO-connectedness (Balachandran *et al.*, 1991)) of topological spaces. Throughout this paper (X, τ) and (Y, σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of (X, τ).The closure of A, the interior of A and the complement of A is denoted by cl(A), int(A) and A^c or X-A respectively. A subset A of X is said to be preopen (Mashhour *et al.*, 1982) if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$. The complement of a preopen set is called preclosed (Mashhour *et al.*, 1982). The intersection of all preclosed (Abd, 1980) sets containing a subset A of (X, τ) is called the preclosure of A and is denoted by p-cl(A). The pre-interior of A is the largest preopen set contained in A and is denoted by p-int(A).

Definition 1.1. A subset A of a space (X, τ) is called:

(i) A generalized closed (briefly, g -closed) (Levine, 1970) set if $\mathrm{cl}(A) \subseteq u$ whenever $A \subseteq u$ and u is open,

(ii) A g*-closed (Kumar, 2000) set if $\mathrm{cl}(A) \subseteq u$ whenever $A \subseteq u$ and u is g-open,

(iii) A generalized preclosed (briefly, gp-closed) (Balachandran and Arokiarain) set if $p-\mathrm{cl}(A) \subseteq u$ whenever $A \subseteq u$ and u is open ,

(iv) A generalized preregular closed (briefly, gpr-closed) (Gnanambal, 1997) set if $p-\mathrm{cl}(A) \subseteq u$ whenever $A \subseteq u$ and u is regular-open .

(v) A strongly generalized preclosed (briefly, strongly gp-closed) (El-Maghrabi *et al.*, 2012) set if p-cl (A) $\subseteq$ U whenever A $\subseteq$ U and u is g-open in (X, τ).

The complement of g-closed (resp.g*-closed, gp-closed, gpr-closed, strongly gp-closed) set is called g-open (resp. g*-open, gp-open, gpr-open, strongly gp-open). The family of all preopen (resp. preclosed, strongly gp-closed, strongly gp-open) subsets of (X, τ) will be as always denoted by PO(X, τ) (resp. PC(X, τ), St.GPC(X, τ), St.GPO(X, τ)).

***Definition 1.2* (El-Maghrabi *et al.*, 2012).** A mapping f: (X, τ) $\rightarrow$ (Y, σ) is called:

(1) Strongly generalized precontinuous (briefly, st.gp-cont.) if the inverse image of every closed set of (Y, σ) is strongly gp-closed in (X, τ),

(2) A strongly generalized pre-irresolute (briefly, strongly gp-irresolute) if $f^{-1}(v)$ is strongly gp-closed in (X, τ), for every strongly gp-closed set v of (Y, σ).

Definition 1.3 A mapping f: (X, τ) $\rightarrow$ (Y, σ) is called :

(1) M-preopen (Mashhour *et al.*, 1984), if, $f(u)$ is a preopen set of (Y, σ), for each U is preopen set in (X, τ),

(2) Pre-strongly gp-closed (El-Maghrabi *et al.*, 2012) if the image of each strongly gp-closed set of (X, τ) is strongly gp-closed in (Y, σ),

(3) Pre-strongly gp-open (El-Maghrabi *et al.*, 2012) if the image of each strongly gp-open) set in (X, τ) is strongly gp-open in (Y, σ),

(4) Super-strongly gp-open (El-Maghrabi *et al.*, 2012) if the image of each strongly gp-open) set of (X, τ) is open in (Y, σ).

Definition 1.4 A space (X, τ) is called strongly compact (Mashhour *et al.*, 1984), if every cover of S by preopen sets of (X, τ) has a finite subcover.

***Lemma 1.1* (El-Maghrabi *et al.*, 2012).** In a topological space (X, τ), then:

(i) Every preopen (resp. g*-open) set is strongly gp-open.

(ii) Every strongly gp-open set is gp-open.

Strongly Generalized Precompact Spaces

This section is devoted to introduce and study the notion of a strongly generalized precompact space. Also, some characterizations of it are discussed.

