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Research Article

VAGUE GROUP WITH MEMBERSHIP AND NON MEMBERSHIP FUNCTION

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ABSTRACT
In this paper we defines Vague groups (VG) and some characterization of them with some numerical examples.

Key Words: *Membership Function, Non Membership Function, Vague Set, Group, Vague Group, Vague - cut, Vague - cut Group*

INTRODUCTION
In his paper, Zadeh (1965) introduced the fuzzy set theory and in his paper Gau and Buehrer (1993) defines vague theory. In an analogous application with groups, Rosenfeld (1971) formulated the elements of a theory of fuzzy groups. In most cases of judgements, evaluation is done by human beings (or by an intelligent agent) where there certainly is a limitation of knowledge or intellectual functionaries. Naturally, every decision- maker hesitates more or less, on every evaluation activity. To judge whether a patient has cancer or not, a doctor (the decision-maker) will hesitate because of the fact that a fraction of evaluation he thinks in favor of truthness, another fraction in favor of falseness and rest part remains undecided to him. This is the breaking philosophy in the notion of vague set theory introduced by Gau and Buehrer (1993). In this paper we introduce a notion of vague algebra by defining vague groups of a group, with some numerical examples.

Preliminaries
In this section, we present now some preliminaries on the theory of vague sets (VS). In his pioneer work, Zadeh (1965) proposed the theory of fuzzy sets. Since then it has been applied in wide verities of fields like Computer Science, Management Science, Medical Sciences, Engineering problems etc. to list a few only.

Let $U = \{x_1, x_2, x_3, ..., x_n\}$ be the universe of discourse. The membership function for fuzzy sets can take any value from the closed interval $[0, 1]$. Fuzzy set A is defined as the set of ordered pairs $A = \{(x, \mu_A(x))\ where\ x \in U\}$ where $\mu_A(x)$ is the grade of membership of elements x in set A. The greater $\mu_A(x)$ the greater is the truth of the statement that 'the element x belong to the set A'.

Definition 2.1
A vague set (or in short VS) A in the universe of discourse U is characterized by two functions given by:
1) A membership function $\mu_A : U \to [0 ,1]$ and
2) A non membership function $\nu_A : U \to [0\,1]$

Where $\mu_A(x)$ is a lower bound of the grade of membership of x derived from the 'evidence for x', and $\nu_A(x)$ is the lower bound on the negation of x derived from the 'evidence against x' and $\mu_A(x) + \nu_A(x) \le 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval [$\mu_A(x),\ 1 - \nu_A(x)$] of [0, 1]. This indicate that if the actual grade of membership is $\eta(x)$, then $\mu_A(x) \le \eta(x) \le 1 - \nu(x)$.

The vague set is written as $A = \{< x, [\mu_A(x), \nu(x)] >: x \in U\}$.

Where the interval [$\mu_A(x),\ 1-\nu_A(x)$] is called the vague value of x in A and is denoted by $V_A(x)$.

For example, consider a universe $U = \{TEACHER,\ DOCTER,\ ENGINEAR\}$.

A vague set A of U could be

$A = \{< TEACHER, [.6,.3] >, < DOCTER, [.3,.5], < ENGINEAR, [.4,.6] >\}$

Definition 2.2

(a) A vague set A of a set U with $\mu_A(x) = 0$ *and* $\nu_A(x) = 1\ \forall x \in U$, is called the zero vague set of U.

(b) A vague set of A of a set U with $\mu_A(x) = 1$ *and* $\nu_A(x) = 0\ \forall x \in U$, is called the unit vague set of U.

Definition 2.3

A vague set A of a set U with $\mu_A(x) = \alpha$ *and* $\nu_A(x) = 1-\alpha\ \forall x \in U$, is called the $\alpha - vague$ Set of U, where $\alpha \in [0,1]$.

Vague Group (VG)

The notion of fuzzy groups defined by Rosenfeld (1971) is the first application fuzzy set theory in Algebra. In this section we defined the notion of vague groups analogous to the idea of Rosenfeld (1971). For our discussion, we shall use the following notations on interval arithmetic.

Notations Let $I[0,1]$ denote the family of all the closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be two elements of $I[0,1]$, we call $I_1 \geq I_2$ if , $a_1 \geq a_2$ and $b_1 \geq b_2$, Similarly we define the relation $I_1 \leq I_2$ if $a_1 \leq a_2$ and $b_1 \leq b_2$.*and* $I_1 = I_2$ if $a_1 = a_2$ and $b_1 = b_2$. The relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely. We define the term 'imax' to mean s the maximum of two intervals as $i\max(I_1, I_2) = [\max(a_1, a_2), \max(b_1, b_2)]$ and $i\min(I_1, I_2) = [\min(a_1, a_2), \min(b_1, b_2)]$

The concept of 'imax' and 'imin' could be extended to define 'isup' and 'iinf' of infinite number of elements of $I[0,1]$.

