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Research Article

EFFECT OF THERMAL FIELD AND MAGNETIC FIELD ON THE PROPAGATION OF EDGE WAVES UNDER INITIAL COMPRESSIVE HYDROSTATIC STRESS

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ABSTRACT
The effect of thermal field and magnetic field on the propagation of edge waves in a homogeneous, isotropic plate of finite thickness under initial compressive hydrostatic stress has been studied in the context of coupled thermoelasticity. The frequency equation of phase velocity of the waves at the edge of a plate of finite thickness has been derived and is approximated for small thickness of the plate. The numerical values of phase velocities of the edge waves have been calculated for different values of initial stress parameters and thermoelastic coupling parameter as well as magnetic pressure number and the results obtained are shown graphically.

Key Words: *Thermal Field, Magnetic Field, Edge Waves, Initial Hydrostatic Stress, Frequency Equation*

INTRODUCTION
When an elastic body is deformed, waves propagate inside and on the edge or surface of the body. The waves, which propagate in the plate of finite thickness, are called edge waves. If the plate is of infinite thickness, these waves are called surface waves. The propagation of elastic waves at an edge of a medium is in general more complex. Due to reflection and refraction of waves at the edge, the velocity of the wave will be different at the edge than that moving inside.

Kumar (1959) obtained the phase velocities of edge waves without considering the initial stress. Initial stresses develop in the medium due to various reasons, and it is of paramount interest to study the effect of these stresses on the propagation of elastic waves. A lot of systematic studies have been made on the propagation of elastic waves. Biot (1965) showed that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. The influence of initial stresses on the propagation of edge waves has been studied by Das and Dey (1970). Lokenath and Roy (1988) studied the propagation of edge waves in a thinly layered laminated medium with stress couples under initial stresses. Based on Biot's theory, Montanaro (1999) investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Abraham and Norris (2000) discussed the existence of the edge waves in a plate under a particular physical condition. Dey and De (2009) showed the effect of initial stress on the velocity of propagation of edge wave in an incompressible anisotropic initially stressed plate of finite thickness, the velocity has been computed for various initial stress parameter and different anisotropy ratio and some particular cases have been discussed to get the velocity in an initially stress free and isotropic medium.

Since deformation of a body can give rise to a temperature variation in space and time, it is important to study the edge wave propagation in presence of thermal field as well in the light of theory of thermoelasticity. The coupling between thermal and strain fields give rise to the coupled theory of thermoelasticity. In the theory of thermoelasticity the governing equations are hyperbolic equation (wave type) of motion and parabolic equation (diffusion type) of heat conduction. When an isotropic elastic medium is subjected to a mechanical and thermal disturbance the effect of both the displacement field and temperature field are felt at infinite distance from the source of disturbance implying an infinite speed of propagation which is physically impossible. In view of this drawback Lord and Shulmon (1967) modified Fourier Heat conduction equation and the constitutive equations by introducing linear harmonic to the thermal waves and the heat flux rate term to formulate the generalized theory of thermoelasticity.

Introducing the concept of relaxation time of thermoelastic process to the generalized theory of heat conduction equation, Lord and Shulmon (1967) gave a hyperbolic type of heat transport equation. These equations include the time needed for acceleration of heat flow and take into account the coupling between temperature field and strain field for isotropic materials. Later the hyperbolic heat conduction equation was further modified by Green and Lindsay (1972) who introduced the concept of two relaxation times of the thermal process along with the temperature rate, among other constitutive variables. These rigorous theories have been found to be more realistic than the conventional theories and are in good agreement with the experimental results. Chandrasekharaiah (1986) referred to this wave like thermal disturbance as "second sound''. A brief review of different thermoelastic models can be found in the paper of Hetnarski and Ignaczak (1999).
In the context of generalized theory of thermoelasticity, Ahmed (2000) has shown how the phase velocity of thermo elastic waves is influenced by the nature of the solid medium and the initial stress present in it. The propagation of Surface waves under initial stress in presence of temperature field has been studied by Addy and Chakraborty (2005"a") and the propagation of Gravity waves in a liquid under the effect of temperature field and initial stress has also been studied by Addy and Chakraborty (2005"b"). Recently S. Gupta *et al.,* (2010) discussed the propagation of Surface waves (S-waves) in a non-homogeneous anisotropic incompressible and initially stressed medium. The reflection and refraction of the elastic waves under the generalized theory of thermoelasticity under the influence of initial stress has been discussed by Chakraborty and Singh (2011).
Magnetic field also influences the wave propagation in elastic medium. Taking into account, the effect of magnetic field and thermal field, Abd-alla (2000), Ezzat and Othman (2000), Ezzat and El-karmany (2003), Acharya and Mondal (2006), Othman and Song (2008), Abo-dahab and Singh (2009), Kumar and Devi (2010) and many others have studied problems of elastic waves. But none considered the effect of initial stress.
The effect of magnetic field in presence of initial stress has been discussed by very few authors such as Abd-Allaa *et al.,* (2009), Gehlot*et al.,* (2011), Singh *et al.,* (2012).
In the present paper, the effect of thermal field and initial compressive hydrostatic stress on the propagation of edge waves in a homogeneous, isotopic plate of finite thickness has been discussed in presence of magnetic field. The frequency equation for the edge waves has been obtained. The frequency equation is approximated and analyzed numerically to observe the effect of initial stress parameter, thermoelastic coupling parameter and magnetic pressure number, on the phase velocity of edge waves.

