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Research Article

A SINGLE PERIOD MODEL WHERE THE LOST SALES RECAPTURE IS A FUNCTION OF $\log_m\left(1+\frac{r}{p}\right)$

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ABSTRACT
We consider a lost sale recapture model in a newsvendor framework. In this paper we analyse how to recapture lost customers in which easier to win back old customers than it is to acquire new customers. We consider a single-period decision of a retailer facing uncertain and price dependent demand. The typical modeling of the problem in a newsvendor framework assumes the unfulfilled demand to be lost once and for all. However, in reality, there may be an opportunity to backlog the lost sales, by offering some incentive for waiting. Nevertheless, the retailer's procurement price may be higher, due to the likely cost increase of the emergency purchase. Further, not all the customers that could not buy in the first instance may avail the rebate offer and buy. The backlog fill rate is modeled as a function of the proportion of the rebate to the price. Then the retailer has to decide ahead of the realization of the demand the quantity to be ordered, the price and the rebate to be offered for backlogged sales that will maximize its expected profit. Numerical examples are presented to highlight model sensitivities to parametric changes. The back log fill rate is modelled as a log function of adding one to the proportion of rebate relative to the price. Sensitivities of optimal rebate to demand errors are carried out with uniform distribution.

Keywords: *Newsvendor Problem, Lost Sales, Rebates, Price Dependent Demand*

INTRODUCTION

This paper considers the buying and ordering policies of a newsvendor-type retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation. The backordering occurs through an emergency purchase of the items in question at some premium over the regular purchasing cost. In turn, the retailer offers to the end-customers left out of the initial sale a rebate incentive upon purchase of each item backordered.

The problem of backordering shortage items has been considered recently by Weng (2004) and Zhou and Wang (2009). Both generalize the newsvendor problem (heretofore NVP) into a two-step decision process. In the first stage, the retailer places the initial order that equates the costs of over- and under-estimation of the demand, as corresponds to the traditional NVP. In the second, the retailer may place a special order from the manufacturer at the end of the selling season. The basic difference between the two models lies in whether the manufacturer (Weng, 2004) or both parties (Zhou and Wang, 2009) pay for the setup costs of the special order.

Our model differs from these two in five fundamental ways. First, we consider a price-dependent demand, with the selling price, p, a decision variable, more in accordance with the main tenets of microeconomic theory (e.g. Arcelus and Srinivasan, 1987). Second, we introduce a rebate-dependent fill rate, Ω, representing the probability of the end-customers returning to satisfy the unfilled demand. This fill rate is a function of the size of the rebate, r, offered relative to the selling price. Third, the policy decisions on the emergency order and on the rebate policy occur up front, along with the remaining ordering and pricing policies, rather than at the end of the season, thereby rendering the resulting formulation into a more traditional one-stage, rather than a two-stage, NVP. Fourth, the decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand. Our model yields a unique profit-maximizing solution, for a family of deterministic mean demand functions and of probability distributions of the demand error that encompasses the vast majority of the models in the existing literature.

The organization of the paper is as follows. The next section presents the formulation of the model, based upon that of Zhou and Wang (2009), to which we add the offering of a price rebate per backordered unit purchased. This paper is similar in lines of Arcelus *et al.*, (2012), and Patel and Gor (2013). Here, we use an entirely different fill rate function than discussed in Patel and Gor (2013). We describe the characteristics of the model, develop the objective function and derive the profit-maximizing optimality conditions that are shown to be unique. Section 3 presents a numerical example. In addition to illustrating the main features of the model and discussing some comparative statics of interest, this section attempts to conjecture the behavioural relationship between various parameters and variables. A conclusions section completes the paper. Table 1 lists the notations used throughout the paper.

Table 1: Notation

p	The selling price per unit (*decision variable*)
v	The salvage value per unsold unit
q	The order quantity (*decision variable*)
r	The rebate per backordered item (*decision variable*)
c	The acquisition cost per unit
s	The shortage penalty per unsold unit
D	The total demand rate per unit of time
g, ε	The deterministic and stochastic components, respectively, of D
a,b	The upper and lower values, respectively, of ε
μ, σ	The mean and standard deviation, respectively, of ε
f, F	The density function and the cumulative distribution function, respectively, of ε
δ_0,δ_1	The intercept and slope, respectively, of the deterministic linear demand function
γ_0, γ_1	The intercept and the demand elasticity, respectively, of the iso-elastic deterministic demand function
Ω	The fill rate of backlogged demand
d	The premium on the purchase price of each backlogged unit acquired
z	The stocking factor
Λ, Φ	The expected number of leftovers and shortages, respectively
e	The price elasticity of demand
I_ε	The generalized failure rate function
$\pi(p,q,r)$	The retailer's profit function
$E(p,q,r)$	The retailer's expected profit function

Model Formulation

In this section, we describe the key characteristics of the model, formulate the retailer's profit-maximizing objective function and derive the optimality conditions. Observe that, in the development of the models, the arguments of the functions are omitted whenever possible, to simplify notation.