It is obvious that $L = \{I[0,1], i\sup, i\inf, \leq\}$ is a lattice with universal bounds [0,0] and [1,1].

Definition 3.1

Let $(X,*)$ be a group. A vague set A of X is called vague group (VG) of X if the following conditions are true:

$\forall x, y \in X, V_A(xy) \geq \min\{V_A(x), V_A(y)\}$, *i.e*

1) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, $1-\nu_A(xy) \geq \min\{1-\nu_A(x), 1-\nu_A(y)\}$, and

2) $\mu_A(x^{-1}) \geq \mu_A(x)$,and $1-\nu_A(x^{-1}) \geq 1-\nu_A(x)$ here the element xy is stand for $x * y$

Example 3.1 Consider the group $X = \{1, \omega, \omega^2\}$ with respect to the binary operation complex number multiplication where ω is the imaginary cube root of unity. Clearly the vague set $A = \{(1,.9,.1), (\omega,.6,.2), (\omega^2,.6,.2)\}$ is a vague group of the group X.

Since $Z_3 \cong X$ (every group of order 3 is cyclic) and hence similarly we can defined vague group corresponding to every group of order 3.

Vague Group Corresponding to Order 4 Group

We know there are two group of order 4 up to isomorphism one is isomorphic to Z_4 and another is isomorphic to $Z_2 \times Z_2$ (i.e a klein 4 group). We define vague group corresponding to order 4 group is as follows.

Example3.2 Consider the group $Z_4 = \{0,1,2,3,+_4\}$ where binary operation is addition modulo 4.

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Define vague set $B = \{(0,.9,.1),(1,.6,.2),(2,.5,.3),(3,.6,.2)\}$. The above set B satisfy all the property of vague group hence B is vague group of Z_4. Similarly we can defined a vague group corresponding to every cyclic group of order 4. Because every cyclic group of order 4 is isomorphic to Z_4.

Example 3.3 Consider the group $Z_2 \times Z_2 = \{(0,0),(0,1),(1,0),(1,1)\}$ where Z_2 is addition modulo 2 group. Define the vague set $C = \{((0,0),.9,.1),((0,1),.6,.2),((1,0),.6,.1),((1,1),.6,.2)\}$. Clearly the vague set C is vague group. Similarly we can defined a vague group corresponding to every group of order 4 which is isomorphic to klein 4 group.

Vague Group Corresponding to Order 5 Group

We know every group of order 5 is cyclic and is isomorphic to Z_5.

Example3.4 Consider the group $Z_5 = \{0,1,2,3,4,+_5\}$ where binary operation is addition modulo 5. Define the vague set $D = \{(0,.9,.1),(1,.6,.2),(2,.7,.2),(3,.7,.2),(4,.6,.2)\}$ and this set D satisfy all the property of vague group and hence vague set D is a vague group corresponding to Z_5. Similarly we can defined a vague group corresponding to every group of order 5.Because every group of order 5 is isomorphic to Z_5.

Vague Group Corresponding to Order 6 Group

We know there are two group of order 6 up to isomorphism one is isomorphic to Z_6 and another is isomorphic to S_3 (the group of permutation on 3 symbol).i.e one is cyclic and another is non Ablain. We define vague group corresponding to order 6 group is as follows.

Example 3.4 Consider the group $Z_6 = \{0,1,2,3,4,5,+_6\}$ where binary operation is addition modulo 6. Define the vague set $E = \{(0,.9,.1),(1,.6,.2),(2,.7,.2),(3,.6,.1),(4,.7,.2),(5,.6,.2)\}$ and this set satisfy all the property of vague group and hence vague set E is vague group corresponding to Z_6.

Similarly we can define a vague group corresponding to cyclic group of order 6.Because every group of order 6 is isomorphic to Z_6.

Example 3.5 Consider the group $S_3 = \{I,(123),(132),(12),(13),(23)\}$ where binary operation is composition of functions. Define the set $F = \{(I,.9,.1),((123),.6,.2),((132),.6,.2),((12),.5,.3),((13),.5,.3),((23),.5,.3)\}$ and this set Satisfy all the property of vague group. Hence above vague set is vague group corresponding to the non Ablain group of order 6. We know every non Ablain group of order 6 is isomorphic to S_3. In the similar way we can defined a vague group corresponding to every non Ablain group of order 6.