Definition of the Problem

We consider a homogeneous, isotropic plate of infinite length and thickness H, under uniform initial compressive hydrostatic stress P, at an initial temperature T_0 as shown in figure 1.

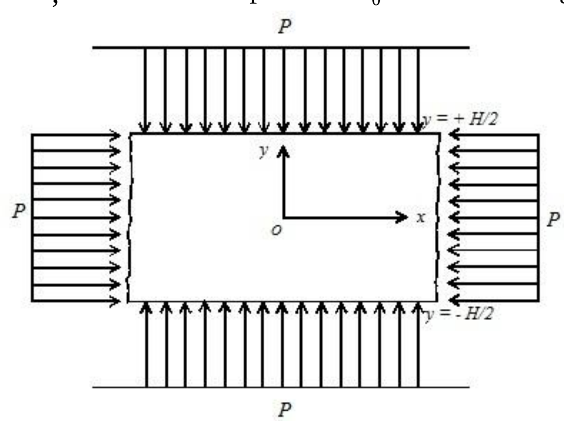

Figure1: A homogeneous plate of infinite length and thickness H, under initial compressive hydrostatic stress P

Formulation of the Problem

Consider a three-dimensional Cartesian co-ordinate system. The origin of the co-ordinate system is located at the middle of the plate. The x-axis is taken in the direction of wave propagation and y-axis is taken vertically upwards. The plate is permeated into a uniform magnetic field $\vec{H}_0 = (0,0,H_3)$ which is parallel to z-axis.

Basic Equations

The dynamical equations of equilibrium under initial hydrostatic stress given by Biot (1965), taking into account the presence of Lorentz force are

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} + F_1 = \rho \frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} + F_2 = \rho \frac{\partial^2 v}{\partial t^2} \tag{1}$$

Where s_{11}, s_{22} and s_{12} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and last one is shear stress component in the xy plane. ρ is the density of the material, u and v are the displacement components along x- and y- axes, respectively. F_1 and F_2 are Lorenz force components along x- and y- axes, respectively.

The stress-strain relations with incremental isotropy given by Biot (1965) are

$$s_{11} = \lambda(e_{xx} + e_{yy}) + 2\mu e_{xx} - \gamma\left(T + t_1 \frac{\partial T}{\partial t}\right)$$
$$s_{22} = \lambda(e_{xx} + e_{yy}) + 2\mu e_{yy} - \gamma\left(T + t_1 \frac{\partial T}{\partial t}\right)$$
$$s_{12} = 2\mu e_{xy} \tag{2}$$

Where $\gamma = (3\lambda + 2\mu)\alpha_t, \alpha_t$ is the coefficient of linear expansion of the material and λ, μ are Lame's constants. T is the incremental change of temperature from the initial state and t_1 is second relaxation time.

The incremental strain components given by Biot (1965) are

$$e_{xx} = \frac{\partial u}{\partial x}, e_{xy} = \frac{\partial v}{\partial y}, e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{3}$$

Where e_{xx} and e_{yy} are the principal strain components and e_{xy} is the shear strain component.

