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Research Article

VAGUE SET THEORETIC APPROACH TO MEDICAL DIAGNOSIS IN HEALTH CARE SYSTEM

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ABSTRACT

In healthcare systems related to government hospitals and private nursing homes, the quality considerations may mean that the quality of services, the quality of professionalism and the quality of management. Patient expectation's is an important issue that is addressed in the total quality management. Patients are expecting more from healthcare units and are increasingly dissatisfied with the services provided by them. So, these units must seek a quality culture in which patient satisfaction should be properly met. From patient's point of view, quality of health service can be considered to have following three different dimensions: science of medicine, the art of treatment and the environment under which a health service is provided. A strict policy for waiting time will definitely not be achievable properly. So, healthcare units require relaxable optimal policy under which units may work properly. We intend to study this problem in this paper and propose a fuzzy linear programming approach to find an optimal policy between the specified and tolerable satisfaction level.

Key Words: *Linear Programming, Fuzzy Linear Programming, Vague Set, Hesitation Function, Triangular Vague Number, Order Function, Soft and Hard Constraints*

INTRODUCTION

Linear programming is an extremely powerful tool for addressing a wide range of applied optimization problems. There are two hidden assumptions associated with linear programming problems (LPP). First is that the data needed for a LPP are described in precise manner. Secondly even a little violation of any constraint in it renders the solution infeasible. Hence, all constraints are considered to be of equal importance.

In real world situations, it often becomes difficult to acquire data in exact or precise form. This leads to the violation of first assumption. Fuzzy Linear Programming (FLP) addresses this type of situations. The fuzziness may be there in the coefficients of objective function and/or in the constraints of the LPP. Zimmermann (1991) introduced yet another type of FLP in which decision maker sets the limits of fuzziness with respect to the objective function and/or constraints. In the literature, this approach to define FLP is called aspiration level approach. This approach is based on Bellmen and Zadeh's (1965) decision making principle. The aspiration level approach permits violation of second assumption to certain tolerable limits and provides method to evaluate a degree of violation of goals and constraints.

1. Problem Formulation

Linear programming models are special kinds of decision models. The decision space is defined by the constraints and the goal is defined by the objective function. The model can be stated as

$$Max\ or\ Min\ \ f(X) = C^T X$$

$$Subject\ to$$

$$AX \le b, \quad X \ge 0$$

$$C, X \in R^n, b \in R^m \quad A \in R^{m\times n}$$

Here coefficients of *A*, *b* and *C* have crisp values.

If linear programming problem is considered in fuzzy environment, it would mean that the decision maker might really not want to actually maximize or minimize the objective function; rather he might want to reach to some aspiration levels that could not be definable crisply. The other possibility may be that the constraint might be vague and smaller violations in the constraint with strict inequalities might well be acceptable. This can happen if the constraints represent aspiration levels. The role of the constraints can be different from that in classical linear programming where the violation of any single constraint by any amount, renders the solution infeasible. The decision maker might accept small violations of constraints but might also attach different degrees of importance to violations of different constraints. Although fuzzy linear programming offers various ways to allow the fuzziness, we shall here define only that case which is used in our model. The fuzzy linear programming problem can be written as:

$$Min\ \ f(X) = C^T X$$
$$Subject\ to$$
$$AX \geq b$$
$$DX \geq b'$$
$$D'X \leq b'' \quad X \geq 0$$
$$C, X \in R^n, b, \in R^m, b' \in R^{m'}, b'' \in R^{m''},$$
$$D \in R^{m_1 \times n_1}, D' \in R^{m_1' \times n_1'}\ A \in R^{m \times n}$$

Where $\geq$ stands for the fuzzy version of the symbol $\geq$ having linguistic interpretation.

