International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (1) January-March, pp. 26-33/Gor and Patel
Research Article

EXTENSION OF A NEWSBOY PROBLEM WITH LOST SALES RECAPTURE AS FUNCTION OF (R/P) AND NORMALLY DISTRIBUTED DEMAND ERROR

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ABSTRACT
We consider a lost sale recapture model in a newsvendor framework. As in real practice, we have considered that there may be an opportunity to backlog the lost sales, by offering some incentive for waiting. Further, not all the customers that could not buy in the first instant may avail the rebate offer and buy. The retailer's decision includes selling price, order quantity and the rebate that will maximize its expected profit. The back log fill rate is modelled as a function of the proportion of rebate relative to the price. Sensitivities of the demand errors in the form of normal distribution rather than the uniform distribution serve as an extension to the previous work by the authors.

Keywords: *Newsvendor Problem, Lost Sales, Rebates, Price Dependent Demand*

INTRODUCTION
This paper considers the buying and ordering policies of a newsvendor-type retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation. The backordering occurs through an emergency purchase of the items in question at some premium over the regular purchasing cost. In turn, the retailer offers to the end-customers left out of the initial sale a rebate incentive upon purchase of each item backordered.
The problem of backordering shortage items has been considered recently by Weng (2004) and Zhou and Wang (2009). Both generalize the newsvendor problem (heretofore NVP) into a two-step decision process. In the first stage, the retailer places the initial order that equates the costs of over- and under-estimation of the demand, as corresponds to the traditional NVP. In the second, the retailer may place a special order from the manufacturer at the end of the selling season. The basic difference between the two models lies in whether the manufacturer (Weng, 2004) or both parties (Zhou and Wang, 2009) pay for the setup costs of the special order.
Our model differs from these two in five fundamental ways. First, we consider a price-dependent demand, with the selling price, p, a decision variable, more in accordance with the main tenets of microeconomic theory (e.g. Arcelus and Srinivasan, 1987). Second, we introduce a rebate-dependent fill rate, Ω, representing the probability of the end-customers returning to satisfy the unfilled demand. This fill rate is a function of the size of the rebate, r, offered relative to the selling price. Third, the policy decisions on the emergency order and on the rebate policy occur up front, along with the remaining ordering and pricing policies, rather than at the end of the season, thereby rendering the resulting formulation into a more traditional one-stage, rather than a two-stage, NVP. Fourth, the decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand. Our model yields a unique profit-maximizing solution, for a family of deterministic mean demand functions and of probability distributions of the demand error that encompasses the vast majority of the models in the existing literature.
The organization of the paper is as follows. The next section presents the formulation of the model, based upon that of Zhou and Wang (2009), to which we add the offering of a price rebate per backordered unit purchased. This paper is similar in lines of Arcelus *et al.,* (2012), and Patel and Gor (2013). Here, we use an entirely different fill rate function than Arcelus *et al.,* (2012) and include sensitivities to the normal

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distribution over and above the one for uniform distribution discussed in Patel and Gor (2013). We describe the characteristics of the model, develop the objective function and derive the profit-maximizing optimality conditions that are shown to be unique. Section 3 presents a numerical example. In addition to illustrating the main features of the model and discussing some comparative statics of interest, this section attempts to conjecture the behavioural relationship between various parameters and variables. A conclusions section completes the paper. Table 1 lists the notations used throughout the paper.

Table 1: Notation

p	The selling price per unit (*decision variable*)
v	The salvage value per unsold unit
q	The order quantity (*decision variable*)
r	The rebate per backordered item (*decision variable*)
c	The acquisition cost per unit
s	The shortage penalty per unsold unit
D	The total demand rate per unit of time
g, ε	The deterministic and stochastic components, respectively, of D
a,b	The upper and lower values, respectively, of ε
μ, σ	The mean and standard deviation, respectively, of ε
f, F	The density function and the cumulative distribution function, respectively, of ε
δ_0,δ_1	The intercept and slope, respectively, of the deterministic linear demand function
γ_0, γ_1	The intercept and the demand elasticity, respectively, of the iso-elastic deterministic demand function
Ω	The fill rate of backlogged demand
d	The premium on the purchase price of each backlogged unit acquired
z	The stocking factor
Λ, Φ	The expected number of leftovers and shortages, respectively
e	The price elasticity of demand
I_ε	The generalized failure rate function
$\pi(p,q,r)$	The retailer's profit function
$E(p,q,r)$	The retailer's expected profit function
U.D	The uniform Distribution
N.D	The Normal Distribution

Model Formulation

In this section, we describe the key characteristics of the model, formulate the retailer's profit-maximizing objective function and derive the optimality conditions. Observe that, in the development of the models, the arguments of the functions are omitted whenever possible, to simplify notation.