Definition 2.1. A collection $\{A_i : i \in I\}$ of strongly gp-open sets in a topological space (X, τ) is called a strongly generalized preopen cover of a subset B of (X, τ) ,if $B \subseteq \cup\{A_i : i \in I\}$.

Definition 2.2. A topological space (X, τ) is strongly generalized precompact (briefly, strongly gp-compact) if every strongly generalized preopen cover of X has a finite subcover.

Definition 2.3. A subset S of a space (X, τ) is strongly gp-compact relative to X, if every cover of S by strongly gp-open sets of X has a finite subcover.

Definition 2.4. A subset B of a topological space (X, τ) is said to be strongly gp-compact if B is strongly gp-compact as a subset of X.

Theorem 2.1. For a space (X, τ), the following statements are equivalent:

(i) (X, τ) is strongly gp-compact,

(ii) Any family of strongly gp-closed subsets of (X, τ) satisfying the finite intersection property.

(ii) Any family of strongly gp-closed subsets of (X, τ) with empty intersection has a finite subfamily with empty intersection.

Proof (i) $\Rightarrow$ (ii). Let $\{F_i : i \in I\}$ be a family of strongly gp-closed subsets of (X, τ) which satisfy the finite intersection property. To prove that $\bigcap_{i\in I} F_i \neq \phi$ .Suppose that the converse $\bigcap_{i\in I} F_i = \phi$. Then $X = \bigcup_{i\in I} F_i^c$ and therefore $\{F^c{}_i : i \in I\}$ is a strongly gp-open cover of (X, τ). Thus by hypothesis, there exists a finite subset I_0 of I such that $X = \bigcup_{i\in I_0} F_i^c$ and hence $\bigcap_{i\in I_0} F_i \neq \phi$ which is a contradiction with the assumption. Therefore, $\bigcap_{i\in I} F_i \neq \phi$.

(ii)$\Rightarrow$(i). Suppose that $\{G_i : i \in I\}$is a strongly gp-open cover of (X, τ) and (X, τ) is not strongly gp-compact. Then there exists a strongly gp-open cover of (X, τ) has no finite subcover. Hence for a finite subset I_0 of I , $\bigcup\limits_{i\in I} G_i \neq X$.Therefore, $\bigcap\limits_{i\in I} G^c{}_i \neq \phi$ and thus the family $\{G^c{}_i : i \in I\}$satisfies the finite intersection property which is a contradiction with the assumption and hence (X, τ) is strongly gp-compact.

(i)$\Rightarrow$(iii). Let $\{F_i : i \in I\}$be a collection of strongly gp-closed subsets of (X, τ) with $\bigcap\limits_{i\in I} F_i = \phi$ and hence, $\bigcup\limits_{i\in I} F_i^{\,c} = X$. Thus, the family$\{F^c{}_i : i \in I\}$is a strongly gp-open cover of (X, τ). Therefore by hypothesis, there exists a finite subset $\{F_{i_k} : k = 1,..., m\}$of (X, τ) such that $X = \bigcup\limits_{k=1}^{m} F^c{}_{i_k}$ and hence $\bigcap\limits_{k=1}^{m} F_{ik} = \phi$.Hence, there exists a finite subfamily with empty intersection.

(iii)$\Rightarrow$(i). Suppose that $\{G_i : i \in I\}$is a collection of strongly gp-open cover of (X, τ) such that $\bigcup\limits_{i\in I} G_i = X$. Hence, $\bigcap\limits_{i\in I} G^c{}_i = \phi$. Thus, there existsa collection of strongly gp-closed subsets $\{G^c{}_i : i \in I\}$ of (X, τ) such that $\bigcap\limits_{i\in I} G^c{}_i = \phi$.Hence by hypothesis, a finite subcollection $\{G_{i_k} : k = 1,...,m\}$of(X, τ) such that $\bigcap\limits_{k=1}^{m} G^c{}_{i_k} = \phi$.So, $X = \bigcup\limits_{k=1}^{m} G_{i_k}$ and therefore (X, τ) is strongly gp-compact.