The membership function for the fuzzy set representing i^{th} fuzzy constraint is as following:

$$\mu_i(X) = \begin{cases} 0 & if\ \ A_i(X) \leq b_i - p_i \\ \dfrac{A_i(X) - (b_i - p_i)}{p_i} & if\ \ b_i - p_i \leq A_i(X) \leq b_i\,. \\ 1 & if\ \ b_i \leq A_i(X) \end{cases}$$

Now we will find the fuzzy set of the optimal solutions. The membership function of the objective values can be obtained by solving following two crisp linear programming problems.

$$Min\ \ f(X) = C^T X$$
$$Subject\ to$$
$$AX \geq b,$$
$$DX \geq b'$$
$$D'X \leq b'' \quad X \geq 0$$

giving the optimal solution Z_u and

$$Min\ \ f(X) = C^T X$$
$$Subject\ to$$
$$AX \geq b - p,$$
$$DX \geq b'$$
$$D'X \leq b'' \quad X \geq 0$$

giving the optimal solution Z_l.

The membership function is given as:

$$\mu_{\tilde{G}} = \begin{cases} 0 & \text{if } Z_u \le C^T X \\ \dfrac{Z_u - C^T X}{Z_u - Z_l} & \text{if } Z_l \le C^T X \le Z_u, \\ 1 & \text{if } C^T X \le Z_l \end{cases}$$

The crisp equivalent of the fuzzy model is defined a

$$\begin{aligned} & Max\ \lambda \\ & Subject\ to \\ & \quad C^T X + \lambda(Z_u - Z_l) \le Z_u \\ & \quad AX - \lambda p \ge b - p, \\ & \quad DX \ge b' \\ & \quad D'X \le b'' \\ & \quad 0 \le \lambda \le 1, X \ge 0 \end{aligned}$$

2. Preliminary Concepts

2.1 Vague set. A vague set $\tilde{A}$ in the universe of discourse X is characterized by two membership functions as:

(1) Truth membership function

$$\mu_{\tilde{A}} : X \to [0,1] \text{ and } \tag{2.1}$$

(2) False membership function

$$\nu_{\tilde{A}} : X \to [0,1]. \tag{2.2}$$

The grade of membership for any element x in the vague set is bounded by a sub interval [$\mu_{\tilde{A}}(x)$, 1- $\nu_{\tilde{A}}(x)$] of [0,1] where the grade $\mu_{\tilde{A}}(x)$ is called the lower bound of membership grade of x derived from favourable evidence for x and $\nu_{\tilde{A}}(x)$ is the lower bound of membership grade on the negation of x derived from the evidence against x, where $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$. The interval, [$\mu_{\tilde{A}}(x)$,1-$\nu_{\tilde{A}}(x)$] is called the vague value of x in $\tilde{A}$. In the extreme case of equality where $\mu_{\tilde{A}}(x)$ = 1- $\nu_{\tilde{A}}(x)$, the vague set reduces to the fuzzy set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however,

$$\mu_{\tilde{A}}(x) \le \text{ exact membership grade of } x \le 1 - \nu_{\tilde{A}}(x). \tag{2.3}$$

Expressions given below can be used to represent a vague set $\tilde{A}$ for finite, countable and uncountable universe of discourse X respectively:

$$\tilde{A} = \sum_{k=1}^{n} [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k,\ \tilde{A} = \sum_{k=1}^{\infty} [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k$$

$$\tilde{A} = \int_{x \in X} [\mu_{\tilde{A}}(x), 1 - \nu_{\tilde{A}}(x)] / x. \tag{2.4}$$

It is worth to mention here that interval valued fuzzy sets are not vague sets.

In interval valued fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the favourable evidence for x only, without considering evidence against x. In vague sets both are independently proposed by the decision maker.

2.2 Triangular Vague Number

We introduce the concept of a triangular vague number on similar lines to Chen [4] who defined triangular vague sets and arithmetic operations between them.