The heat conduction equation given by

$$\delta \nabla^2 T = \rho S\left(\frac{\partial T}{\partial t} + t_0 \frac{\partial^2 T}{\partial t^2}\right) + T_0 \gamma\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + t_0 \delta_{ij} \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] \tag{4}$$

Where S is the specific heat per unit mass at constant strain, δ is the thermal conductivity of the material, t_0 is the first relaxation time and δ_{ij} is kronecker delta.

Taking into account the absence of displacement current, the simplified linear Maxwell equations of electrodynamics of slowly moving medium having perfect electric conductivity are

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$$\text{curl}\,\vec{h} = \vec{J}, \text{curl}\,\vec{E} = -\mu_e \frac{\partial \vec{h}}{\partial t}, \text{div}\,\vec{h} = 0 \text{ and div}\,\vec{E} = 0 \tag{5}$$

Where $\vec{h} = \text{curl}\left(\vec{u} \times \vec{H}_0\right)$ (6)

Here $\vec{h}$ and $\vec{E}$ are the induced magnetic field and induced electric field, respectively, $\vec{J}$ is the electric current density vector, $\vec{u}$ is the displacement vector and μ_e is the magnetic permeability.
The equation of Lorentz force is

$$\vec{F} = \mu_e\left(\vec{J} \times \vec{H}_0\right) \tag{7}$$

Substituting (6) in (5) we get the value of $\vec{J}$ and putting the value of $\vec{J}$ in (7), we get the components of Lorentz force as

$$F_1 = \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) \text{and}\, F_2 = \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) \tag{8}$$

Maxwell stress components are given by

$$T_{ij} = \mu_e\left[H_i h_j + H_j h_i - (H_k h_k)\delta_{ij}\right], (\text{where}\, i, j, k = 1,2,3) \tag{9}$$

Equation (9) gives

$$T_{22} = \mu_e H_3^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \text{and}\, T_{12} = 0 \tag{10}$$

Solution of the Problem
Equation (1) with the help of (2), (3) and (8) take the forms

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial y^2} + \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = \rho\frac{\partial^2 u}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + t_1\frac{\partial^2 T}{\partial t \partial x}\right) \tag{11a}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial x^2} + \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \rho\frac{\partial^2 v}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial y} + t_1\frac{\partial^2 T}{\partial t \partial y}\right) \tag{11b}$$

In Lord-Shulman theory $t_1 = 0, t_0 \rangle 0$ and $\delta_{ij} = 1$.
In Green-Lindsay theory $t_1 \rangle t_0 \rangle 0$ and $\delta_{ij} = 0$.
Here we use classical dynamical coupled theory in which $t_0 = 0, t_1 = 0$ and $\delta_{ij} = 0$. So Equations (4) and (11) change to

$$\delta\nabla^2 T = \rho S\frac{\partial T}{\partial t} + T_0\gamma\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{12}$$

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial y^2} + \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = \rho\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial x}(\gamma T) \tag{13a}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial x^2} + \mu_e H_3^2\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \rho\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial x}(\gamma T) \tag{13b}$$

The displacement components u and v may be expressed in terms of the potential functions ϕ and ψ as:

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$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y} \text{ and } v = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \tag{14}$$

From equations (13) and (14), we get the following wave equations:

$$\nabla^2 \phi = \frac{\rho}{\left(\lambda + 2\mu + \mu_e H_3^2\right)} \frac{\partial^2 \phi}{\partial t^2} + \frac{\gamma T}{\left(\lambda + 2\mu + \mu_e H_3^2\right)} \tag{15a}$$

$$\nabla^2 \psi = \frac{\rho}{\mu} \frac{\partial^2 \psi}{\partial t^2} \tag{15b}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Putting (14) in (12), we get

$$\nabla^2 T - \frac{S\rho}{\delta} \frac{\partial T}{\partial t} - \frac{\gamma T_0}{\delta} \nabla^2 \frac{\partial \phi}{\partial t} = 0 \tag{16}$$

Eliminating T froms (15a) and (16), we get

$$\left(\nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2}\right)\left(\nabla^2 - \frac{S\rho}{\delta} \frac{\partial}{\partial t}\right)\phi - \beta \eta_0 \nabla^2 \left(\frac{\partial \phi}{\partial t}\right) = 0 \tag{17a}$$