Characteristics of the model

***Characteristic 1**: **Key properties of the demand function**.*

The random single-period total demand, D (p,ε), is of the form:

$$D(p,\varepsilon) = \begin{cases} g(p)+\varepsilon, & \text{if additive error} \\ g(p)\varepsilon, & \text{if multiplicative error} \end{cases} \qquad (1)$$

g(p) has an IPE or increasing price elasticity, e, which satisfies the following condition:

$$e'_p = \frac{\partial e}{\partial p} \geq 0, \quad where \quad e = \frac{\partial g}{\partial p}\frac{p}{g}$$

ε has a GSIFR or generalized strictly increasing failure rate, I_ε, since

$$I'_{\varepsilon} = \partial I_{\varepsilon} / \partial \varepsilon \geq 0, \quad where \quad I_{\varepsilon} = \varepsilon f /(1-F)$$

Observe in (1) that the total demand includes a deterministic component of *g* units, denoted as the mean demand; and a stochastic element, denoted by ε units. Following the customary conventions of the literature on the subject, the relationship between *g* and ε is assumed to be either additive (Mills, 1958) or multiplicative (Karlin and Carr, 1962), with the former (latter) exhibiting a constant (variable) error variance and a variable (constant) coefficient of variation. Chan, *et al.*, (2004), Lau, *et al.*, (2007), Petruzzi and Dada (1999), Yao (2002) and Yao, *et al.*, (2006) discuss the implications of these assumptions and provide a review of the extant works on the field.

Furthermore, unless otherwise stated, there is no need to identify a functional form of the mean demand, *g(p)*. The results presented here are applicable to all the demand distributions normally used in the sales-promotion field, i.e. linear, iso-elastic, log-concave or concave in *p* and the like (Yao, 2002; Yao, *et al.*, 2006). Detailed discussions can be found in, Arcelus *et al.*, (2012) and Patel and Gor (2013).

Characteristic 2: A fill rate, Ω, given by the following expression:

$$\Omega = \log_m\left(1+\frac{r}{p}\right), \quad where \quad 0<r<p, \quad 0<\Omega<1, \; 2<m<\infty, \; m>\left(1+\frac{r}{p}\right) \qquad (2)$$

The fill rate, Ω, measures the fraction of end-customers who wish to fulfill their demand from the emergency order. Its functional form in (2) is rooted on the empirical literature on the subject and satisfies several properties of interest. First, it is a function of the value of the rebate relative to the selling price, $\log_m\left(1+\frac{r}{p}\right)$. Second, the value of Ω falls between 0 and 1, but does not approach either value as *0<r<p*. Also, as m→ $\left(1+\frac{r}{p}\right)$, Ω →*1* and as *m*→∞, Ω →*0.* Only in the absence of the rebate i.e. *r*=0, Ω =*0*. This reflects empirical findings implying that, if there is no rebate, buying of lost sales will not take place, unless the product enjoys a monopoly. Arcelus, Gor and Srinivasan (2012) have developed a lost sale recapture model validating Bawa and Shoemaker (1989) that there is still some "exposure effect" to the original sale that leads some end-customers to purchase, even in the absence of a coupon, i.e. even when *r*=0, Ω =*0*. On the other hand, in this model, as $\left(1+\frac{r}{p}\right)$→m, Ω →1 indicating the possibility of every lost sale converting if the product is offered at a rebate equal to the selling price i.e almost absolutely free.

Characteristic 3: The stocking factor, z

$$z = q-g, \quad if \; additive$$
$$= q/g, \quad if \; multiplicative$$

$$\Phi = \int_z^B (\varepsilon - z) f(\varepsilon) d\varepsilon \qquad (3)$$

$$\Lambda = \int_A^z (z-\varepsilon) f(\varepsilon) d\varepsilon = \Phi + z - \mu$$

In (3), *Φ* and *Λ* represent the expected number of shortages and leftovers, respectively, as a result of demand fluctuations. The shortage level is expected to decrease with the rebate incentive. With respect to the stocking variable, *z*, it was introduced by Petruzzi and Dada (1999) and subsequently used by Arcelus, *et al.*, (2005), among many others, as a replacement for another decision variable, namely the order quantity. It represents the expected level of leftover and shortages, generated by the demand uncertainty and by the retailer's optimal policies. Its inclusion simplifies the interpretation of the findings of the current study and the derivations of the optimality conditions.

