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Research Article

RADIATION AND MASS TRANSFER EFFECTS ON MHD FREE CONVECTION FLOW PAST A VERTICAL PLATE WITH VARIABLE TEMPERATURE AND CONCENTRATION

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ABSTRACT

In this assertion, the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field has been investigated. The fluid is considered here as absorbing/emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and the concentration level near the plate is raised to C'_∞.
We use proper transformations to make the governing equations dimensionless. Then the dimensionless governing equations are reduced to a set of ordinary differential equations. We solve these equations with the help of transformed boundary conditions. The effect of various parameters such as Gr (Grashof number), Gm (Mass Grashof number), Sc (Schmidt number), Pr (Prandtl number), R (Radiation parameter), M (Magnetic field parameter), t (time) and a (accelerating parameter) on velocity profiles, temperature profiles, concentration profiles, skin friction profiles, rate of heat transfer profiles and rate of mass transfer profiles are shown graphically.

Key Words: *Grash of number, Mass Grash of number, Radiation parameter, MHD, Vertical plate*

INTRODUCTION

Natural convection induced by the simultaneous action of buoyancy forces from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. The effect of the magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gases. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. When free convection flows occur at high temperature, radiation effects on the flow become significant. Many processes in engineering areas occur at high temperatures and knowledge of radiative heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles are examples of such engineering areas.

Muthucumaraswamy *et al.,* (2008) studied mass transfer effects on exponentially accelerated isothermal vertical plate. Recently the thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion has been studies by Muthucummaraswamy and Vislakshi (2011).

Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate and solve the governing equations analytically. Muthucumaraswamy and Janakiraman (2006) studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Suneetha and Bhaskar (2011) formulated in an (x, y, t) coordinate system with appropriate boundary conditions. Effects on boundary layer flow and heat transfer of a fluid with variable viscosity along a symmetric wedge is presented here by Mukhopadhyay (2009). Rajput and Surendra (2011) studied the MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Theoretical solution of unsteady flow past uniformly accelerated infinite vertical plate has been presented Muthucumaraswamy *et al.,* (2009) in the presence of variable temperature and uniform mass diffusion. Rajesh (2010) discussed the effect of a uniform transverse magnetic field on the free convection and mass- transform flow of an electrically- conducting fluid past an exponentially accelerated infinite vertical plate through a porous medium. Sing *et al.,* (2003) studied the two dimensional free convection and mass transfer flow of an incompressible viscous and a continuously moving infinite vertical porous plate in the presence of heat source, thermal diffusion, large suction and under the influence of uniform magnetic field applied normal to the flow is studied and perturbation technique is used to solve the governing equation. Mehmood and Ali (2007) studied the effect of the wall slip on velocity field. By Rajesh and Vijaya (2009) an analytical study is performed to study the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field.

We study the effects of thermal radiation and mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate with variable temperature and concentration in the presence of magnetic field. We use proper transformations to make the governing equations dimensionless. Then the dimensionless governing equations are reduced to a set of ordinary differential equation. We solve these equations with the help of transformed boundary conditions.\

MATERIALS AND METHODS

Mathematical Analysis

The unsteady flow of an incompressible and electrically conducting viscous fluid past an infinite vertical plate with variable temperature and variable concentration has been considered. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed be in x'-direction which is taken along the vertical plate in the up ward direction and y'-axis is taken to be normal to the plate. Initially the plate and the fluid are assumed at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all points. At time $t'>0$, the plate is exponentially accelerated with a velocity $u=u_0e^{a't'}$ is its own plane and the plate temperature and concentration are raised linearly with the time t and the fluid considered here is a absorbing or emitting radiation but a non-scattering medium. The viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2u'}{\rho} \quad (1)$$

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'} \quad (2)$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

with the boundary conditions

$$\begin{aligned}&t'>0,\ u'=u_0\exp(a't'),\ T'=T'_\infty+(T'_w-T'_\infty)At',\ C'=C'_\infty+(C'_w-C'_\infty)At' \text{ at } y'=0\\ &u'=0, T'\to T'_\infty, C'\to C'_\infty \text{ as } y'\to\infty\end{aligned} \quad (4)$$