Characteristics of the model

***Characteristic 1**: **Key properties of the demand function**.*

The random single-period total demand, D (p,ε), is of the form:

$$D(p,\varepsilon) = g(p) + \varepsilon, \quad if \;\; additive\, error \qquad g(p)\varepsilon, \quad if \;\; multiplicative\, error \tag{1}$$

g(p) has an IPE or increasing price elasticity, e, which satisfies the following condition:

$$e'_p = \frac{\partial e}{\partial p} \geq 0, \quad where \quad e = \frac{\partial g}{\partial p}\frac{p}{g}$$

ε has a GSIFR or generalized strictly increasing failure rate, I_ε, since

$$I'_\varepsilon = \partial I_\varepsilon / \partial\varepsilon \geq 0, \quad where \quad I_\varepsilon = \varepsilon f/(1-F)$$

Observe in (1) that the total demand includes a deterministic component of *g* units, denoted as the mean demand; and a stochastic element, denoted by ε units. Following the customary conventions of the literature on the subject, the relationship between *g* and ε is assumed to be either additive (Mills, 1958) or multiplicative (Karlin and Carr, 1962), with the former (latter) exhibiting a constant (variable) error variance and a variable (constant) coefficient of variation. Chan *et al.*, (2004), Lau *et al.*, (2007), Petruzzi and Dada (1999), Yao (2002) and Yao *et al.*, (2006) discuss the implications of these assumptions and provide a review of the extant works on the field.
Furthermore, unless otherwise stated, there is no need to identify a functional form of the mean demand, *g(p)*. The results presented here are applicable to all the demand distributions normally used in the sales-promotion field, i.e. linear, iso-elastic, log-concave or concave in *p* and the like (Yao, 2002; Yao, *et al.*, 2006). The only requirements for the deterministic demand function are that *g* be downward sloping and at least twice differentiable, with respect to *p*.
Similarly, there is no need either to identify a probabilistic distribution for the stochastic demand component, ε. All that is needed is that it be defined over a finite range *[a,b]* and have a mean of μ, a standard deviation of σ, a density function of *f(.)* and a cumulative density function of *F(.)*. If needed, rescaling the error to produce a different mean value is straightforward. Furthermore, as Yao (2002) and Yao *et al.*, (2006) indicate, the probability distributions belonging to the *GSIFR* class include the most widely used in the literature such as uniform, normal, beta, gamma and the like.
Characteristics 1.2 and 1.3 (e.g. Aydin and Porteus, 2009; Yao, 2002; Yao *et al.*, 2006) represent considerable generalizations from the current practice in the revenue management literature of using specific demand and probabilistic distribution functions. The model is general enough to be applicable to all distributions, satisfying Characteristic 1. Yao, et al. (2006) lists the studies where the mean demand distribution is IPE and the random error distribution, GSFIR, regardless of whether the resulting total demand is modelled in the multiplicative or additive way. Further, as Theorem 1 of Yao, et al. (2006) demonstrates, the use of this family of distributions ensures the uniqueness of the resulting optimal policies. Detailed proofs of these results appear in Yao (2002).

***Characteristic 2**: **A fill rate, Ω, given by the following expression**:*

$$\Omega = \left(\frac{r}{p}\right)^m, \quad where \quad 0 < r < p, \quad 0 < \Omega < 1, \; 0 < m < \infty \qquad (2)$$

The fill rate, Ω, measures the fraction of end-customers who wish to fulfill their demand from the emergency order. Its functional form in (2) is rooted on the empirical literature on the subject and satisfies several properties of interest. First, it is a function of the value of the rebate relative to the selling price, *r/p*. Second, the value of Ω falls between 0 and 1, but does not approach either value as $0<r<p$. Also, as $m\to 0$, $\Omega \to 1$ and as $m\to\infty$, $\Omega \to 0$. Only in the absence of the rebate i.e. $r=0$, $\Omega = 0$. This reflects empirical findings implying that, if there is no rebate, buying of lost sales will not take place, unless the product enjoys a monopoly. Arcelus *et al.*, (2012) have developed a lost sale recapture model validating Bawa and Shoemaker (1989) that there is still some "exposure effect" to the original sale that leads some end-customers to purchase, even in the absence of a coupon, i.e. even when $r=0, \Omega > 0$. On the other hand, in this model, as $r/p \to 1$, $\Omega \to 1$indicating the possibility of every lost sale converting if the product is offered at a rebate equal to the selling price i.e almost absolutely free.

