International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2013 Vol. 3 (4) October-December, pp.15-22/Kalaimaran
Research Article

THEOREM ON INTERSECTION OF TWO CHORDS OF AN ELLIPSE

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ABSTRACT
Ellipse is one of the conic sections obtained when the cutting plane is inclined to the axis of a cone ($>$ $0°$ and $< 90°$). It is also defined as the locus of a point which moves in such a way that the ratio of its distance to the two fixed points (foci) is constant. The size of the ellipse is determined by the sum of these two distances which is equal to the length of the major axis. A new property for the intersection of two chords of an ellipse has been developed and presented in this article with necessary analytic geometric equations, drawings and illustrated with an example.
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Key Words: *Ellipse, Foci, Chord, Intersection of Two Chords of an Ellipse, Major Axis and Minor Axis and Co-ordinate Geometry*

INTRODUCTION

Some Definitions

Chord: A line segment connecting any two points of an ellipse (Eric, 2003).
Secant: A line segment intersecting the ellipse at two points (Eric, 2003).
Centre: The mid-point of the line segment connecting the two foci of the ellipse.
Major Axis: The longest diameter of an ellipse passing through the foci (Eric, 2003).
Minor Axis: The shortest diameter of an ellipse passing through the centre (Eric, 2003).
Foci: The set of points which is related to the construction and properties of conic sections (Eric, 2003)
Eccentricity: A quantity defined for a conic section, which can be given in terms of semi-major ‘a’ and semi-minor axes ‘b’ (Eric, 2003).

An ellipse is the set of all points in a plane such that the sum of the distances from two fixed points called foci is a constant. In the 17th century, a mathematician Mr. Johannes Kepler discovered that the orbits of all the planets around the sun are elliptical with sun as one of the foci. Later, Isaac Newton explained that this as a corollary of his law of universal gravitation. One of the physical properties of ellipse is that sound or light rays emanating from one focus will reflect back to the other focus and vice versa. This property is made use of in medicine, parabolic reflectors used in solar concentrators, automobile industry etc. Many properties of the ellipse have already been discussed and demonstrated. In this article necessary equations with analytical geometry have been discussed and a new property for the secant generated at a fixed point either on the major or minor axis of an ellipse has been developed.

New Theorem

If two chords $\overline{AB}$ *and* $\overline{CD}$ *of an ellipse of which co-ordinates are* $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ *and* $D(x_4, y_4)$*intersecting at a point* $P(x_p,\ y_p)$ *then the following statement is always true (Ref: Fig.1)*

$$\left[b^2(x_p - x_1)^2 + a^2(y_p - y_1)^2\right] \times \left[b^2(x_p - x_2)^2 + a^2(y_p - y_2)^2\right]$$
$$= \left[b^2(x_p - x_3)^2 + a^2(y_p - y_3)^2\right] \times \left[b^2(x_p - x_4)^2 + a^2(y_p - y_4)^2\right]$$

Where, ‘a’ and ‘b’ are the semi-major and semi-minor axis of the ellipse respectively.

$$x_p = \frac{x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[(x_4 - x_3)(y_3 - y_1) - x_3(y_4 - y_3)]}{(x_4 - x_3)(y_2 - y_1) - (x_2 - x_1)(y_4 - y_3)}$$

$$y_p = \frac{y_1(y_4 - y_3)(x_2 - x_1) + (y_2 - y_1)[(y_4 - y_3)(x_3 - x_1) - y_3(x_4 - x_3)]}{[(x_2 - x_1)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_1)]}$$