The retailer's profit-maximizing objective

The retailer profit function is decomposable into two parts, depending upon whether the retailer order quantity exceeds or understates the demand for the product. If the first, then *q* exceeds *D* and the retailer sells *D* units at *p* per unit, disposes of the rest at a salvage value of *v* per unit and incurs an acquisition cost of *c* for each of the *q* units ordered. If the second, *q* is below *D*, in which case the retailer buys and sells the *q* units at a profit margin of (*p-c*) per unit, acquires a fraction *Ω* of the shortage demand at a premium *d* per unit, sells it at (*p-r*), the regular selling price, *p*, net of the per unit rebate offered, *r*, and pays a shortage penalty of *s* per unit on the rest of the merchandise. Formally, the functional form of the retailer's profit function, *π(p,q,r)*, is as follows:

$$\begin{aligned}\pi(p,q,r) &= pD - cq + v(q-D), \quad \text{if } q \ge D \\ &= (p-c)q + \left[(p-r)-(c+d)\right]\Omega(D-q) - s(1-\Omega)(D-q), \quad \text{if } q \le D\end{aligned} \tag{4}$$

The objective is to find the levels of *p*, *q* and *r* that maximizes *E(p,q,r)*, the retailer's expected profit. Using (3) and (4), it can be readily seen that *E* may be written as follows:

$$\begin{aligned}E(p,q,r) &= (p-c)(g+\mu) - (c-v)\Lambda - \left[(p-c+s)(1-\Omega) + \Omega(r+d)\right]\Phi, \ \text{if additive} \\ &= (p-c)g\mu - g(c-v)\Lambda - g\left[(p-c+s)(1-\Omega) + \Omega(r+d)\right]\Phi, \quad \text{if multiplicative}\end{aligned} \tag{5}$$

First-order optimality conditions:

To simplify the explanation, only the additive-error/linear-demand case will be discussed. The multiplicative case can be developed along the same lines. Let $E_i^{'} = \partial E / \partial i, i = p,r,Q$ be the first derivative of the expected profit with respect to each of the decision variables. Setting these derivatives to zero, we obtain the following first-order optimality conditions.

$$\begin{aligned}E_p^{'} &= 0 = (g+\mu) + g_p^{'}(p-c) - (1-\Omega)\Phi + (p-c+s-r-d)\Phi\Omega_p^{'} \\ E_r^{'} &= 0 = \Phi\Omega_r^{'}(p-c+s-r-d) - \Phi\Omega \\ E_z^{'} &= 0 = -(c-v) - \Phi_z^{'}\left[(p-v+s) - \Omega(p-c+s-r-d)\right]\end{aligned} \tag{6}$$

Where $\Omega_p^{'}$ and $\Omega_r^{'}$ are defined in (3). The optimality conditions in (6) have straightforward economic interpretations. All represent tradeoffs between profit gains and losses associated with unit changes in *p, r* and *q,* respectively. With respect to the first, a one-dollar increase in price generates (i) a profit increase of \$$(g+\mu)$ from the units sold: (ii) minus a loss of \$ $g_p^{'}(p-c)$, from the decrease in demand caused by the price increase; (iii) minus an opportunity cost of the shortages not sold even with the emergency order; and (iv) opportunity cost on the decrease of the fill rate due to the price increase. As for the second, a one-dollar increase in the in the shortage rebate, *r*, results in (i) an increase in profits from the associated rise in the fill rate, $\Omega_r^{'} > 0$, from (3); and in (ii) an increase in the rebate costs from the back-logged end-customers purchasing from the emergency order. The third condition indicates that a one-dollar increase in the stocking factor results from the marginal profit changes in the expected leftovers, together with the opposite weighted marginal profits in the expected shortages, with the weights representing the percentage of returning and not returning customers.

Numerical Analysis

This section presents a numerical illustration of key properties of the model just described, to highlight the main features of the various solutions proposed in the paper. Given the central objective of the paper, our numerical analysis centers on the impact of fluctuations in base *m* of the fill rate function, upon the fill rate, *Ω*, and through it, upon the retailer's profit-maximizing pricing, ordering, rebate policies. All computations were carried out with MAPLE's Optimization toolbox.