Theorem 2.2. A strongly gp-closed subset of a strongly gp-compact space is strongly gp-compact.

Proof. Assume that A is a strongly gp-closed subset of (X, τ) which is strongly
gp-compact space. Then, X-A is a strongly gp-open cover of A by strongly gp-open subset of (X, τ).Then $X = (\bigcup\limits_{i\in I} G_i) \cup (X - A)$.Hence, $\{X - A, G_i : i \in I\}$is a strongly gp-open cover of (X, τ) which is strongly gp-compact and therefore there exists a finite subset I_0 of I such that $X = (\bigcup\limits_{i\in I_o} G_i) \cup (X - A)$ but A and its complement are disjoint. Hence, $A \subseteq \bigcup\limits_{i\in I_0} G_i$ which proves that A is strongly gp-compact.

Next, we introduce many further properties on strongly gp-compact spaces.

Theorem 2.3. A strongly gp-continuous image of a strongly gp-compact space is compact.

Proof. Let f: (X, τ)→(Y, σ) be a strongly gp-continuous mapping from a strongly gp-compact space (X, τ) onto a topological space (Y, σ) and suppose that $\{A_i : i \in I\}$ is open cover of (Y, σ). Then, $\{f^{-1}(A_i) : i \in I\}$is a strongly gp-open cover of (X, τ).But,(X, τ) is strongly gp-compact ,then there exists a finite subcover$\{f^{-1}(A_1),...,f^{-1}(A_n)\}$,but f is onto, hence f(X)=Y=$\{A_1,...,A_n\}$. Hence (Y, σ) is compact.

Theorem 2.4. The image of a strongly gp-compact subset under a strongly gp-irresolute mapping is strongly gp-compact.

Proof. Let f: (X, τ)→(Y, σ) be a strongly gp-irresolute mapping and$\{u_i : i \in I\}$be any family strongly gp-open cover of a subset f(G) of Y. Since, f is strongly gp-irresolute, then $\{f^{-1}(u_i) : i \in I\}$ is strongly gp-open cover of a subset G of X. But, G is strongly gp-compact, then there exists a finite subset I_0 of I such that $G \subseteq \cup_{i=1}\{f^{-1}(u_i) : i \in I_o\}$ which implies that $G \subseteq f^{-1}(\bigcup_{i\in I} u_i)$ and therefore $f(G) \subseteq \cup_{i=1}\{u_i : i \in I_0\}$. This shows that f(G) is strongly gp-compact.

Corollary 2.1. If a mapping f: (X, τ)→(Y, σ) is surjection strongly gp-irresolute and (X, τ) is a strongly gp-compact space, then(Y, σ) is strongly gp-compact.

Proof. Suppose that $\{G_i : i \in I\}$ is a collection of strongly gp-open cover of (Y, σ).Then, $\{f^{-1}(G_i) : i \in I\}$ is a strongly gp-open cover of (X, τ) . But, (X, τ) is strongly gp-compact, hence there exists a finite subset I_0 of I such that X= $\cup_{i=1}\{f^{-1}(G_i) : i \in I_0\}$ and therefore Y= $\cup_{i=1}\{G_i : i \in I_o\}$. So, (Y, σ) is strongly gp-compact.

Theorem 2.5. Let f: (X, τ)→(Y, σ) be a bijective pre-strongly gp-open mapping and (Y, σ) be a strongly gp-compact space . Then (X, τ) is strongly gp-compact.

Proof. Assume that $\{u_i : i \in I\}$ is a strongly gp-open cover of (X, τ). and f is a pre-strongly gp-open mapping, then $\{f(u_i) : i \in I\}$ is a strongly gp-open cover of (Y, σ). But, (Y, σ) is strongly gp-compact, hence there exists a finite subset I_0 of I such that Y= $\cup_{i=1}\{f(u_i) : i \in I_0\}$, so by a bijective of, f, X= $\cup\{u_i : i \in I_0\}$ and therefore (X, τ) is strongly gp-compact.