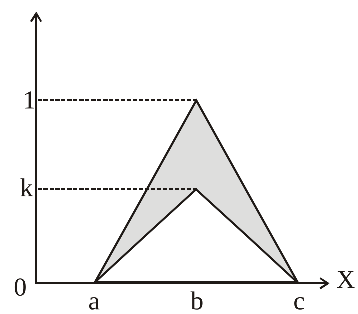
A triangular vague number $\tilde{A}$ denoted by [(a, b, c); k; 1] is characterized by a pair of membership functions: a lower membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{k(x-a)}{b-a}, a \le x \le b \\ \dfrac{k(c-x)}{c-b}, b \le x \le c \\ 0, \quad otherwise \end{cases} \tag{2.5}$$

and an upper membership function

$$\mu'_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x \le b \\ \dfrac{c-x}{c-b}, b \le x \le c \\ 0, otherwise \end{cases} \tag{2.6}$$

where $\mu_{\tilde{A}}'(x) = 1 - \nu_{\tilde{A}}(x)$ and $k \in [0,1]$. Fig. shows a triangular vague number.



A triangular vague number reduces to a triangular fuzzy number for k = 1. In what follows now onwards, we shall use vague number for a triangular vague number.

***2.3 Hesitation Function*:** The true and false membership functions are related to each other by the expression $t_{\tilde{A}}(x) + f_{\tilde{A}}(x) \le 1$. The remaining unknown part

$$\pi_A(x) = 1 - t_A(x) - f_A(x) \tag{2.7}$$

is usually referred as the *hesitation function*. This function may be considered as the degree of uncertainty that could not be resolved through any type of evidences. If $\pi_A(x) = 0$, for every x then the vague set reduces to a fuzzy set. For example, a vague value $[0 \cdot 4, 0 \cdot 7]$, means $t_{\tilde{A}}(x) = 0 \cdot 4, 1 - f_{\tilde{A}}(x) = 0 \cdot 7$. It can be interpreted as "the degree of the element x belongs to vague set $\widehat{A}$ is $0 \cdot 4$; the degree of the element x does not belong to vague set $\widehat{A}$ is 0.3 and the degree of unresolved uncertainty is 0.3". In a voting model, it can be interpreted as "the vote for resolution is 4 in favor, 3 against and 3 abstentions".

***2.4 Order Function*:** Let us look into the voting model more deeply. It may be possible that some members in the abstained group may be inclined to vote for the proposal; some may be clearly against while the rest may be indecisive. If a criterion could be established to have such classification in abstentions then the degree in favor or against could be increased by reducing the indecisive part. Order function mainly tries to reduce the degree of uncertainty by reducing the hesitation function $\pi_A(x)$ in proportion of $t_A(x)/f_A(x)$ in the favor of true/false membership function. Zhou *et al.*, [15] have used following order function in favour of true value.

$$O_1(t_A(x)) = t_A(x) + t_A(x)\pi_A(x) \tag{2.8}$$

We define a new order function $O_2(t_A(x))$ by first reducing the hesitation function in favor of false membership function as given below:

$$O(f_A(x)) = f_A(x) + f_A(x)\pi_A(x), \tag{2.9}$$

$$O_2(t_A(x)) = 1 - O(f_A(x)). \tag{2.10}$$

In the present work we shall use a new order function $O(t_A(x))$ that combines the order functions.

3. Formulation of the Model

When we talk about the waiting times of the out patients, it is not always proper to define a maximum waiting time common to all cases. It should be optimized for better satisfactory check up in the healthcare units. Comprehensive waiting time targets are most frequently and conveniently defined in comparison to the maximum waiting time. In this respect one standard is observed that at least half of the patients should be checked-up within 12 minutes of their appointed time, 65% within half 25 hour and with no more than 2% waiting for over an hour. Nevertheless these targets are arbitrary and nobody knows how many patients would be satisfied if such targets are met and for those satisfied patients, how much they are satisfied.

Huang and Thomson (1995) gave a linear programming model for patient's waiting time. They defined the statements that what percentage of patients should wait no longer than what amount of time. They determined the patients waiting time targets in the form of following assumptions:

1. The probability of waiting times being "very satisfactory" to patients would be higher than a specified level.

2. The probability of waiting times being "satisfactory or very satisfactory" to patients would be higher than another specified level.