The following propositions are obvious.

Proposition 3.1 If A is the vague group of a group X, then $\forall x \in X, V_A(x^{-1}) = V_A(x)$, *i.e* $\mu_A(x^{-1}) = \mu_A(x)$ and $1-\nu_A(x^{-1}) = 1-\nu_A(x)$.

Proposition 3.2 Zero vague set, unit vague set and α vague set of the group X are trivial VGs of X.

Proposition 3.3 A necessary and sufficient conditions for a vague set of group X to be a vague group of X is that $V_A(xy^{-1}) \geq \min\{V_A(x), V_A(y)\}$.

Proof: Let A be a vague group of the group X. Then $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y^{-1})\}$, $\geq \min\{\mu_A(x), \mu_A(y)\}$

Similarly

$1-\nu_A(xy^{-1}) \geq \min\{1-\nu_A(x), 1-\nu_A(y)\}$

For the converse part, suppose that A be a VS of the group X of which e is the identity element.

Now $\mu_A(yy^{-1}) \geq \min\{\mu_A(y), \mu_A(y)\}$ $\mu_A(e) \geq \mu_A(y)$. (1)

Now $\mu_A(ey^{-1}) \geq \min\{\mu_A(e), \mu_A(y)\}$ $\mu_A(y^{-1}) \geq \mu_A(y)$. (2)

From equation (1) and (2) we get $\mu_A(y^{-1}) = \mu_A(y)$

Also $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y^{-1})\}, \geq \min\{\mu_A(x), \mu_A(y)\}$

Similarly it can be proved that

$1-\nu_A(x^{-1}) \geq 1-\nu_A(x)$ and $1-\nu_A(xy) \geq \min\{1-\nu_A(x), 1-\nu_A(y)\}$.

Proposition 3.4 If A and B are two vague groups of a group X. Then $A \cap B$ is also a vague group of X.

Proof: $\mu_{A\cap B}(xy^{-1}) = \min\{\mu_A(xy^{-1}), \mu_B(xy^{-1})\}$ $\geq \min\{\min\{\mu_A(x),, \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}$

$= \min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Proposition 3.5 If $A = (x, \mu_A, \nu_A)$ is a vague group of group X, then the following holds.

1) μ_A is a fuzzy group of X ;

2) $1-\nu_A$ is a fuzzy group of X ;

Proposition 3.6 A necessary and sufficient condition for a Vague set A of a group X to be a vague group is that μ_A and $1-\nu_A$ are fuzzy groups of X.

For $\alpha, \beta \in [0,1]$ we define (α, β)-cut and $\alpha - cut$ of vague set.

Definition 3.2 $(\alpha, \beta) - cut$ **or vague-cut.**

Let A be a vague set of universe X with membership function μ_A and the non membership function ν_A. The $(\alpha, \beta) - cut$ of the vague set A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by $A_{(\alpha,\beta)} = \{x : x \in X, V_A(x) \geq [\alpha, \beta]\}$

Clearly $A_{(0,0)} = X$. The $(\alpha, \beta) - cut$ are also called vague-cuts of the vague set A.

Definition 3.3 $\alpha,-cut$ **of a vague set.**

The $\alpha,-cut$ of the vague set A is a crisp subset A_α of the set X given by $A_\alpha = A_{(\alpha,\alpha)}$.

Thus $A_0 = X$ and if $\alpha \geq \beta$ then $A_\beta \subseteq A_\alpha$ *and* $A_{(\alpha,\beta)} = A_\alpha$

Equivalently, we can define the $\alpha,-cut$ as $A_\alpha = \{x : x \in X, \mu_A(x) \geq \alpha\}$

CONCLUSION

Group theory has much application in physics, chemistry and computer science problems. In this paper we define vague groups and studied some properties of vague groups. The concept is analogous to notion of fuzzy groups introduced by Rosenfeld (1971). If the undecided part $[1-\mu_A(x)-\nu_A(x)]$ is zero $\forall x$ (of the group X), then the vague group A reduces to a Rosenfeld's fuzzy group (Rosenfeld, 1971). It is also justified here that interval-valued fuzzy sets (Zadeh, 1975) are not vague sets. In interval-valued fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the 'evidence for u' only, without considering 'evidence against u', which is not the case in vague set theory. Also we can define a vague group corresponding to every finite group.

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