Now we consider the following transformations to make non dimensional of the equations (1) to (3) with boundary condition (4)

$$\begin{aligned}&u=\frac{u'}{u_0}, t=\frac{t'u_0^2}{\nu}, y=\frac{y'u_0}{\nu}, \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}, Gr=\frac{g\beta\nu(T'_w-T'_\infty)}{u_0^3}, \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty},\\ &Gm=\frac{g\beta^*\nu(C'_w-C'_\infty)}{u_0^3}, \Pr=\frac{\mu C_p}{k}, Sc=\frac{\nu}{D}, M=\frac{\sigma B_0^2\nu}{\rho u_0^2}, R=\frac{16a^*\nu^2\sigma T'_\infty}{ku_0^2},\\ &a=\frac{a'\nu}{u_0^2}\end{aligned} \quad (5)$$

Here, u', u_0, ν, T'_w, g , β, β^* , C', C'_w, C'_∞, μ, D, w and ∞ are velocity of the fluid in the x'-direction, velocity of the plate, kinematic viscosity, temperature of the plate, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of expansion with concentration, species concentration in the fluid, concentration of the plate, concentration of the fluid far away from the plate, coefficient of viscosity, chemical molecular diffusivity, conditions on the wall and free stream conditions respectively.

where $A = e^{a't'}/t$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma(T_\infty'^4 - T'^4) \tag{6}$$

where, q_r, y', $a*$, σ and T' are radiative heat flux in the y' direction, coordinate axis normal to the plate, absorption coefficient, electric conductivity and temperature of the fluid near the plate respectively.

It is assumed that the temperature differences with in the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \tag{7}$$

By using equations (6) and (7), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma T_\infty'^\infty (T'_\infty - T') \tag{8}$$

where, ρ, C_p, t' and k are density of the fluid, specific heat at constant pressure, time and thermal conductivity of the fluid respectively.

Thus using the transformations of (5) the reduced dimensionless equations (1) to (3) take the following form

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \tag{11}$$

Now the boundary conditions (4) are also transformed to the following form

$$t > 0: u = \exp(at), \theta = \exp(at), \phi = \exp(at) \text{ at } y = 0 \tag{12}$$

$$u = 0, \theta \to 0, \phi \to 0 \text{ as } y \to \infty$$

Let us consider the solution of the equations (9) to (11) be of the form $u = u_0 e^{at}$, $\theta = \theta_0 e^{at}$ and $\phi = \phi_0 e^{at}$ respectively. By using the above u, θ and ϕ, the equations (9) to (11) become

$$u_0'' - (M + a)u_0 + Gr\theta_0 + Gm\phi_0 = 0 \tag{13}$$

$$\theta_0'' - \Pr(\frac{R}{\Pr} + a)\theta_0 = 0 \tag{14}$$

$$\frac{1}{Sc}\phi_0'' - a\phi_0 = 0 \tag{15}$$

The boundary conditions of the equation (12) is reduced to the following forms

$$t > 0: u_0 = 1, \theta_0 = 1, \phi_0 = 1 \text{ at } y = 0$$

$$u_0 = 0, \theta_0 = 0, \phi_0 = 0 \text{ as } y \to \infty$$

The equations (13) to (15) are simply ordinary differential equations. The solutions of these equations can be found easily.

Let us consider m_1 = Pr(R/Pr+a) and m_2 = (M+a) then the solutions of equations (13) to (15) can be written as

$$u_0 = (1 + \frac{Gr}{m_1 - m_2} + \frac{Gm}{aSc - m_2})e^{-\sqrt{m_2}y} - Gr\frac{1}{m_1 - m_2}e^{-\sqrt{m_1}y} - Gm\frac{1}{aSc - m_2}e^{-\sqrt{aSc}y}$$

$$\theta_0 = e^{-\sqrt{m_1}\,y}$$

$$\phi_0 = e^{-\sqrt{aSc}\,y}$$

Finally the velocity field, temperature field and concentration field are as follows