Characteristic 3**: **The stocking factor, z

$$\begin{aligned} z &= q - g, \quad if\ additive \\ &= q/g, \quad if\ multiplicative \end{aligned}$$

$$\Phi = \int_z^B (\varepsilon - z) f(\varepsilon) d\varepsilon \qquad (3)$$

$$\Lambda = \int_A^z (z - \varepsilon) f(\varepsilon) d\varepsilon = \Phi + z - \mu$$

In (3), Φ and Λ represent the expected number of shortages and leftovers, respectively, as a result of demand fluctuations. The shortage level is expected to decrease with the rebate incentive. With respect to the stocking variable, *z*, it was introduced by Petruzzi and Dada (1999) and subsequently used by Arcelus *et al.*, (2005), among many others, as a replacement for another decision variable, namely the order quantity. It represents the expected level of leftover and shortages, generated by the demand uncertainty and by the retailer's optimal policies. Its inclusion simplifies the interpretation of the findings of the current study and the derivations of the optimality conditions.

The retailer's profit-maximizing objective

The retailer profit function is decomposable into two parts, depending upon whether the retailer order quantity exceeds or understates the demand for the product. If the first, then *q* exceeds *D* and the retailer sells *D* units at *p* per unit, disposes of the rest at a salvage value of *v* per unit and incurs an acquisition cost of *c* for each of the *q* units ordered. If the second, *q* is below *D*, in which case the retailer buys and sells the *q* units at a profit margin of (*p-c*) per unit, acquires a fraction Ω of the shortage demand at a premium *d* per unit, sells it at (*p-r*), the regular selling price, *p*, net of the per unit rebate offered, *r*, and pays a shortage penalty of *s* per unit on the rest of the merchandise. Formally, the functional form of the retailer's profit function, $\pi(p,q,r)$, is as follows:

$$\begin{aligned}\pi(p,q,r) &= pD - cq + v(q-D), \quad \text{if } q \geq D \\ &= (p-c)q + \left[(p-r)-(c+d)\right]\Omega(D-q) - s(1-\Omega)(D-q), \quad \text{if } q \leq D\end{aligned} \tag{4}$$

The objective is to find the levels of *p*, *q* and *r* that maximizes *E(p,q,r)*, the retailer's expected profit. Using (3) and (4), it can be readily seen that *E* may be written as follows:

$$\begin{aligned}E(p,q,r) &= (p-c)(g+\mu)-(c-v)\Lambda - \left[(p-c+s)(1-\Omega)+\Omega(r+d)\right]\Phi, \ \text{if additive} \\ &= (p-c)g\mu - g(c-v)\Lambda - g\left[(p-c+s)(1-\Omega)+\Omega(r+d)\right]\Phi, \quad \text{if multiplicative}\end{aligned} \tag{5}$$

First-order optimality conditions:

To simplify the explanation, only the additive-error/linear-demand case will be discussed. The multiplicative case can be developed along the same lines. Let $E_i^{'} = \partial E / \partial i, i = p,r,Q$ be the first derivative of the expected profit with respect to each of the decision variables. Setting these derivatives to zero, we obtain the following first-order optimality conditions.

$$\begin{aligned}E_p^{'} &= 0 = (g+\mu) + g_p^{'}(p-c) - (1-\Omega)\Phi + (p-c+s-r-d)\Phi\Omega_p^{'} \\ E_r^{'} &= 0 = \Phi\Omega_r^{'}(p-c+s-r-d) - \Phi\Omega \\ E_z^{'} &= 0 = -(c-v) - \Phi_z^{'}\left[(p-v+s) - \Omega(p-c+s-r-d)\right]\end{aligned} \tag{6}$$

Where $\Omega_p^{'}$ and $\Omega_r^{'}$ are defined in (3). The optimality conditions in (6) have straightforward economic interpretations. All represent tradeoffs between profit gains and losses associated with unit changes in *p, r* and *q*, respectively. With respect to the first, a one-dollar increase in price generates (i) a profit increase of $\$(g+\mu)$ from the units sold: (ii) minus a loss of $\$ g_p^{'}(p-c)$, from the decrease in demand caused by the price increase; (iii) minus an opportunity cost of the shortages not sold even with the emergency order; and (iv) opportunity cost on the decrease of the fill rate due to the price increase. As for the second, a one-dollar increase in the in the shortage rebate, *r*, results in (i) an increase in profits from the associated rise in the fill rate, $\Omega_r^{'} > 0$, from (3); and in (ii) an increase in the rebate costs from the back-logged end-customers purchasing from the emergency order. The third condition indicates that a one-dollar increase in the stocking factor results from the marginal profit changes in the expected leftovers, together with the opposite weighted marginal profits in the expected shortages, with the weights representing the percentage of returning and not returning customers.