Base-case numerical structure

The starting point consists of two sets of examples that serve as the base-case for the analysis of this section. The first (second) set, denoted by *AL (MI)*, assumes the deterministic demand, *g*, to be linear (iso-elastic) and its error, additive (multiplicative), i.e.

$$D(p) = \delta_0 - \delta_1 p + \varepsilon, \quad \delta_0 > 0, \quad \delta_1 > 0, \quad \text{for AL total demand}$$
$$\gamma_0 p^{-\gamma_1}\varepsilon, \quad \gamma_0 > 0, \quad 0 > \gamma_1 > 1, \quad \text{for MI total demand} \tag{7}$$

For comparability purposes, this section operates with the parameter values of Patel and Gor (2013) to which suitable values for the remaining parameters have been added. These values appear in Tables 2. In this way, any sensitivity analysis can be carried out by adroit manipulation of the appropriate parameter values for any of the components of the base-case.

Further for maximum comparability among probability distributions, all cases are related to a random variable uniformly distributed over the interval (-3,500, 1,500), for the AL demand model and (0.7, 1.1), for its MI counterpart. Either support interval describes the uniform distribution completely.

Base-case numerical results

Having described the nature of the numerical structure that gives rise to the parameter values of the *AL* and *MI* components of the base case, we now discuss the numerical results. Unless otherwise stated, we concentrate our remarks on the *AL* demand case. As mentioned latter on in this section, the results for the *MI* case can be interpreted in similar fashion.

Table 2.Numerical Analysis: Base Case Optimal Policies

Distribution	Support, mean and Standard deviation			
Uniform Distribution	support [A,B]			
Additive Error and Linear Demand **A > -a**	[-3500, 1500] , Mean = -1000, SD = 1440			
Multiplicative Error and Iso-elastic demand **A>0**	[0.7, 1.1], Mean = 0.9 , SD = 0.06			
Additive Error Linear Demand				
Parameter values: γ_0 =100000 ; γ_1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	***Λ***	***Φ***
333909	50.22	23276	444	836
Multiplicative Error Iso-Elastic Demand				
Parameter values: γ_0 = 500000000; γ_1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	***Λ***	***Φ***
356419	61.41	15496	988	713

Numerical Example and Interpretations

The optimal results using MAPLE for the fill rate model with varied bases on $\log_m\left(1+\frac{r}{p}\right)$ are shown in Table 3. Both the cases Additive Error Linear Demand and Multiplicative Error Iso-elastic Demand are showcased to highlight the variations in the optimal solutions too. The following observations and interpretations are made:

(a) The optimal policy for the fill rate model with *m*=2, as shown in row 1 of Table 3 in Additive Error Linear Demand case, consists of the retailer acquiring *q**=23125 units at a unit cost of *c*=$35 and selling them at a unit price of *p**=$50.25. With respect to the fill rate, approximately Ω^*= 4% of the shortages are recaptured at an extra purchasing cost of *d*=$3.00 to the retailer, who allows a rebate of *r**=$7.36 per unit backlogged. Afterwards, all unsold units, i.e. $[(1-\Omega^*)(D-q^*)]$, will be assigned a unit shortage penalty of *s*=$3.

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On the other hand, when demand falls below the q*=23,125 units ordered and all purchased at the cost of c=$35 per unit, D units are sold at the regular unit price of p*=$50.25 and the remaining, at the salvage value of v=$10.00 per unit.

The resulting optimal policy is π*[p*, q*, r*] = $336828 [50.26, 22975, 5.08].

As show in Table 2, these results contrast with the optimal solution for the *AL* certainty case of π*[p*; q*] =$333,909 [$50.22; 23,276]

(b) Similar interpretation follows for the other models in the Additive Error Linear Demand case, where the base on $\log_m\left(1+\frac{r}{p}\right)$ increases as shown in Table 2. The increase in the power of the fill rate function tends to increase the optimal order quantity and the rebate, whereas decreases the selling price and profits.

(c) Table 3 also gives results for the *MI* case. Observe though that unlike its Additive Error Linear Demand counterpart, in this case, increase in the base of the fill rate function, tends to increase the order quantity and the rebate and also the selling price. Profits decrease with the increase in the base of the fill rate function.