$$u=(1+\frac{Gr}{m_1-m_2}+\frac{Gm}{aSc-m_2})e^{at-\sqrt{m_2}\,y}-Gr\frac{1}{m_1-m_2}e^{at-\sqrt{m_1}\,y}-Gm\frac{1}{aSc-m_2}e^{at-\sqrt{aSc}\,y} \quad (16)$$

$$\theta=e^{at-\sqrt{m_1}\,y} \quad (17)$$

$$\phi=e^{at-\sqrt{aSc}\,y} \quad (18)$$

Now we want to calculate the skin friction, the rate of heat transfer (Nusselt number) and the rate of mass transfer (Sherwood number). For this purpose we differentiate u, θ and ϕ with respect to y and then for $y = 0$ we can write, where y is the dimensionless coordinate axis normal to the plate

$$\text{The skin friction } \tau=-\sqrt{m_2}(1+\frac{Gr}{m_1-m_2}+\frac{Gm}{aSc-m_2})e^{at}+\frac{Gr\sqrt{m_1}}{m_1-m_2}e^{at}+\frac{Gm\sqrt{aSc}}{aSc-m_2}e^{at} \quad (19)$$

$$\text{The rate of heat transfer } Nu=-\sqrt{m_1}\,e^{at} \quad (20)$$

$$\text{The rate of mass transfer } Sh=-\sqrt{aSc}\,e^{at} \quad (21)$$

RESULTS AND DISCUSSION

The numerical values of the velocity, temperature, concentration, skin friction, rate of heat transfer and rate of mass transfer are computed for different physical parameters like Magnetic field parameter (M), Radiation parameter (R), Schmidt number (Sc), Prandtl number (Pr) , Thermal Grashof number (Gr), Mass Grashof number (Gm), accelerating parameter (a) and time (t). The purpose of the calculations are given here is to study the effects of the parameters M, R, Sc, Pr, Gr, Gm, a and t upon the nature of the flow and transport.

The velocity profiles for different parameters M, R, t, Sc, Pr and Gr are presented in figures 1 to 6 for the cases of heating (Gr<0, Gm<0) and cooling (Gr>0, Gm>0) of the plate. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient. Figure 1 represents variation in the velocity field for different values of M in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plates with Pr = 0.71, R = 4, a = 0.5, Sc = 2.01 and t = 0.2. It is observed that for an externally cooled plate, an increase in M, the velocity field decreases. For an externally heated plate the results are observed in reverse order. In the Figure 2 we see that increasing values of R decrease the velocity field in the cooling plate for R = 2, 15, 25, 35 and the velocity goes to its maximum position near the plate and then decays to zero asymptotically as $y\to\infty$. But the reverse results are shown in the heating plate. It is clear in the Figure 3 that in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plates and for M = 1, Pr = 0.71, Sc = 2.01, a = 0.5 and R = 4 the velocity field is increasing and decreasing respectively with respect to the increasing values of t. The variation of velocity field for various values of Sc in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plate with M = 1, Pr =0.71, a = 0.5, R = 4 and t = 0.2 are given in Figure 4. Here we choose Sc = 0.22 (Hydrogen), Sc = 0.30(Helium), Sc = 0.60(water vapor) and Sc = 0.78(Ammonia). It is noted that velocity field is decreasing with the increasing values of Sc in the cooling plate. But in the heating plate the velocity profile is increasing for the increasing values of Sc. We observe in Figure 5 that velocity field is decreasing with an increase in Pr in the cooling plate. The reverse effects are shown in the heated plate for Pr = 0.01, 0.05, 0.71 and 7.00. Figure 6 depicts the variation of velocity field with respect to the increasing values of Gr both in the cooled and heated plate with M =1, Pr = 0.71, Sc = 3.01, a = 0.5, R = 4, t = 0.2 and Gm = 10. Here we see that velocity is increasing with an increase in Gr for the cooling plate. The reverse effects are seen in the heated plate.