Where $c_1^2 = \frac{\left(\lambda + 2\mu + \mu_e H_3^2\right)}{\rho}, \beta = \frac{\gamma}{\left(\lambda + 2\mu + \mu_e H_3^2\right)}$ and $\eta_0 = \frac{\gamma T_0}{\delta}$

Equation (15b) takes the form

$$\left(\nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{17b}$$

Where $c_2^2 = \frac{\mu}{\rho}$

For plane harmonic waves moving along the x- axis, the solutions of (17a) and (17b) can be taken in the form

$$\phi(x, y, t) = \frac{1}{\alpha^2} f(\alpha y) \exp i(\alpha x - \omega t) \tag{18a}$$

$$\psi(x, y, t) = \frac{1}{\alpha^2} g(\alpha y) \exp i(\alpha x - \omega t) \tag{18b}$$

Where ω is frequency of oscillation and α is the wave number

Putting (18a) in (17a) and (18b) in (17b), we obtain the following equations:

$$\left[\frac{\partial^2}{\partial (\alpha y)^2} - \lambda_1^2\right]\left[\frac{\partial^2}{\partial (\alpha y)^2} - \lambda_2^2\right]\alpha^4 f(\alpha y) = 0 \tag{19a}$$

$$\left[\frac{\partial^2}{\partial (\alpha y)^2} - \nu_1^2\right]\alpha^2 g(\alpha y) = 0 \tag{19b}$$

Where $\alpha^2 \lambda_1^2 = \alpha^2 - k_1^2, \alpha^2 \lambda_2^2 = \alpha^2 - k_2^2$ and $\alpha^2 {\nu_1}^2 = \alpha^2 - \tau^2$

Here $\tau^2 = \frac{\omega^2}{c_2^2}$ and k_1^2 and k_2^2 are roots of the biquadratic equation:

$$k^4 - k^2\left[\sigma^2 + q(1+\varepsilon)\right] + \sigma^2 q = 0 \quad (20)$$

Where $k^2 = -\nabla^2$ and the roots k_1^2 and k_2^2 are given by

$$k_1^2 = q\left[1 - \frac{q\varepsilon}{\sigma^2 - q}\right] \text{and } k_2^2 = \sigma^2\left[1 + \frac{q\varepsilon}{\sigma^2 - q}\right] \quad (21)$$

Also

$\sigma^2 = \frac{\omega}{c_1^2}, q = \frac{i\omega S\rho}{\delta}$ and $\varepsilon = \frac{\gamma^2 T_0}{S\rho\left(\lambda + 2\mu + \mu_e H_3^2\right)}$, is thermoelastic coupling parameter.

The requirement that the stresses and hence the functions ϕ and ψ vanish as $\left(x^2 + y^2\right)$ tends to infinity leads to the following solutions of equations (19):

$$f(\alpha y) = A\cos h\lambda_1(\alpha y) + B\cos h\lambda_2(\alpha y) \quad (22a)$$

$$g(\alpha y) = C\sin h\nu_1(\alpha y) \quad (22b)$$

Putting (22a) in (18a) and (22b) in (18b), we get

$$\phi(x,y,t) = \frac{1}{\alpha^2}\left[A\cos h\lambda_1(\alpha y) + B\cos h\lambda_2(\alpha y)\right]\exp i(\alpha x - \omega t) \quad (23a)$$

$$\psi(x,y,t) = \frac{1}{\alpha^2}\left[C\sin h\nu_1(\alpha y)\right]\exp i(\alpha x - \omega t) \quad (23b)$$

Equation (15a) gives

$$T = \frac{\left(\lambda + 2\mu + \mu_e H_3^2\right)}{\gamma}\left[\nabla^2\phi - \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2}\right] \quad (24)$$

Using (23a) and (24), we get

$$T = \frac{\left(\lambda + 2\mu + \mu_e H_3^2\right)}{\gamma}\frac{1}{\alpha^2}\left[\eta_1 A\cos h\lambda_1(\alpha y) + \eta_2 B\cos h\lambda_2(\alpha y)\right]\exp i(\alpha x - \omega t) \quad (25)$$

Where $\eta_1 = \sigma^2 - k_1^2$ and $\eta_2 = \sigma^2 - k_2^2$

Boundary Conditions

The boundary conditions on the plane $y = \pm\frac{H}{2}$ are given by

$$\Delta f_x = s_{12} - P\frac{\partial v}{\partial x} + T_{12} = 0 \quad (26a)$$

$$\Delta f_y = s_{22} + P\frac{\partial u}{\partial x} + T_{22} = 0 \quad (26b)$$

$$\frac{\partial T}{\partial y} + h'T = 0 \quad (26c)$$

Where Δf_x and Δf_y are incremental boundary forces per unite initial area and h' is the ratio of heat transfer coefficient and thermal conductivity.