Theorem 2.6. If f: (X, τ)→(Y, σ) is a bijective M-preopen mapping and (Y, σ) is a strongly gp-compact space, then (X, τ) is strongly compact.

Proof. Let $\{G_i : i \in I\}$ be a preopen cover of (X, τ). Then, $\{f(G_i), i \in I\}$ is a preopen cover of (Y, σ), hence by Lemma 1.1, $\{f(G_i) : i \in I\}$ is a strongly gp-open cover of (Y, σ). Since, (Y, σ) is a strongly gp-compact space, then there exists a finite subset I_0 of I such that Y= $\cup_{i=1}\{f(G_i) : i \in I_0\}$ and thus X= $\cup_{i=1}\{G_i : i \in I_0\}$. So, (X, τ) is strongly compact.

Theorem 2.7. Let A , B be two subsets of a space (X, τ), A be strongly gp-compact relative to (X, τ) and B be a strongly gp-closed subset of (X, τ).Then $A \cap B$ is strongly gp-compact relative to (X, τ).

Proof. Let $\{V_i : i \in I\}$ be a strongly gp-open cover of $A \cap B$ and X-B be a strongly gp-open subset of (X, τ). Then $(X - B) \cup_{i=1}\{V_i : i \in I\}$ is strongly gp-open cover of A which is strongly gp-compact relative to (X, τ), hence there exists a finite subset I_0 of I such that $A \subseteq \cup_{i=1}\{V_i : i \in I_0\} \cup (X - B)$. Therefore $A \cap B \subseteq \cup_{i=1}\{V_i : i \in I_0\}$ and hence $A \cap B$ is strongly gp-compact relative to (X, τ).

Theorem 2.8. If $\{A_j : j \in J\}$ is a finite family of strongly gp-compact subsets relative to a space (X, τ), then $\cup_{i=1}\{A_j : j \in J\}$ is a strongly gp-compact relative to (X, τ).

Proof. Suppose that $\{V_i : i \in I\}$ is a strongly gp-open cover of $\cup_{j=1}\{A_j : j \in J\}$.Then, $\{V_i : i \in I\}$ is a strongly gp-open cover of A_j, for each $j \in J$. Hence, there exists a finite subset I_0 of I such that $A_j \subseteq \cup_{i=1}\{V_i : i \in I_0\}$, for each $j \in J$. Therefore, $\cup_{j=1} A_j \subseteq \cup_{i=1}\{V_i : i \in I_0\}$. Thus, $\cup_{j=1}\{A_j : j \in J\}$ is a strongly gp-compact relative to (X, τ).

Strongly Generalized Preconnected Spaces

The concepts of strongly generalized pre-separated and strongly generalized preconnected spaces are presented in this section. Also, many properties of these notions are investigated.

Definition 3.1. Two non-empty subsets A and B in a space (X, τ) are called strongly generalized pre-separated (briefly, strongly gp-separated) if and only if $A \cap st.gp-cl(B) = \phi$ and $st.gp-cl(A) \cap B = \phi$.

Remark 3.1. For a topological space (X, τ), the following statements are hold:

(i) Each separation is strongly gp-separation by the fact that $st.gp-cl(A) \subseteq cl(A)$, for each $A \subseteq X$.

(ii) Any two strongly gp-separated sets are always disjoint but the converse in general is not true as shown by the following example.

Example 3.1. If X= {a, b, c, d} with a topology, τ ={X, ϕ ,{c},{a, b},{a, b, c}}, then two subsets {a, c} and {b, d} of τ are disjoint but not strongly gp-separated.