The temperature profiles for different parameters R and Pr are presented in figures 7 to 8. By Figure 7 we observe that an increase in R decreases the temperature field and then goes to zero for large values of y. Figure 8 represents variation in the temperature field for various values of Pr. It is observed that an increase in Pr the temperature field decreases. It is interesting to note that the temperature decreases rapidly with increase in Pr.

The concentration profiles for various parameters Sc and a are presented in figures 9 to 10. In the Figure 9 we see that concentration field is decreasing with an increasing value of Sc. Physically, the increase of Sc means decrese of molecular diffusivity (D).

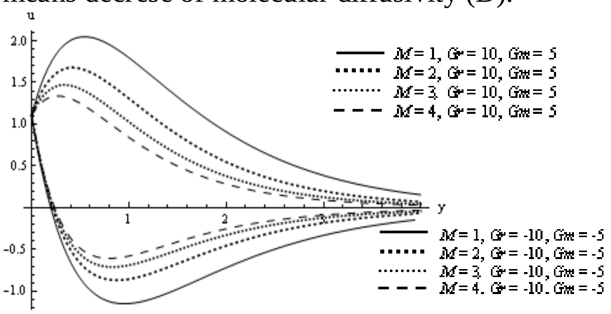

Figure 1: Velocity profile u for different values of M with R = 4, P r= 0.71, a = 0.5, Sc = 2.01 and t = 0.2 against y

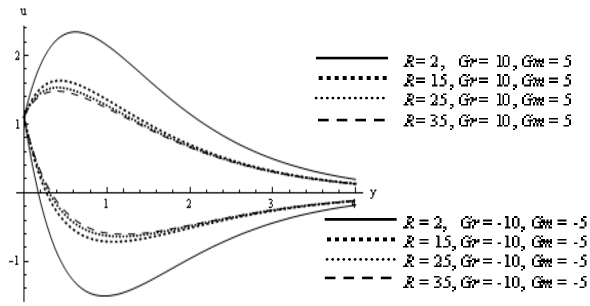

Figure 2: Velocity profile u for different values of R with M = 1, Pr = 0.71, a = 0.5, Sc = 2.01 and t = 0.2 against y

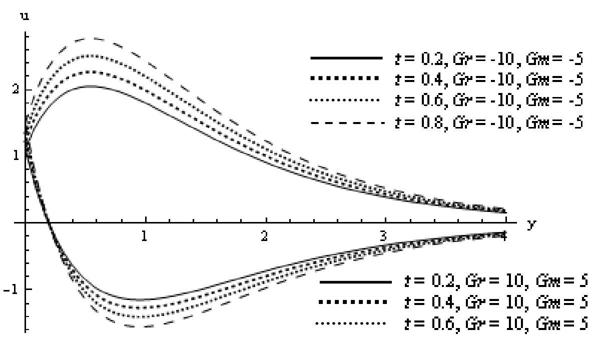

Figure 3: Velocity profile u for different values of t with M = 1, Pr = 0.71, Sc = 2.01, a = 0.5, and R = 4 against y

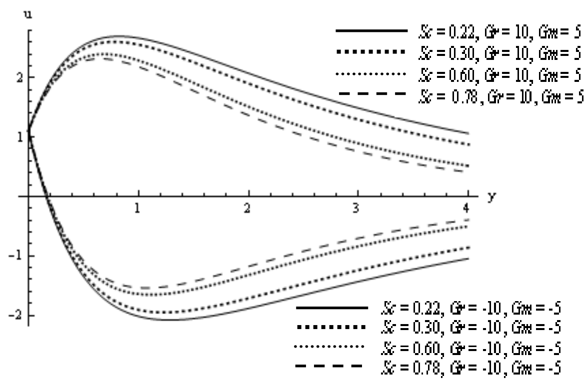

Figure 4: Velocity profile u for different values of Sc with M = 1, Pr = 0.71, a = 0.5, R = 4 and t = 0.2 against y