3. The probability of waiting times being "unsatisfactory or very unsatisfactory" to patients would not exceed another specified level.

Work done in (Huang and Thomson, 1995) was based on sharp constraints of patient's satisfaction. This results in a strict policy under which a healthcare unit should perform. Many times healthcare units may fail to function according this strict policy due to shortage of resources like doctors, nurses, technicians etc or due to the disturbance created by the medical representatives in meeting the doctors during check-up time of the patient. Real life situations, therefore, require little flexibility in the specified standards. Keeping this in view, we must have a soft policy for the healthcare units than this strict policy. For this purpose we use the lower tolerance limit of the specified aspiration of the constraint.

Further there is a common agreement that patients waiting times should be counted from a patient's appointment time instead of his arrival time, unless they arrive late. Hence, patients who arrive early would be treated as if they came on time. The treatment of late patients is not as simple as it seems. It may be argued that patients should have to wait as long as necessary if they miss their appointments. It is obviously not reasonable to keep a patient waiting for 2 hours, should the appointment be missed by few minutes. Therefore, attitudes towards waiting among the late patients should also be considered. There is evidence that patients who miss their appointments would be willing to wait longer but not indefinitely (Huang, 1994).

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We use following notations to formulate the model:

x_j j^{th} waiting time limits where $j = \{1,2,\ldots n\}$;

x_m the maximum time that a patient can possibly wait;

P_{1j} the minimum probability that punctual patients do not have to wait longer than $x_j\,(j=1,2,\ldots,n-1)$;

P_{1n} the maximum probability that punctual patients have to wait longer than x_n;

P_{2j} the minimum probability that late patients do not have to wait longer than $x_j\,(j=1,2,\ldots,n-1\}$;

P_{2n} the maximum probability that punctual patients have to wait longer than x_n;

$\tilde{A}$ represents the fuzzy set for the concept that the waiting time is "very satisfactory";

$\tilde{B}$ represents the fuzzy set for the concept that the waiting time is " satisfactory or very satisfactory";

$\tilde{C}$ Represents the fuzzy set for the concept that the waiting time is "at least little satisfactory (little satisfactory or satisfactory or very satisfactory";

f_A specified minimum probability that waiting times are "very satisfactory" for patients;

f_B specified minimum probability that waiting times are "satisfactory or very satisfactory" for the patients;

f_C specified minimum probability that waiting times are "at least little satisfactory" for the patients;

p_{A1} The lower tolerance of the probability that waiting times are "very satisfactory" for the punctual patients;

p_{A2} The lower tolerance of the probability that waiting times are "very satisfactory" for the late patients;

p_{B1} The lower tolerance of the probability that waiting times are "satisfactory or very satisfactory" for the punctual patients;

p_{B2} The lower tolerance of the probability that waiting times are "satisfactory or very satisfactory" for the late patients;

p_{C1} The lower tolerance of the probability that waiting times are "at least little satisfactory (little satisfactory or satisfactory or very satisfactory)" for the punctual patients;

p_{C2} The lower tolerance of the probability that waiting times are "at least little satisfactory (little satisfactory or satisfactory or very satisfactory)" for the late patients;

Following probabilities are also used to measure the degrees of satisfaction for punctual and late patients:

$P_1(A)$ The probability that waiting times would be "very satisfactory" to punctual patients if the waiting time targets are achieved;

$P_1(B)$ The probability that waiting times would be "satisfactory or very satisfactory" to punctual patients if the waiting time targets are achieved;

$P_1(C)$ The probability that waiting times would be "at least little satisfactory" to punctual patients if the waiting time targets are achieved;

$P_2(A)$ The probability that waiting times would be "very satisfactory" to late patients if the waiting time targets are achieved;

$P_2(B)$ The probability that waiting times would be "satisfactory or very satisfactory" to late patients if the waiting time targets are achieved;

$P_2(C)$ The probability that waiting times would be "at least little satisfactory" to late patients if the waiting time targets are achieved;