Using (2),(10),(14),(23a) and (23b), the first boundary condition (26a) becomes

$$2i(1-\varsigma)\lambda_1 \sin\left(h\lambda_1\beta'\right)\cdot A + 2i(1-\varsigma)\lambda_2 \sin\left(h\lambda_2\beta'\right)\cdot B + \left[\frac{\tau^2}{\alpha^2} - 2(1-\varsigma)\right]\sin\left(h\nu_1\beta'\right)\cdot C = 0 \quad (27a)$$

Using (2),(10),(14),(23a),(23b) and (25), the second boundary condition (26b) becomes

$$\left[2(1-\varsigma) - \frac{\omega^2\rho}{\alpha^2\mu}\right]\cos\left(h\lambda_1\beta'\right)\cdot A + \left[2(1-\varsigma) - \frac{\omega^2\rho}{\alpha^2\mu}\right]\cos\left(h\lambda_2\beta'\right)\cdot B + 2i\nu_1(1-\varsigma)\cos\left(h\nu_1\beta'\right)\cdot C = 0 \quad (27b)$$

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Using (25) and $h' = 0$, i.e. for the case thermal insulation, the third boundary condition (26c) becomes

$$\lambda_1\eta_1 \sin(h\lambda_1\beta')\cdot A + \lambda_2\eta_2 \sin(h\lambda_2\beta')\cdot B = 0 \tag{27c}$$

Where $\varsigma = \dfrac{P}{2\mu}$, is the initial stress parameter in dimensionless form and $\beta' = \dfrac{\alpha H}{2}$.

Frequency Equation

Eliminating the constants A, B and C from (27a), (27b) and (27c), we get

$$\begin{vmatrix} 2i(1-\varsigma)\lambda_1 & 2i(1-\varsigma)\lambda_2 & \left[\dfrac{\tau^2}{\alpha^2} - 2(1-\varsigma)\right] \\ \left[2(1-\varsigma) - \dfrac{\omega^2\rho}{\alpha^2\mu}\right]\cot(h\lambda_1\beta') & \left[2(1-\varsigma) - \dfrac{\omega^2\rho}{\alpha^2\mu}\right]\cot(h\lambda_2\beta') & 2i\nu_1(1-\varsigma)\cot(h\nu_1\beta') \\ \lambda_1\eta_1 & \lambda_2\eta_2 & 0 \end{vmatrix} = 0 \tag{28}$$

Expanding the determinant (28) and simplifying, we get

$$\begin{aligned} &4(1-\varsigma)^2 \lambda_1\lambda_2\nu_1(\eta_2 - \eta_1)\cot(h\nu_1\beta') \\ &+ \left[2(1-\varsigma) - \frac{\omega^2\rho}{\alpha^2\mu}\right]\left[\frac{\tau^2}{\alpha^2} - 2(1-\varsigma)\right]\left[\lambda_2\eta_2\cot(h\lambda_1\beta') - \lambda_1\eta_1\cot(h\lambda_2\beta')\right] = 0 \end{aligned} \tag{29}$$

Let $\lambda_1^2 = 1 - \dfrac{k_1^2}{\alpha^2} = \beta_1^2$ and $\lambda_2^2 = 1 - \dfrac{k_2^2}{\alpha^2} = \beta_2^2$ (30)

Also

$$\cot(h\lambda_1\beta') \to \frac{1}{\lambda_1\beta'}, \cot(h\lambda_2\beta') \to \frac{1}{\lambda_2\beta'} \text{ and } \cot(h\nu_1\beta') \to \frac{1}{\nu_1\beta'}, \text{ as } H \to 0, \beta' \to 0 \tag{31}$$