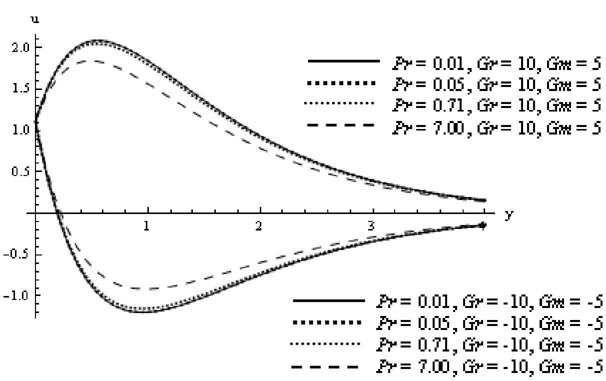

Figure 5: Velocity profile u for different values of Pr with M = 1, Sc = 2.01, a = 0.5, R = 4 and t = 0.2 against y

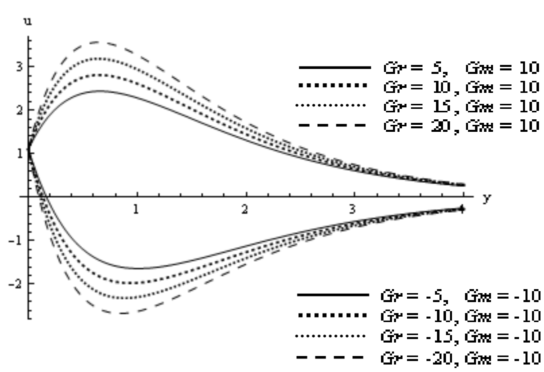

Figure 6: Velocity profile u for different values of Gr with M = 1, Pr = 0.71, Sc = 2.01, a = 0.5 , R = 4 and t = 0.2 against y

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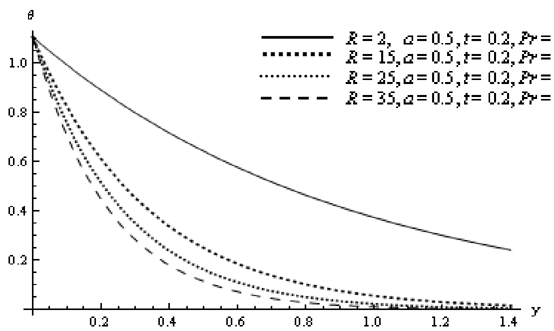

Figure 7: Dimensionless temperature profiles Θ for different values of R against y

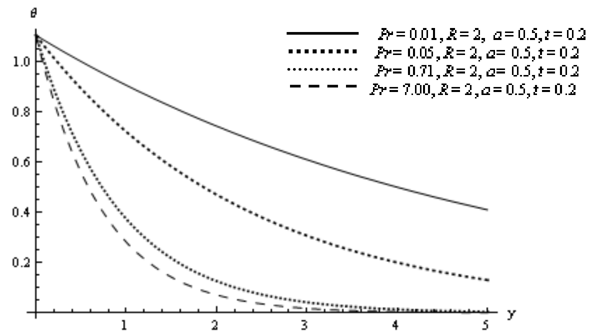

Figure 8: Dimensionless temperature profiles Θ for different values of Pr against y

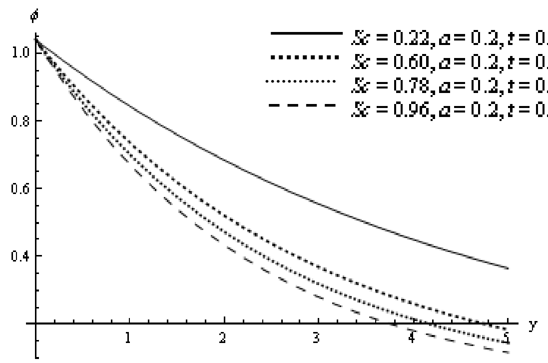

Figure 9: Dimensionless concentration profiles ϕ for different values of Sc against y