Now in order to form a fuzzy mathematical model our aim is to minimize (relax) the patient's waiting time targets keeping attention on the patient satisfaction as well as healthcare units' performance. The probabilities $P_{ij}\,(i=1, 2; j = 1, 2,\ldots n)$ are the decision variables. The fuzzy model is described as

$$\text{Minimize } \sum_{i=1}^{2} P_{i1} + w_{i1}P_{i1} + w_{i1}P_{i1} - w_{i1}P_{i1}$$

Where w_{ij} is a weight that indicates the relative importance of reducing a unit of P_{ij} to the relaxation of waiting time targets?

3.1 Soft Constraints

The fuzzy constraints in the model to compute the probabilities P_{ij} on the basis of the patients' satisfactions are specified as follow

$P_1(A) \leq f_{A'}, \quad P_2(A) \geq f_{A'}, \; P_1(B) \leq f_{B'}, P_2(B) \geq f_{B'}, P_1(C) \leq f_{C'}, P_2(C) \geq f_{C'},$

3.2 Hard Constraints

Except these constraints on patients' satisfactions, there are other constraints also which need to be considered. Having defined the P_{ij} as decision variables, we have crisp constraints

$$P_{i(j-1)} - P_{ij} \leq 0 \; (i = 1,2; j = 1,2,\ldots,n-1).$$

To make the waiting time standards consistent in the sense that waiting time targets should be set in such way that punctual patients will not be negatively discriminated in any circumstances, we have

$$P_{1j} - P_{2j} \geq 0 \; (j = 1,2,\ldots,n-1),$$
$$P_{1n} - P_{2n} \leq 0,$$
$$P_{1(n-1)} + P_{in} \leq 1 \quad (i = 1,2) \text{ and } P_{ij} \geq 0, \; P_{ij} \leq 1,$$

So, we have formulated the fuzzy model consisting the objective function, soft constraints and hard constraints. Now we give the fuzzy sets for the linguistic terms "very satisfactory", "satisfactory or very satisfactory" and "at least little satisfactory".

3.3 Determination of Linguistic Terms

The membership functions of the linguistic terms **"**very satisfactory", "satisfactory or very satisfactory" and "little satisfactory or satisfactory or very satisfactory" for punctual and late patients can be approximated by the semi normal functions [5, 12]. These are given as follows:

The membership function of "very satisfactory" for punctual patient is defined by

$$\mu^1_{\tilde{A}}(x) = \begin{cases} 1 & if \; x \leq 2 \\ e^{\left[-0.001204(x-2)^2\right]} & if \; x > 2 \end{cases}.$$

The membership function of "satisfactory or very satisfactory" for punctual patient is defined by

$$\mu^1_{\tilde{B}}(x) = \begin{cases} 1 & if \; x \leq 5 \\ e^{\left[-0.000794(x-5)^2\right]} & if \; x > 5 \end{cases}.$$

The membership function of "unsatisfactory or very unsatisfactory" for punctual patient is defined by

$$\mu^1_{\tilde{C}}(x) = \begin{cases} 1 & if \; x \leq 13 \\ e^{\left[-0.000398(x-13)^2\right]} & if \; x > 13 \end{cases}.$$

The membership function of "very satisfactory" for late patient is defined by

$$\mu^2_{\tilde{A}}(x) = \begin{cases} 1 & if \; x \leq 4 \\ e^{\left[-0.000395(x-4)^2\right]} & if \; x > 4 \end{cases}.$$

The membership function of "satisfactory or very satisfactory" for late patient is defined by

$$\mu^2_{\tilde{B}}(x) = \begin{cases} 1 & if \; x \leq 13 \\ e^{\left[-0.000372(x-13)^2\right]} & if \; x > 13 \end{cases}.$$

The membership function of "unsatisfactory or very unsatisfactory" for late patient is defined by

$$\mu^2_{\tilde{C}}(x) = \begin{cases} 1 & if \; x \leq 20 \\ e^{\left[-0.000213(x-20)^2\right]} & if \; x > 20 \end{cases}.$$

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4. Numerical Computations

In the present section an example has been used to demonstrate the application of the fuzzy linear programming approach to set waiting time targets for our patients in a management model for Medical Diagnosis.