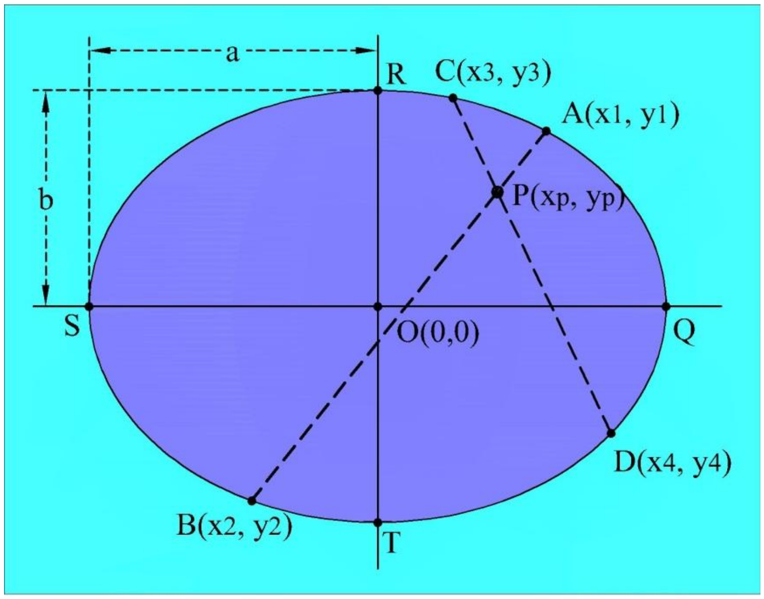

Figure 1: An ellipse with two chords intersecting at point P

Derivation of Equations and Proof of the Theorem

Figure 2a & 2 bare the drawings of an ellipse whose vertices are QRST is drawn. Point O the origin, $\overline{\mathbf{AB}}$ **and** $\overline{\mathbf{CD}}$are two chords drawn inside of the ellipse. Point P is the intersection of$\overline{\mathbf{AB}}$ **and** $\overline{\mathbf{CD}}$. Circle is drawn through Q and S. Points$\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}'$, $\mathbf{D}'$, $\mathbf{P}'$, $\mathbf{R}'$**and** , $\mathbf{T}'$are theprojection of A, B, C, D, P, R and T to the circle respectively. The ellipse can also be defined as the front view of the circle which is in x-y plane and rotated about its x-axis or y-axis.

Figure 2. a is the side view of the ellipse. $\mathbf{OR'} = \mathbf{OR''}, \mathbf{OA'} = \mathbf{OA''}, \mathbf{OT'} = \mathbf{OT''}, \Delta\mathbf{ORR''} = \Delta\mathbf{OAA''}$.

In right angled triangle ORR$''$, $\angle ORR'' = 90°$ and $\angle ROR'' = \theta°$.

$$\cos\theta° = \frac{OR}{OR''} = \frac{OR}{OR'} = \frac{b}{a} \text{ and } \cos\theta° = \frac{OA}{OA''} = \frac{OA}{OA'} \text{ Similarly } \cos\theta° = \frac{OB}{OB'} = \frac{OC}{OC'} = \frac{OD}{OD'} = \frac{OP}{OP'}$$

$$\text{Therefore, } OA' = \frac{OA}{\cos\theta°}, OB' = \frac{OB}{\cos\theta°}, OC' = \frac{OC}{\cos\theta°}, OD' = \frac{OD}{\cos\theta°} \text{ and } OP' = \frac{OP}{\cos\theta°}$$

Ref. fig. 2. In right triangle OAA$''$, $\cos\theta° = \frac{OA}{OA''} = \frac{OA}{OA'}$

$$\text{Therefore, } OA' = \frac{OA}{\cos\theta°} \text{ Similarly, } OB' = \frac{OB}{\cos\theta°}, \quad OC' = \frac{OC}{\cos\theta°} \text{ and } OP' = \frac{OP}{\cos\theta°}$$

Let co ordinates of A $= (x_1, y_1)$

Therefore, coordinates of A$'$ $= \left(x_1, \frac{y_1}{\cos\theta°}\right)$ ———————————————————(1)

Let, coordinates of B $= (x_2, y_2)$

Therefore, coordinates of B$'$ $= \left(x_2, \frac{y_2}{\cos\theta°}\right)$ ———————————————————(2)

Let, coordinates of C $= (x_3, y_3)$

Therefore, coordinates of C$'$ $= \left(x_3, \frac{y_3}{\cos\theta°}\right)$ ———————————————————(3)

Let, coordinates of D $= (x_4, y_4)$

Therefore, coordinates of D$'$ $= \left(x_4, \frac{y_4}{\cos\theta°}\right)$ ———————————————————(4)

Let, coordinates of P $= (x_p, y_p)$

Similarly, coordinates of $P' = \left(x_p, \frac{y_p}{\cos\theta°}\right)$ ------------------- (5)

$$\overline{A'P'} = \sqrt{\left(x_p - x_1\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_1}{\cos\theta°}\right)^2} \quad \text{---------------------(6)}$$