Numerical Analysis

This section presents a numerical illustration of key properties of the model just described, to highlight the main features of the various solutions proposed in the paper. Given the central objective of the paper, our numerical analysis centers on the impact of fluctuations in power *m* of the fill rate function, upon the fill rate, Ω, and through it, upon the retailer's profit-maximizing pricing, ordering, rebate policies. All computations were carried out with MAPLE's Optimization toolbox.

Base-case numerical structure

The starting point consists of two sets of examples that serve as the base-case for the analysis of this section. The first (second) set, denoted by *AL (MI)*, assumes the deterministic demand, *g*, to be linear (iso-elastic) and its error, additive (multiplicative), i.e.

$$D(p) = \delta_0 - \delta_1 p + \varepsilon, \quad \delta_0 > 0, \quad \delta_1 > 0, \quad \textit{for AL total demand}$$
$$\gamma_0 p^{-\gamma_1}\varepsilon, \quad \gamma_0 > 0, \quad 0 > \gamma_1 > 1, \quad \textit{for MI total demand} \qquad (7)$$

For comparability purposes, this section operates with the parameter values of Patel and Gor (2013), to which suitable values for the remaining parameters have been added. These values appear in Table 2 (N. D.). In this way, any sensitivity analysis can be carried out by adroit manipulation of the appropriate parameter values for any of the components of the base-case.

Further for maximum comparability among probability distributions, all cases are related to a random variable uniformly distributed and normal distributed over the interval (-3,500, 1,500), for the AL demand model and (0.7, 1.1), for its MI counterpart. Either support interval describes the normal distribution completely.

Base-case numerical results

Having described the nature of the numerical structure that gives rise to the parameter values of the *AL* and *MI* components of the base case, we now discuss the numerical results. Unless otherwise stated, we concentrate our remarks on the *AL* demand case. As mentioned latter on in this section, the results for the *MI* case can be interpreted in similar fashion.

Table 2: Numerical Analysis: Base Case Optimal Policies (Normal distribution)

DISTRIBUTION		Support, mean and Standard deviation		
NORMAL DISTRIBUTION		support [A,B]		
Additive Error and Linear Demand		[-3500, 1500] , Mean = -1000, SD = 1440		
Multiplicative Error and Iso-elastic demand		[0.7, 1.1], Mean = 0.9 , SD = 0.07		
Additive Error Linear Demand				
Parameter values: γ_0=100000 ; γ_1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	$\boldsymbol{\Lambda}$	$\boldsymbol{\Phi}$
346866	50.36	23295	245	399
Multiplicative Error Iso-Elastic Demand				
Parameter values: γ_0= 500000000; γ_1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	$\boldsymbol{\Lambda}$	$\boldsymbol{\Phi}$
377413	59.90	16290	538	452

Numerical Example and Interpretations

The optimal results using MAPLE for the fill rate model with varied powers on r/p are shown in Table 3, (N.D.). The reader can refer to Patel and Gor (2013) for comparability purposes with the uniform distribution case. Both the cases Additive Error Linear Demand and Multiplicative Error Iso-elastic Demand are showcased to highlight the variations in the optimal solutions too. The following observations and interpretations are made:

(a) The optimal policy for the fill rate model with *m*=0.5, as shown in row 1 of Table 3(N. D.) in Additive Error Linear Demand case, consists of the retailer acquiring q^*=23,223 units at a unit cost of *c*=$35 and selling them at a unit price of p^*=$50.39. With respect to the fill rate, approximately Ω^*= 23% of the shortages are recaptured at an extra purchasing cost of *d*=$3.00 to the retailer, who allows a rebate of r^*=$2.79 per unit backlogged. Afterwards, all unsold units, i.e. $[(1-\Omega^*)(D-q^*)]$, will be assigned a unit shortage penalty of *s*=$3.