Table 3: Optimal Policies for lost sale recapture model with fill rate Ω= $\log_m\left(1+\frac{r}{p}\right)$

Additive Error Linear Demand							
m	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
2	335256	50.25	23125	7.36	0.04	321	1027
3	334747	50.23	23183	7.36	0.02	367	950
4	334570	50.23	23202	7.36	0.02	383	924
5	334477	50.23	23213	7.35	0.02	392	911
Multiplicative Error Iso-Elastic Demand							
m	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
2	359274	61.27	15351	12.53	0.12	726	1017
3	358172	61.32	15411	12.55	0.07	828	889
4	357795	61.34	15430	12.56	0.06	863	848
5	357599	61.35	15440	12.57	0.05	881	827

Sensitivity Analysis

Table 4 describes the sensitivities of the optimal policies to the change in the salvage and shortage costs in the Additive Error and Linear Demand case. Corresponding results for the Iso-elastic demand and multiplicative error case can be easily computed. The primary objective to carry out the sensitivity analysis is to observe the directional change in the shortages and the leftover values. Observe that, even though, in all of the examples of Table 3, the expected number of leftovers, Λ^*, never exceeds the expected shortages, Φ^*, the relationship between these two is parameter specific, since sensitivity analysis shows that we can construct numerous examples, where $\Lambda^* > \Phi^*$.

Table 4: Sensitivities to the salvage and shortage costs in Additive Error Linear Demand Case for *m*=1

Linear Demand Additive Error Case for *m*=2								
v	**s**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
18	**3**	339232	50.32	23495	7.40	0.04	511	750
19	**3**	339867	50.33	23556	7.40	0.04	546	709
20	**3**	340542	50.34	23621	7.41	0.04	585	666
21	**3**	341262	50.35	23690	7.41	0.04	627	622
22	**3**	342030	50.36	23765	7.42	0.04	675	576
23	**3**	342852	50.37	23847	7.42	0.04	728	529

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Linear Demand Additive Error Case for *m*=2								
10	**19**	327213	50.36	23604	14.70	0.23	593	657
10	**20**	326872	50.36	23626	15.14	0.24	607	643
10	**21**	326544	50.37	23647	15.58	0.25	620	629
10	**22**	326228	50.37	23667	16.02	0.26	633	616
10	**23**	325923	50.37	23687	16.46	0.27	645	604
10	**24**	325630	50.38	23706	16.90	0.28	657	593

Next, we perform sensitivity analysis to the change in the support values [A,B] for the Uniform distribution for the fill rate model with base *m*=2. Similar sensitivities can be performed for various other values of m, as well as support structures.

Table 5: Sensitivities to the Uniform Distribution Support Changes: CASE *m*=2

SUPPORT	**Mean**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
Linear Demand and Additive Error								
-5500,1500	-2000	310281	49.81	22549	7.15	0.04	438	1461
-3500,1500	-1000	335256	50.25	23125	7.36	0.04	321	1027
-1500,3500	1000	366432	50.92	24153	7.68	0.05	335	1003
1500,3500	2500	406259	51.57	24972	7.99	0.05	139	392
1500,5500	3500	412526	51.81	25452	8.10	0.05	282	778
Iso-elastic Demand and Multiplicative Error								
.6,1.0	0.8	314599	61.68	13442	12.72	0.12	724	993
.6,1.2	0.9	338223	63.00	14461	13.33	0.12	1077	1380
.7,1.1	0.9	363087	60.83	15299	5.29	0.37	726	1017
.8,1.2	1.0	403994	60.94	17263	12.38	0.11	728	1037
.8,1.4	1.1	427276	62.03	18259	12.89	0.12	1084	1460

Some Concluding Comments

The primary contribution of this paper has been to consider a completely new lost sale recapture function than discussed in Patel and Gor (2013), the impact upon the ordering and pricing policies of a newsvendor-type, profit-maximizing retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation, by offering some rebate incentives for waiting. The backordering occurs through an emergency purchase of the items in question at some premium over the regular purchasing cost. In turn, the retailer offers to the end-customers left out of the initial sale a rebate incentive upon purchase of each item backordered, quite aware that not all the customers that could not buy in the first instant may avail themselves of the rebate offer and buy. The backlog fill rate, representing the probability of the end-customers returning to satisfy their unfilled demand, is modelled as a function of the size of the rebate offered relative to the selling price. Further, the policy decisions on the emergency order and on the rebate policy occur up front, along with the remaining ordering and pricing policies, rather than at the end of the season. Then the retailer has to decide, ahead of the realization of the demand, the profit-maximizing ordering, pricing and rebate policies. The decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand.

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