Theorem 3.1. Let A and B be non-empty subsets of a space (X, τ). Then the following statements are hold:

(i) If A and B are strongly gp-separated sets, A_1 and B_1 are non-empty sets such that $A_1 \subseteq A$ and $B_1 \subseteq B$, then A_1 and B_1 are also strongly gp-separated.

(ii) If $A \cap B = \phi$ and either both A and B are strongly gp-open or strongly gp-closed, then A and B are strongly gp-separated.

(iii) If both A and B are either strongly gp-open or strongly gp-closed and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G are strongly gp-separated.

Proof. (i) Immediately.

(ii) Let A, B $\in St.GPO(X,\tau)$.Then X-A and X-B are strongly gp-closed sets. But, A, B are disjoint, then $A \subseteq X - B$ which implies that $st.gp-cl(A) \subseteq st.gp-cl(X-B) = X - B$ and so, $st.gp-cl(A) \cap B = \phi$.

Similarly, $st.gp-cl(B) \cap A = \phi$.Hence, A and B are strongly gp-separated. For the case of strongly gp-closed sets A and B given the result directly.

(iii) Assume that A and B are strongly gp-open sets. Then X-A and X-B are strongly gp-closed. Since, $H \subseteq X - B$, hence $st.gp-cl(H) \subseteq st.gp-cl(X-B)$ =X-B and so, $st.gp-cl(H) \cap B = \phi$. Therefore, $st.gp-cl(H) \cap G = \phi$.

Similarly. We can prove that $st.gp-cl(G) \cap H = \phi$.Since, A and B are strongly gp-closed sets, then $A = st.gp-cl(A)$ and $B = st.gp-cl(B)$. But, $H \subseteq X - B$, then $H \cap st.gp-cl(B) = \phi$ and hence $st.gp-cl(G) \cap H = \phi$.

Similarly. We can show that $st.gp-cl(H) \cap G = \phi$. Therefore, in both cases H and G are strongly gp-separated.

Theorem 3.2. If U and V are strongly gp-open sets in (X, τ) such that $A \subseteq U, B \subseteq V$ and $A \cap V = \phi, B \cap U = \phi$, then A and B are non empty strongly gp-separated sets in (X, τ).

Proof. Suppose that U and V are strongly gp-open sets such that $A \subseteq U, B \subseteq V$ and $A \cap V = \phi, B \cap U = \phi$.Then, X-V and X-U are strongly gp-closed sets and hence $st.gp-cl(A) \subseteq X - V \subseteq X - B$ and $st.gp-cl(B) \subseteq$ X-U $\subseteq$ X-A. Therefore, $st.gp-cl(A) \cap B = \phi$ and $st.gp-cl(B) \cap A = \phi$.Thus, A and B are non empty strongly gp-separated sets.

Definition 3.2. A space (X, τ) is called a strongly generalized preconnected (briefly, strongly gp-connected) space if X cannot be written as a disjoint union of two non-empty strongly gp-open sets.

Definition 3.3. A subset S of a space (X, τ) is said to be strongly generalized preconnected (briefly, strongly gp-connected) relative to (X, τ) if there is no subsets A and B are strongly gp-separated relative to (X, τ) and $S = A \cup B$. In other words, a subset S cannot be expressed as the union of two non-empty strongly gp-separated sets A, B.

Remark 3.2. For a space (X, τ), every strongly gp-connected set is connected (resp.GO-connected) , but the converse may not be true as shown by the following examples.

Example 3.2. If $X=\{a,b,c,d\}$ with a topology $\tau=\{X, \phi, \{a\},\{b\},\{a,b\},\{a,b,c\}\}$, then the subset $A=\{a, c\}$ is connected but not strongly gp-connected.

Example 3.3. If $X= \{a,b,c,d\}$ with a topology $\tau=\{X, \phi, \{b,d\},\{a,b,d\},\{b,c,d\}\}$, then the subset $B=\{a,b,d\}$ is GO-connected but not strongly gp-connected.

Remark 3.3. Every strongly gp-connected space is connected (GO-connected).