Expressing the quantities $\lambda_1, \lambda_2, \eta_1$ and η_2 in terms of quantities β_1 and β_2 and using (31),we find that (29) reduces to the form:

$$4(1-\varsigma)^2\left[\frac{c^2}{c_1^2} + \beta_1^2 + \beta_2^2 - \beta_1^2\beta_2^2 - 1\right] = \left[2(1-\varsigma)\left(\frac{\tau^2}{\alpha^2} + \frac{c^2}{c_2^2}\right) - \frac{c^2}{c_2^2}\frac{\tau^2}{\alpha^2}\right]\left(\frac{c^2}{c_1^2} + \beta_1^2 + \beta_2^2 - 1\right) \tag{32}$$

Where $c^2 = \dfrac{\omega^2}{\alpha^2}$

From (20), we get

$$k_1^2 + k_2^2 = \sigma^2 + q(1+\varepsilon) \text{ and } k_1^2 \cdot k_2^2 = \sigma^2 q \tag{33}$$

Using (30) and (33), we get

$$\beta_1^2 + \beta_2^2 = 2 - \frac{c^2}{c_1^2} - \frac{ic^2}{fc_1^2}(1+\varepsilon) \text{ and } \beta_1^2\beta_2^2 = 1 - \frac{c^2}{c_1^2} - \frac{ic^2}{fc_1^2}\left(1 + \varepsilon - \frac{c^2}{c_1^2}\right) \tag{34}$$

Where $f = \dfrac{\delta\omega}{S\rho c_1^2}$, is the reduced frequency.

Putting (34) in (32), we get a frequency equation of Edge waves:

$$4(1-\varsigma)^2\frac{c^2}{c_1^2}-4(1-\varsigma)^2\frac{ic^4}{fc_1^4}=\left[4(1-\varsigma)-\frac{c^2}{c_2^2}\right]\frac{c^2}{c_2^2}-\left[4(1-\varsigma)-\frac{c^2}{c_2^2}\right]\frac{ic^4}{fc_1^2c_2^2}(1+\varepsilon) \tag{35}$$

Equating imaginary parts of (35), we get

$$4(1-\varsigma)^2\frac{c_2^2}{c_1^2(1+\varepsilon)}=4(1-\varsigma)-\frac{c^2}{c_2^2} \tag{36}$$

Putting $\frac{c_2^2}{c_1^2(1+\varepsilon)}=\Omega$ and $\frac{c^2}{c_2^2}=V$, (36) reduces to

$$4(1-\varsigma)^2\Omega=4(1-\varsigma)-V \tag{37}$$

Where $V=\frac{c^2}{c_2^2}$ represent the phase velocity of Edge waves

$$\text{Also }\ \Omega=\frac{c_2^2}{c_1^2(1+\varepsilon)}=\frac{c_2^2}{c_0^2(1+R_H)(1+\varepsilon)} \tag{38}$$

Where $R_H=\frac{c_A^2}{c_0^2}, c_A^2=\frac{\mu_e H_3^2}{\rho}, c_0^2=\frac{\lambda+2\mu}{\rho}, c_2^2=\frac{\mu}{\rho}$. Here R_H, c_A, c_0, c_2, magnetic pressure number, Alfven wave velocity, isothermal dilatational and rotational wave velocity respectively.

RESULTS AND DISCUSSION

The data for aluminium is used for numerical calculations. Following (Abd-alla et al., 2003), we take the following material constants for aluminium:

$\lambda=57.75\times10^9$ Nm^{-2}, $\mu=26.43\times10^9$ Nm^{-2}, $\alpha_t=23\times10^{-6}$ K^{-1}, $\rho=2700$ Kgm^{-3},

$S=900$ J Kg^{-1}K^{-1}, $\delta=237$ Wm^{-1}K^{-1}

The numerical values of phase velocity of edge waves have been computed from the equation (37), for different values of initial stress parameter, magnetic pressure number and thermoelastic coupling parameter. These results have been plotted in the following graphs:

Thermoelastic Coupling Parameter is Kept Constant

(a) Variation of phase velocity with initial stress parameters for different values of magnetic pressure number:

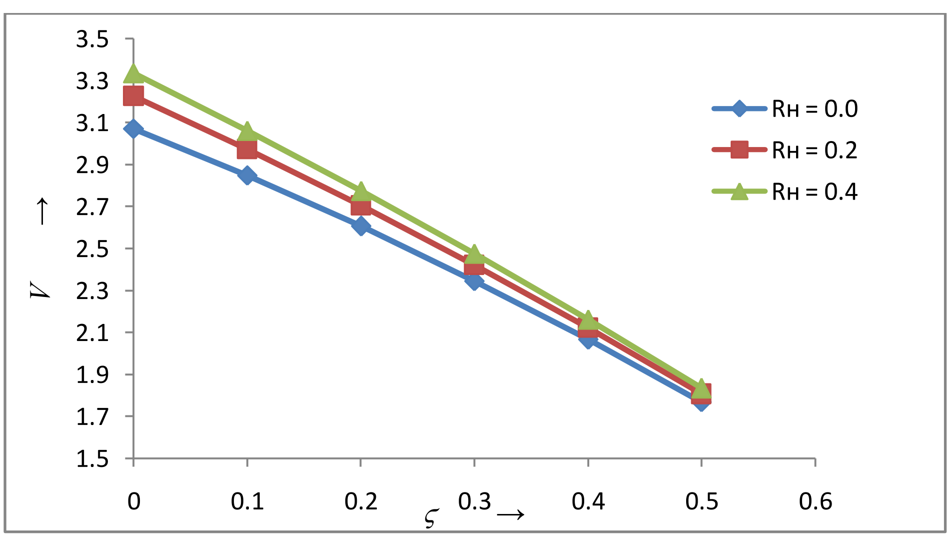

Figure 2: Variation of phase velocity (V) of edge waves with initial stress parameter (ς) for different values of magnetic pressure number (R_H) keeping thermoelastic coupling parameter constant ($\varepsilon = 0.03$).

Figure 2 shows that the phase velocity of edge waves decreases sharply with the increase in the initial stress parameter. Phase velocity is also higher for higher magnetic stress parameter.

(b) Variation of phase velocity with magnetic pressure number for different values of initial stress parameters:

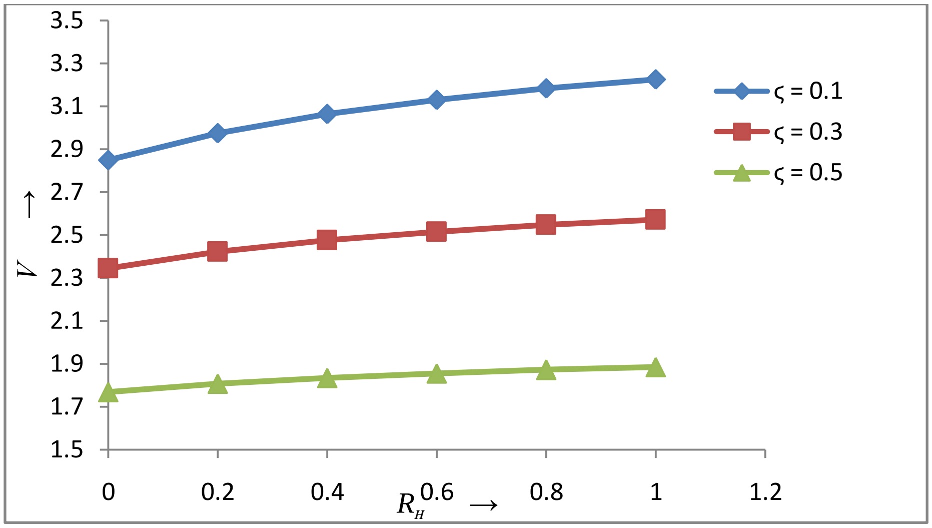

Figure 3: Variation of phase velocity (V) of edge waves with magnetic pressure number (R_H) for different values of initial stress parameter (ς) keeping thermoelastic coupling parameter constant ($\varepsilon = 0.03$).

Figure 3 shows that the phase velocity of edge waves increases slowly with the increase in the magnetic pressure number. Graph also shows that the magnitude of phase velocity decreases as the initial stress parameter increases.