In the model formulated in above, let us first assume that waiting time limits are specified as $x_1 = 15$ minutes; $x_2 = 30$ minutes; $x_3 = 45$ minutes; $x_4 = 60$ minutes; and $x_m = 120$ minutes.

The weights are chosen as $w_{i1} = 4$, $w_{i2} = 3$, $w_{i3} = 2$, $w_{i4} = 1$ for both punctual and late patients $(i = .1,2)$.

We assume that patient waiting times are uniformly distributed within each interval over [0, x_m] as divided by x_j $(j = 1,2,...,n)$, that is, $[0, x_1]$, $[x_1, x_2]$, $[x_2, x_3]$,…, $[x_n, x_m]$. The probabilities, $P_i(A)$, $P_i(B)$, $P_i(C)$ used to measure the different satisfaction levels are determined by using the following relations:

$$P_i(A) = \frac{P_{i1}}{x_1}\int_0^{x_1} \mu_{\tilde{A}}^i(x)\,dx + \frac{P_{i2}-P_{i1}}{x_2-x_1}\int_{x_1}^{x_2} \mu_{\tilde{A}}^i(x)\,dx + \dots$$
$$\dots + \frac{P_{i(n-1)}-P_{i(n-2)}}{x_{n-1}-x_{n-2}}\int_{x_{n-2}}^{x_{n-1}} \mu_{\tilde{A}}^i(x)\,dx + \frac{1-P_{i(n-1)}-P_{in}}{x_n-x_{n-1}}\int_{x_{n-1}}^{x_n} \mu_{\tilde{A}}^i(x)\,dx,$$
$$+\frac{P_{in}}{x_m-x_n}\int_{x_n}^{x_m} \mu_{\tilde{A}}^i(x)\,dx \qquad (i=1,2)$$

$$P_i(B) = \frac{P_{i1}}{x_1}\int_0^{x_1} \mu_{\tilde{B}}^i(x)\,dx + \frac{P_{i2}-P_{i1}}{x_2-x_1}\int_{x_1}^{x_2} \mu_{\tilde{B}}^i(x)\,dx + \dots$$
$$\dots + \frac{P_{i(n-1)}-P_{i(n-2)}}{x_{n-1}-x_{n-2}}\int_{x_{n-2}}^{x_{n-1}} \mu_{\tilde{B}}^i(x)\,dx + \frac{1-P_{i(n-1)}-P_{in}}{x_n-x_{n-1}}\int_{x_{n-1}}^{x_n} \mu_{\tilde{B}}^i(x)\,dx$$
$$+\frac{P_{in}}{x_m-x_n}\int_{x_n}^{x_m} \mu_{\tilde{B}}^i(x)\,dx \qquad (i=1,2)$$

$$P_i(C) = \frac{P_{i1}}{x_1}\int_0^{x_1} \mu_{\tilde{C}}^i(x)\,dx + \frac{P_{i2}-P_{i1}}{x_2-x_1}\int_{x_1}^{x_2} \mu_{\tilde{C}}^i(x)\,dx + \dots$$
$$\dots + \frac{P_{i(n-1)}-P_{i(n-2)}}{x_{n-1}-x_{n-2}}\int_{x_{n-2}}^{x_{n-1}} \mu_{\tilde{C}}^i(x)\,dx + \frac{1-P_{i(n-1)}-P_{in}}{x_n-x_{n-1}}\int_{x_{n-1}}^{x_n} \mu_{\tilde{C}}^i(x)\,dx.$$
$$+\frac{P_{in}}{x_m-x_n}\int_{x_n}^{x_m} \mu_{\tilde{C}}^i(x)\,dx \qquad (i=1,2)$$

Obviously, it is difficult to solve the integrals for the above defined membership functions. We compute the integrals by replacing the original membership function with its Taylor series expansion about the midpoint of the corresponding integral interval. The higher the order of the Taylor series, more precisely the membership function is approximated within interval. The membership function is approximated as follows:

Let x_0 be the middle point of the interval $[x_L, x_H]$ i.e. $x_0 = \frac{1}{2}[x_L, x_H]$.