Example 3.4. Let $X=\{a, b, c\}$ with a topology $\tau=\{X, \phi, \{a, b\}\}$. Then, (X, τ) is connected but it is not strongly gp-connected. Since, $\{b, c\}$ is both strongly gp-open and strongly gp-closed.

Example 3.5. Let $X= \{a,b,c,d\}$ with topology $\tau=\{X, \phi, \{b\}, \{c\}, \{b, c\},\{b,d\},\{b,c,d\}\}$. Then (X, τ) is GO-connected but it is not strongly gp-connected. Since, $\{d\}$ is both strongly gp-open and strongly gp-closed.

Theorem 3.3. In a space (X, τ) if E is strongly gp-connected, then st.gp-cl(E) is also.

Proof. Assume that st.gp-cl(E) is strongly gp-disconnected, then there exist two non-empty strongly gp-separated sets G and H in (X, τ) such that st.gp-cl(E)=$G \cup H$. Now, $E = (G \cap E) \cup (H \cap E)$ and $st.gp-cl(G \cap E) \subseteq st.gp-cl(G)$,
$st.gp-cl(H \cap E) \subseteq st.gp-cl(H)$ and $G \cap H = \phi$,this implies that $st.gp-cl(G \cap E) \cap H= \phi$.Thus, $st.gp-cl(G \cap E) \cap (H \cap E) = \phi$.

Similarly, we can prove that $st.gp-cl(H \cap E) \cap (G \cap E) = \phi$ and hence E is strongly gp-disconnected.

Theorem 3.4. For a space (X, τ), the following are equivalent:

(i) (X, τ) is strongly gp-connected,

(ii) The only subsets of (X, τ) which are both strongly gp-open and strongly gp-closed are the empty set ϕ and X.

(iii) Each strongly gp-continuous mapping of (X, τ) into a discrete space(Y, σ) with at least two points is a constant mapping.

Proof. (i)$\Rightarrow$(ii). Let G be strongly gp-open and strongly gp-closed of (X, τ).Then, X-G is both strongly gp-open and strongly gp-closed. Since, X is the disjoint union two strongly gp-open sets G and X-G one of these must be empty, that is, $G = \phi$ or $G = X$.

(ii)$\Rightarrow$(i). Suppose that $X = C \cup D$, where C and D are disjoint non-empty strongly gp-open and strongly gp-closed subsets of (X, τ), then C is both strongly gp-open and strongly gp-closed, by hypothesis, $C = \phi$ or X.

(ii)$\Rightarrow$(iii). Assume that f: $X \to Y$ be a strongly gp-continous map. Then, X is covered by strongly gp-open and strongly gp-closed covering $\{f^{-1}(y), y \in Y\}$, by assumption, $f^{-1}(y) = \phi$ or X, for each $y \in Y$.If $f^{-1}(y) = \phi$ for all $y \in Y$, then fails to be map. Then there exists only point $y \in Y$ such that $f^{-1}(y) \neq \phi$ and hence $f^{-1}(y) = X$.Thus, shows that f is a constant map.

(iii)$\Rightarrow$(ii). Let G be both strongly gp-open and strongly gp-closed in (X, τ).Suppose $G \neq \phi$ and f: X$\rightarrow$ Y be a strongly gp-continuous map defined by $f(G)=\{y\}$ and $f(X-G)=\{w\}$ for some distinct points y and w in Y, by assumption, f is constant. Therefore, G=X.

Theorem 3.5. For a surjective mapping f: (X, τ)→(Y, σ), the following statements are hold:

(i) If f is strongly gp-continuous and X is strongly gp-connected , then Y is connected.
(ii) If f is strongly gp-irresolute and X is strongly gp-connected ,then Y is strongly gp-connected.

Proof. (i) Suppose that (Y, σ) is not connected and $Y=A\cup B$, where A and B are disjoint non-empty open sets in (Y, σ).Since, f is surjection strongly gp-continuous, then $X=f^{-1}(A)\cup f^{-1}(B)$,where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty strongly gp-open sets in (X, τ) which is a contradiction with the fact that (X, τ)is strongly gp-connected. Hence, (Y, σ) is connected.