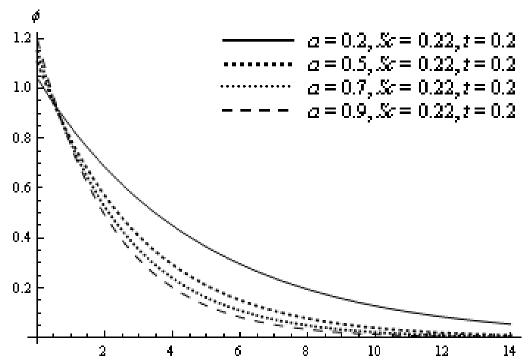

Figure 10: Dimensionless concentration profiles ϕ for different values of a against y

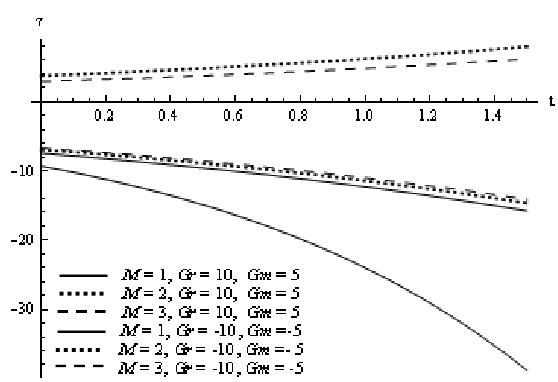

Figure 11: The skin friction profile τ for different values of M with R = 4, a = 0.5, Sc = 0.22 and Pr = 0.71 against t

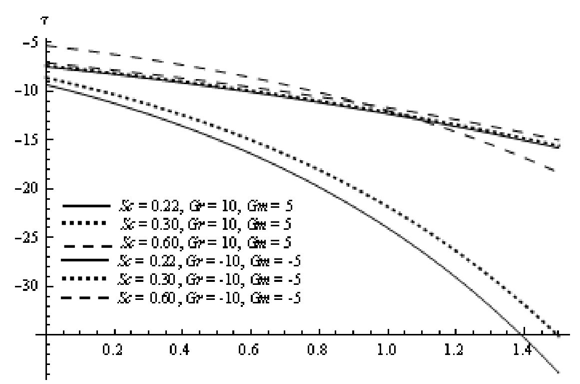

Figure 12: The skin friction profile τ for different values of Sc with R = 4, M = 1, a = 0.5 and Pr = 0.71 against t

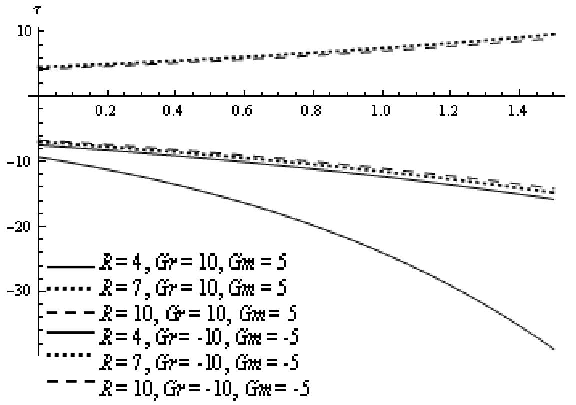

Figure 13: The skin friction profile τ for different values of R with a = 0.5, M = 1, Sc = 0.22 and Pr = 0.71 against t

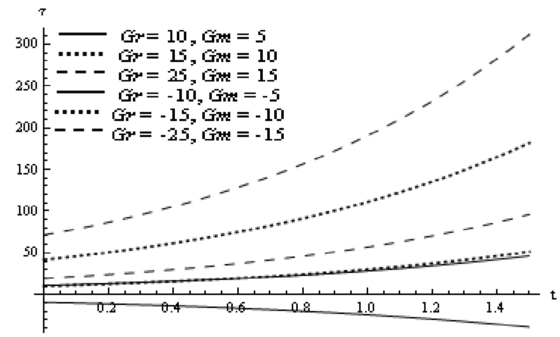

Figure 14: The skin friction profile τ for different values of Gr, Gm with R = 4, M = 1, Sc = 0.22, Pr = 0.71 and a = 0.5 against t