On the other hand, when demand falls below the *q**=23,223 units ordered and all purchased at the cost of *c*=$35 per unit, D units are sold at the regular unit price of *p**=$50.39 and the remaining, at the salvage value of *v*=$10.00 per unit.

The resulting optimal policy is π*[p*, q*, r*]=$347405 [50.39, 23223, 2.79].

As shown in Table 2, these results contrast with the optimal solution for the *AL* certainty case of π*[p*; q*] =$346866 [$50.36; 23,295]

(b) Similar interpretation follows for the other models in the Additive Error Linear Demand case, where the power on r/p increases Table 3(N. D.). The increase in the power of the fill rate function tends to increase the optimal order quantity and the rebate, whereas decreases selling price as well as profits.

(c) Table 3(N. D.) also gives results for the *MI* case. Observe though that unlike its Additive Error Linear Demand counterpart, in this case, increase in the power of the fill rate function, tends to increase the order quantity and the rebate and also the selling price. Profits decrease with the increase in the power of the fill rate function.

Table 3: Optimal Policies for lost sale recapture model with fill rate $\Omega=(r/p)^m$ (N.D)

Additive Error Linear Demand							
M	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
0.5	347405	50.39	23223	2.79	0.23	230	420
1	347006	50.37	23273	4.18	0.08	241	404
2	346866	50.36	23295	0.00	0.00	245	399
3	346866	50.36	23295	0.00	0.00	245	399
Multiplicative Error Iso-Elastic Demand							
M	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
0.5	380499	59.77	16178	8.25	0.37	442	556
1	378631	59.88	16225	12.44	.020	499	490
2	377704	59.91	16263	16.60	0.07	528	460
3	377498	59.91	16279	18.68	0.03	535	454

Sensitivity Analysis

Table 4 (N.D.) describes the sensitivities of the optimal policies to the change in the salvage and shortage costs in the Additive Error and Linear Demand case. Corresponding results for the Iso-elastic demand and multiplicative error case can be easily computed. The primary objective to carry out the sensitivity analysis is to observe the directional change in the short ages and the leftover values. Observe that, through Table 6 and 7, we have tried to construct examples where the relationship between shortages and leftovers is $\Lambda^* > \Phi^*$ as well as $\Lambda^* < \Phi^*$.

Table 4: Sensitivities to the salvage and shortage costs in Additive Error Linear Demand Case for =1 (N.D.)

Linear Demand Additive Error Case for *m*=1								
v	**s**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
16	**3**	348908	50.40	23349	7.70	0.152	292	341
17	**3**	349207	50.40	23370	7.70	0.152	305	327
18	**3**	349520	50.40	23393	7.70	0.152	320	313
19	**3**	349848	50.41	23417	7.70	0.152	335	298
20	**3**	350192	50.41	23242	7.70	0.152	352	283
21	**3**	350554	50.42	23470	7.71	0.152	371	267
Linear Demand Additive Error Case for *m*=1								
10	**10**	345207	50.40	23346	11.20	0.22	295	338
10	**11**	344949	50.40	23358	11.70	0.23	303	330
10	**12**	344700	50.41	23370	12.20	0.24	311	322
10	**13**	344460	50.41	23381	12.70	0.25	318	314

10	**14**	344229	50.41	23392	13.20	0.26	326	307
10	**15**	344006	50.41	23403	13.70	0.27	333	300

Next, as shown in Table 5, we perform sensitivity analysis to the change in the support values [A,B] for the Normal distribution for the fill rate model with power *m*=1. Similar sensitivities can be performed for various other values of m, as well as support structures.

Table 5: Sensitivities to the Normal Distribution Support Changes: CASE *m*=1

SUPPORT	**Mean**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
Linear Demand and Additive Error								
-3500,1500	-1000	347346	50.37	23245	7.68	0.15	232	418
1500,3500	2500	411085	51.61	25006	8.30	0.16	101	166
1500,5500	3500	421745	51.90	25518	8.45	0.16	214	344
-5500,1500	-2000	325830	49.98	22731	7.49	0.14	339	632
-1500,3500	1000	378773	51.04	24257	8.02	0.15	239	408
Iso-elastic Demand and Multiplicative Error								
.7,1.1	0.9	378631	59.88	16225	12.44	0.20	499	490
.8,1.2	1.0	423471	59.71	18146	12.35	0.20	500	495
.6,1.0	0.8	333803	60.08	14305	12.54	0.20	497	483
.6,1.2	0.9	367858	60.61	15769	12.80	0.21	709	670
.8,1.4	1.1	457448	60.17	19605	12.58	0.20	714	690

Table 6, shows the percentage change in the optimal policies when for capturing the demand errors, the normal distribution is used instead of the uniform distribution (Patel and Gor, 2013).