(ii) Let (Y, σ) be not strongly gp-connected. Then, there exists two non-empty disjoint strongly gp-open sets A and B, where $Y=A\cup B$. Since, f is strongly gp-irresolute, then $X=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty strongly gp-open sets in (X, τ) which is contradictions with fact that (X, τ) is strongly gp-conneced. Hence, (Y, σ)is strongly gp-connected.

Theorem 3.6. For a bijective mapping f: (X, τ) →(Y, σ), the following statements are hold:

(i) If f is pre-strongly gp-closed and K is strongly gp-connected relative to (Y, σ), then $f^{-1}(K)$ is strongly gp-connected relative to (X, τ).

(ii) If f is super strongly gp-open and K is connected relative to (Y, σ), then $f^{-1}(K)$ is strongly gp-connected relative to (X, τ).

Proof: (i) Let $f^{-1}(K)$ be not strongly gp-connected relative to (X, τ).

Then there exists disjoint non-empty strongly gp-open sets A and B where $f^{-1}(K)=A\cup B$ Since, f is bijective pre-strongly gp-open, then $K=f(A)\cup f(B)$, where $f(A)$ and $f(B)$ are non-empty disjoint strongly gp-closed sets in (Y, σ) which is a contradiction with the fact that K is strongly gp-connected. Hence, $f^{-1}(K)$ is strongly gp-connected relative to (X, τ).

(ii) Suppose that $f^{-1}(K)$ is not strongly gp-connected relative to (X, τ). Then, there exist disjoint non-empty strongly gp-open sets A and B where $f^{-1}(K)=A\cup B$. Since, f is bijective super strongly gp-open map and $K=f(A)\cup f(B)$ which is a contradiction with the fact that K is connected relative to(Y, σ). Therefore, $f^{-1}(K)$ is strongly gp-connected relative to(X, τ).

REFERENCES

Abd El-Monsef ME (1980). Studies on some pretopological concepts, Ph. D. Thesis Tanta University.
Arhangelskii AV and Wiegandt R (1975). Connectedness and disconnectedness in topology, *Topology and its Applications* (5) 9-33.
Balachandran K and Arokiarain (No Date). On generalized preclosed sets, Preprint.
Balachandra K, Sundaram P and Maki H (1991). On generalized continuous maps in topological spaces, Memoirs of the Faculty of Science. Kochi University (12) 5-13.
El-Maghrabi AI, AL-Subhi B and Jafari S (2012). On strongly generalized precontinuous mappings. *Fasciculi Mathematici* (48) 23-34.

Research Article

Gnanambal Y (1997). On generalized preregular closed sets in topological spaces. *Indian Journal of Pure and Applied Mathematics* **28**(3) 351-360.

Kumar M (2000). Between closed sets and g-closed sets. Memoirs of the Faculty of Science, Kochi University (Math.) (21)1-9.

Levine N (1970). Generalized closed sets in topology. *Rendiconti del Circolo Matematico di Palermo* **19**(2) 89-46.

Mashhour AS, Abd El-Monsef M and Eand EL-Deeb SN (1982). On precontinuous and weak Precontinuous mappings, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences,* Egypt (53) 47-53.

Mashhour AS, Abd El-Monsef ME and Hasanien IA (1984). On pretopological spaces, *Bulletin of the Mathématique de la Société des Sciences Mathématiques de la République Socialiste de Roumanie* **28** (76) 39-45.

Mashhour AS, Abd El-Monsef ME, Hasanien IA and Noiri T (1984). Strongly compact spaces, *Delta Journal of Science* **8**(1) 30-46.

Reilly IL and Vamanamurthy MK (1984). Connectedness and strong semi-continuity, Časopisproěstovani Matematiky, Roć. (109) 261-265.