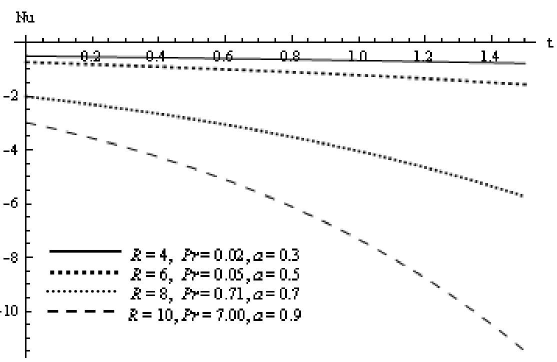

Figure 15: Rate of heat transfer profiles Nu for different values of R, Pr and a against t

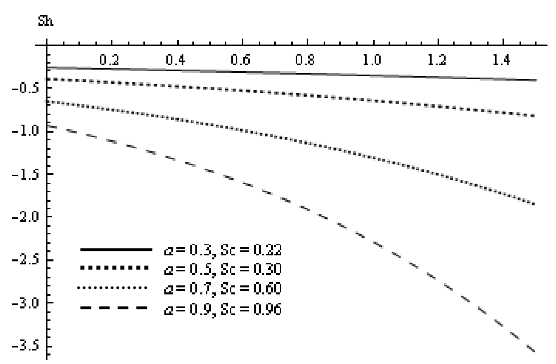

Figure 16: Rate of mass transfer Sh for different values of a and Sc against t

That results in decrease of concentration boundary layer. Hence the concentration of the species is higher for small values of Sc and lower for large values of Sc. The maximum values of concentration are found at the plate. Figure 10 represents that an increase in a increases the concentration field at the plate to $y<0.5$, but for $y \geq 0.5$ plate, the concentration field is decreasing with the increasing values of a and then decays to zero as $y \rightarrow \infty$.

The skin friction profiles for different parameters M, Sc, R, Gr and Gm are presented in figures 11 to 14 for the cases of heating (Gr<0, Gm<0) and cooling (Gr>0, Gm>0) of the plate. In Figure 11 it is shown that skin friction is increasing with the increasing values of M. But at M = 2 the skin fiction goes to its maximum position. These effects are shown in the cooling plate (Gr = 10, Gm = 5) with the increasing values of M. The same effects are also shown in the heating plate (Gr = -10, Gm = -5) with the increasing values of M. Figure 12 depicts the effects of Sc on skin friction profiles. Here we choose Sc = 0.22, 0.30 and 0.60 for both in cooling and heating plate. Here the skin frictions increase with an increase in Sc. These effects are same both in cooling and heating plate. In the cooling plate skin frictions increase with an increase in R. But at R = 7 skin friction increases more than at R = 10. The skin friction increases randomly in the heating plate with an increase in R. All these effects are observed at Figure 13 We conclude from Figure 14 that skin friction increases for increasing values of Gm and Gr in the cooling plate. The same effects are also shown in the heated plate. In the Figure 15 we see that for increasing values of R, Pr and a the rate of heat transfer profiles are decreased and gradually goes to infinity as $t \to \infty$. In the Figure 16 we observe that the rate of mass transfer is decreased with an increase in a and Sc.

Conclusion

In this assertion, the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated vertical plate with mass transfer in the presence of magnetic field are studied. In this study, the following conclusions are set out.

In case of cooling of the plate (Gr, Gm >0), the velocity decreases with an increase in M, R, Sc and Pr. But the velocity is increases with an increase in Gr and t. In case of heating of the plate (Gr, Gm <0), the velocity increases with an increase in M, R, Sc and Pr. But the velocity is decreases with an increase in Gr and t.

The temperature decreases with an increase in the value of R and Pr.

The concentration decreases with an increase in the values of Sc and a.

Skin friction increases with an increase in M, Sc, R, Gr and Gm for cooling of the plate. The same effects are also shown for heating of the plate.

The Nusselt number decreases with an increase in the value of R, Pr and a.

The Sherwood number decreases with an increase in the values of a and Sc.

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