In our model, the membership functions defined are of type

$$f(x) = e^{-k(x-a)^2}$$

and the Taylor series expansion of any function about x_0 is

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2} f'(x_0) + \ldots$$

$$e^{-k(x-a)^2} = e^{-k(x-a)^2} - 2k(x_0 - a)e^{-k(x-a)^2}(x - x_0)$$
$$+ k(x_0 - a)e^{-k(x-a)^2}[2k(x_0 - a)^2 - 1](x - x_0)^2 + \ldots$$

Using third order approximation and integrating the function we get

$$\int_{x_L}^{x_H} e^{-k(x-a)^2} dx = e^{-k(x-a)^2}$$
$$\times \left\{ (x_H - x_L) + \frac{k}{3}[2k(x_0 - a)^2 - 1][(x_H - x_0)^3 - (x_L - x_0)^3 \right\}$$

We have obtained the values of the integrals defined for $P_i(A)$, $P_i(B)$, $P_i(C)$). We present below the calculation of probability $P_1(A)$:

$$P_1(A) = \frac{P_{11}}{x_1}\int_0^{x_1} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{12} - P_{11}}{x_2 - x_1}\int_{x_1}^{x_2} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{13} - P_{12}}{x_3 - x_2}\int_{x_2}^{x_3} \mu_{\tilde{A}}^1(x)\,dx$$
$$+ \frac{1 - P_{13} - P_{14}}{x_4 - x_3}\int_{x_3}^{x_4} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{14}}{x_m - x_4}\int_{x_4}^{x_m} \mu_{\tilde{A}}^1(x)\,dx$$

In our model

$$x_1 = 15, x_2 = 30, x_3 = 45, x_4 = 60, x_m = 120,$$

$$P_1(A) = \frac{P_{11}}{15}\int_0^{15} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{12} - P_{11}}{30 - 15}\int_{15}^{30} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{13} - P_{12}}{45 - 30}\int_{30}^{45} \mu_{\tilde{A}}^1(x)\,dx$$
$$+ \frac{1 - P_{13} - P_{14}}{60 - 45}\int_{45}^{60} \mu_{\tilde{A}}^1(x)\,dx + \frac{P_{14}}{120 - 60}\int_{60}^{120} \mu_{\tilde{A}}^1(x)\,dx$$

The membership function is given by

$$\mu_{\tilde{A}}^1(x) = 1 \qquad if\ x \le 2, \quad \mu_{\tilde{A}}^1(x) = e^{\left[-0.001204(x-2)^2\right]} \quad if\ x > 2$$

$$\int_0^2 1\,dx = 2$$

Similarly other calculations can be made.
Putting the values of the integrals in above relation, we get

$$P_1(A) = \frac{P_{11}}{15} \times 2 + \frac{P_{11}}{15} \times 12.1668 + \frac{P_{12} - P_{11}}{15} \times 9.0438 + \frac{P_{13} - P_{12}}{15} \times 3.4405$$
$$+ \frac{1 - P_{13} - P_{14}}{15} \times 0.7767 + \frac{P_{14}}{60} \times 0.03950,$$

$$P_1(A) = 0.1333P_{11} + 0.811P_{11} + (P_{12} - P_{11}) \times 0.6029 + (P_{13} - P_{12}) \times 0.2294$$
$$+ (1 - P_{13} - P_{14}) \times 0.05178 + 0.000658P_{14}.$$

Hence we have,

$$P_1(A) = 0.3415P_{11} + 0.3735P_{12} + 0.1776P_{13} - 0.05110P_{14} + 0.05178.$$

Similarly, we compute other probabilities.