Table 6: % change in the Base Case Optimal Policieswhen N. D. used instead of U. D.

Dist	Profit	p	q	Λ	Φ
Additive Error Linear Demand					
U. D.	339096	50.22	23276	444	836
N. D.	346866	50.36	23295	245	399
% CHANGE	2.29↑	0.28↑	0.08↑	44.81↓	44.81↓
Multiplicative Error Iso-Elastic Demand					
Dist	Profit	p	q	Λ	Φ
U. D.	356420	61.42	15496	988	713
N. D.	377413	59.90	16290	538	452
% CHANGE	5.88↑	2.47↓	5.12↑	45.54↓	36.6↓

Some Concluding Comments

The primary contribution of this paper has been to consider the impact upon the ordering and pricing policies of a newsvendor-type, profit-maximizing retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation, by offering some rebate incentives for waiting. The backlog fill rate, representing the probability of the end-customers returning to satisfy their unfilled demand, is modelled as a function of the size of the rebate offered relative to the selling price. The decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand. Sensitivities of the demand errors in the form of normal distribution rather than the uniform distribution serve as an extension to the previous work by the authors.

REFERENCES

Alford BL and Biswas A (2002). The effects of discount level, price consciousness and sale proneness on consumers' price perception and behavioral intentions. *Journal of Business Research* **55** 775-783.

Arcelus FJ and Srinivasan G (1987). Inventory policies under various optimizing criteria and variable markup rates. *Management Science* **33** 756-762.

Arcelus FJ, Kumar S and Srinivasan G (2005). Retailer's response to alternate manufacturer's incentives under a single period, price-dependent, stochastic-demand framework. *Decision Sciences* **36** 599-626.

Arcelus FJ, Ravi Gor and Srinivasan G (2012). Price, rebate and order quantity decisions in a newsvendor framework with rebate dependent recapture of lost sales. *International Journal of Production Economics* **140(**1) 473-482.

Aydin G and Porteus EL (2009). Manufacturer-to-Retailer versus Manufacturer-to-Consumer Rebates in a Supply Chain. In N. with the left-skewed (right-skewed) the next lowest (highest) and the symmetrical, Agrawal and S. Smith (Eds.), *Retail Supply Chain Management* (New York: Kluwer).

Bawa K and Shoemaker RS (1989). Analyzing Incremental Sales from a Direct Mail Coupon Promotion. *Journal of Marketing* **53** 66-78.

Chan LMA, Shen ZJM, Simchi-Levi D and Swan JL (2004). Coordination of pricing and inventory decisions: A survey and classification. In: D. Simchi-Levi, S.D. Wu and Z.J.M. Shen, (Eds.), *Handbook of Quantitative Analysis: Modeling in the E-Business Era* (New York: Kluwer) 335-392.

Karlin S and Carr CR (1962). Prices and optimal inventory policy. In *Studies in Applied Probability and Management Science edited by* KJ Arrow and H Scarf (Palo Alto, CA: Stanford University Press) 159-172.

Lau AHL, Lau HS and Wang JC (2007). Some properties of buyback and other related schemes in a newsvendor product supply-chain with price-sensitive demand. *Journal of the Operational Research Society* **58** 491-504.

Mills ES (1958). Uncertainty and price theory. *Quarterly Journal of Economics* **73** 116-130.

Patel A and Gor Ravi (2013). A single period model where the lost sales recapture is a function of (r/p). *Applied Mathematical Sciences* **7(**103) 5103-5114.

Petruzzi NC and Dada M (1999). Pricing and the newsboy problem: A review with extensions. *Operations Research* **47** 183-194.

Weng ZK (2004). Coordinating order quantities between the manufacturer and the buyer: A generalized newsvendor model. *European Journal of Operational Research* **156** 148-161.

Yao L, Chen Y and Yan H (2006). The newsvendor problem with pricing: Extensions. *International Journal of Management Science and Engineering Management* **1** 3-16.

Yao, L. (2002). Supply chain modeling: Pricing, contracts and coordination. Ph.D. Thesis, Chinese University of Hong Kong.

Zhou YW and Wang SD (2009). Manufacturer-buyer coordination for newsvendor-type products with two ordering opportunities and partial backorders. *European Journal of Operational Research* **198** 958-974.