Magnetic Pressure Number is Kept Constant

(a) Variation of phase velocity with initial stress parameters for different values of thermoelastic coupling parameter:

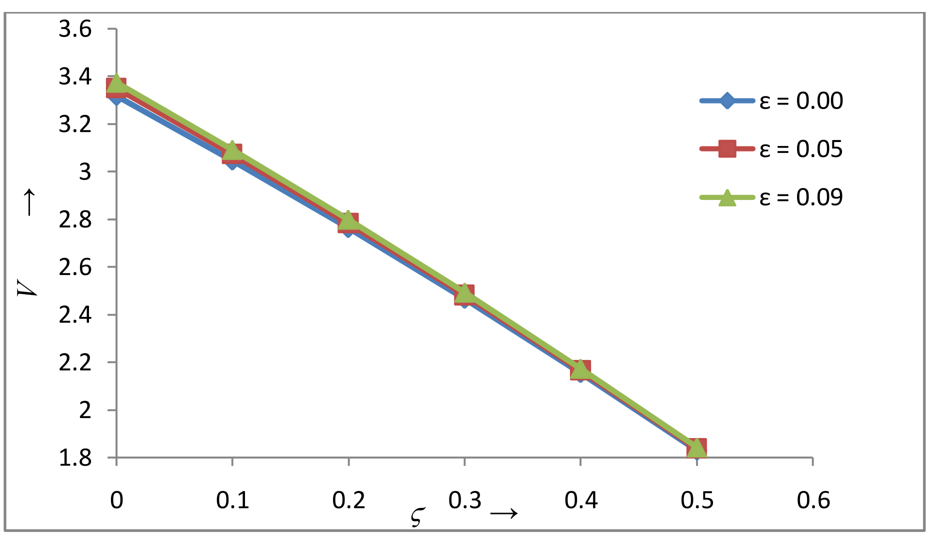

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Figure 4: Variation of phase velocity (V) of edge waves with initial stress parameter (ς) for different values of thermoelastic coupling parameter (ε) keeping magnetic pressure number constant ($R_H = 0.4$).

Figure 4 shows that the phase velocity of edge waves decreases sharply with the increase in the initial stress parameter and effect of thermal variation on the phase velocity is negligible.

(b) Variation of phase velocity with thermoelastic coupling parameter for different values of initial stress parameters:

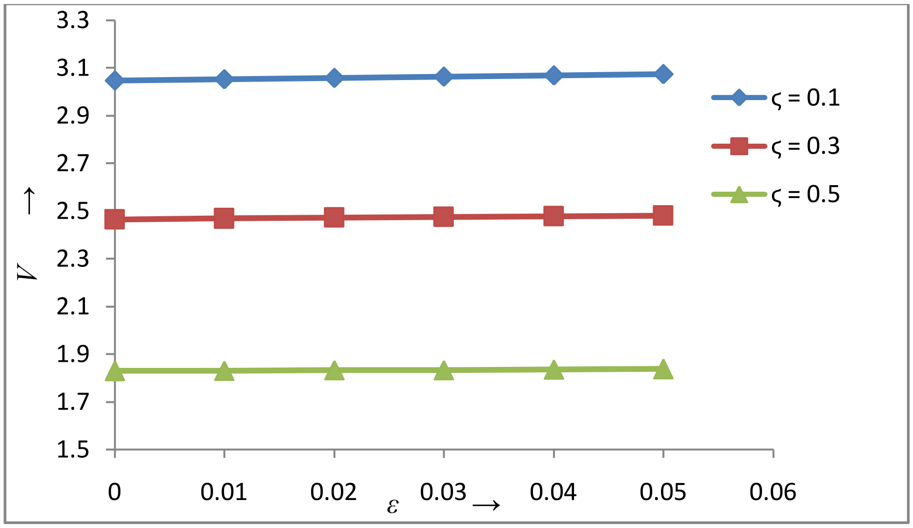

Figure 5: Variation of phase velocity (V) of edge waves with thermoelastic coupling parameter (ε) for different values of initial stress parameter (ς) keeping magnetic pressure number constant ($R_H = 0.4$).

Figure 5 shows that the phase velocity of edge waves practically remains constant with the increase in the thermoelastic coupling parameter. Moreover the magnitude of phase velocity is less when initial stress is more.

Conclusion

From the above study, it can be concluded that the initial compressive hydrostatic stress as well as the magnetic field have significant effect on the phase velocity of edge waves. This study also shows that the thermoelastic coupling parameter has a small influence on the phase velocity of edge waves. This study may be applied to solve problems where the factors like elastic field, thermal field, magnetic field and initial stress coexist such as the seismic wave propagation inside the earth, geophysics, nuclear devices etc.

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