$$P_2(A) = 0.1197P_{21} + 0.2274P_{22} + 0.2440P_{23} - 0.3126P_{24} + 0.3974,$$
$$P_1(B) = 0.2046P_{11} + 0.3415P_{12} + 0.2635P_{13} - 0.1621P_{14} + 0.1731,$$
$$P_2(B) = 0.03918P_{21} + 0.1639P_{22} + 0.23649P_{23} - 0.4082P_{24} + 0.5603,$$
$$P_1(C) = 0.0419P_{11} + 0.1736P_{12} + 0.2461P_{13} - 0.4019P_{14} + 0.5384,$$
$$P_2(C) = 0.0047P_{21} + 0.0616P_{22} + 0.1368P_{23} - 0.4201P_{24} + 0.7968.$$

Our aim is to get patients waiting time targets subjected to the following:

(i) The probability that waiting times are "very satisfactory" to patients should be essentially greater than or equal to *0.5*.

(ii) The probability that waiting times are "satisfactory or very satisfactory" to patients should be essentially greater than or equal to *0.75*.

(iii) The probability that waiting times are "at least little satisfactory" essentially greater than or equal to 0.97.

Now the fuzzy linear model for waiting time targets is given as

Minimize $4P_{11} + 3P_{12} + 2P_{13} - P_{14} + 4P_{21} + 3P_{22} + 2P_{23} - P_{24}$

Subject to

$$0.3415P_{11} + 0.3735P_{12} + 0.1776P_{13} - 0.05110P_{14} \geq 0.4482,$$
$$0.1197P_{21} + 0.2274P_{22} + 0.2440P_{23} - 0.3126P_{24} \geq 0.1026,$$
$$0.2046P_{11} + 0.3415P_{12} + 0.2635P_{13} - 0.1621P_{14} \geq 0.5769,$$
$$0.0392P_{21} + 0.1639P_{22} + 0.2365P_{23} - 0.4082P_{24} \geq 0.1897,$$
$$0.0419P_{11} + 0.1736P_{12} + 0.2461P_{13} - 0.4019P_{14} \geq 0.4316,$$
$$0.0047P_{21} + 0.0616P_{22} + 0.1368P_{23} - 0.4201P_{24} \geq 0.1732,$$

$P_{i(j-1)} - P_{ij} \leq 0$ $(i = 1,2;\ j = 1,2,\ldots,n-1)$,
$P_{1j} - P_{2j} \geq 0$ $(j = 1,2,\ldots,n-1)$,
$P_{1n} - P_{2n} \leq 0$,
$P_{1(n-1)} + P_{in} \leq 1$ $(i = 1,2)$,
and $P_{ij} \geq 0$, $P_{ij} \leq 1$,

solving the above fuzzy program similar to above calculations. We get the optimal solution, $Z_u = 9.9088$ and by taking the lower tolerance limits $p_{A1} = p_{A2} = p_{B1} = p_{B2} = p_{C1} = p_{C2} = 0.05$ and similar the optimal solution, $Z_l = 6.1428$.

CONCLUSION

In the present paper, we have discussed a fuzzy linear programming model to improve the quality management of medical diagnosis for providing better service to patients. Constraints for the satisfaction level of patients about their waiting time are considered in fuzzy sense. To make the waiting time standards consistent, some crisp constraints are also considered. All these constraints are taken for punctual and late patients simultaneously. The result of our model as illustrated in the numerical example reflects an optimal policy of waiting time for patients when they come on their appointment time or become late. This suggests the healthcare units that almost all punctual patients should be checked up within half an hour and most of the late patients should be checked up within thirty nine minutes. The vague model proposed in this paper is more realistic than the crisp one given for the reason that sharp boundaries of the constraint for patient's satisfaction level is hard to implement. Our model uses an aspiration level which is within the tolerable limits of patient's satisfaction. Therefore our model is more credible and reliable for management in medical diagnosis.

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