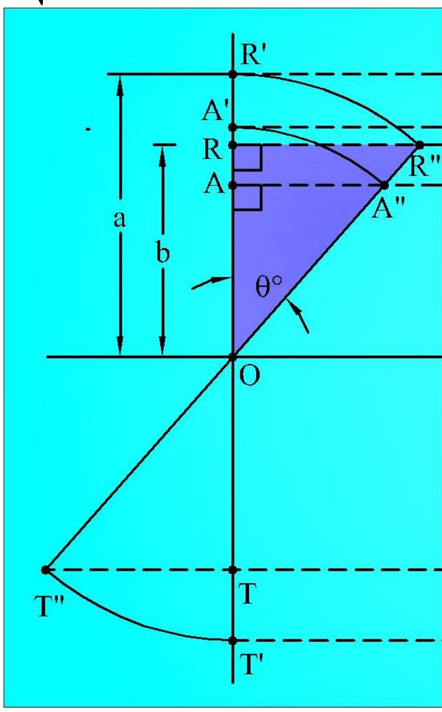

Figure 2a: Side view of the Ellipse

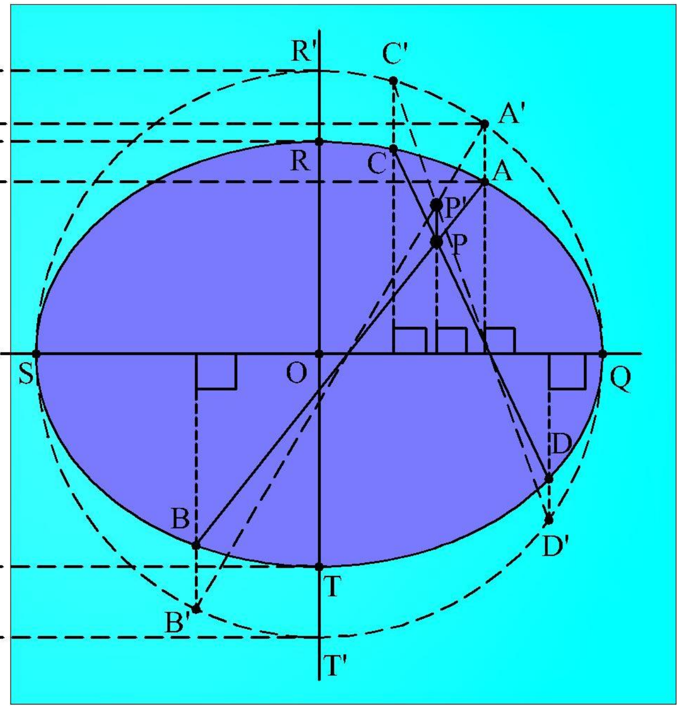

Figure 2b: Front view (elevation) of the Ellipse

$$\overline{B'P'} = \sqrt{\left(x_p - x_2\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_2}{\cos\theta°}\right)^2} \quad \text{---------------------(7)}$$

$$\overline{C'P'} = \sqrt{\left(x_p - x_3\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_3}{\cos\theta°}\right)^2} \quad \text{---------------------(8)}$$

$$\overline{D'P'} = \sqrt{\left(x_p - x_4\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_4}{\cos\theta°}\right)^2} \quad \text{---------------------(9)}$$

Points A', B', C', D' are on the circle of radius a. Point P' isthe intersection of $\overline{A'B'}$ and $\overline{C'D'}$. According to the "Intersecting chord theorem of a circle",

$$\overline{A'P'} \times \overline{B'P'} = \overline{C'P'} \times \overline{D'P'} \quad \text{-------------------------(10)}$$

$$\overline{A'P'} \times \overline{B'P'} = \sqrt{\left(x_p - x_1\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_1}{\cos\theta°}\right)^2} \times \sqrt{\left(x_p - x_1\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_1}{\cos\theta°}\right)^2} \quad \text{--(11)}$$

$$\overline{C'P'} \times \overline{D'P'} = \sqrt{\left(x_p - x_3\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_3}{\cos\theta°}\right)^2} \times \sqrt{\left(x_p - x_4\right)^2 + \left(\frac{y_p}{\cos\theta°} - \frac{y_4}{\cos\theta°}\right)^2} \quad \text{--(12)}$$

Substituting (11) &(12) is (10)

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$$\sqrt{(x_p-x_1)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_1}{\cos\theta^\circ}\right)^2}\times\sqrt{(x_p-x_1)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_1}{\cos\theta^\circ}\right)^2}$$
$$=\sqrt{(x_p-x_3)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_3}{\cos\theta^\circ}\right)^2}\times\sqrt{(x_p-x_4)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_4}{\cos\theta^\circ}\right)^2}\quad --------(13)$$

$$\Rightarrow\left[(x_p-x_1)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_1}{\cos\theta^\circ}\right)^2\right]\times\left[(x_p-x_2)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_2}{\cos\theta^\circ}\right)^2\right]$$
$$=\left[(x_p-x_3)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_3}{\cos\theta^\circ}\right)^2\right]\times\left[(x_p-x_4)^2+\left(\frac{y_p}{\cos\theta^\circ}-\frac{y_4}{\cos\theta^\circ}\right)^2\right]\quad ---(14)$$

$$\Rightarrow\left[\frac{\cos^2\theta^\circ(x_p-x_1)^2+(y_p-y_1)^2}{\cos^2\theta^\circ}\right]\times\left[\frac{\cos^2\theta^\circ(x_p-x_2)^2+(y_p-y_2)^2}{\cos^2\theta^\circ}\right]$$
$$=\left[\frac{\cos^2\theta^\circ(x_p-x_3)^2+(y_p-y_3)^2}{\cos^2\theta^\circ}\right]\times\left[\frac{\cos^2\theta^\circ(x_p-x_4)^2+(y_p-y_4)^2}{\cos^2\theta^\circ}\right]\quad ----(15)$$

$$\Rightarrow\left[\cos^2\theta^\circ(x_p-x_1)^2+(y_p-y_1)^2\right]\times\left[\cos^2\theta^\circ(x_p-x_2)^2+(y_p-y_2)^2\right]$$
$$=\left[\cos^2\theta^\circ(x_p-x_3)^2+(y_p-y_3)^2\right]\times\left[\cos^2\theta^\circ(x_p-x_3)^2+(y_p-y_3)^2\right]------(16)$$

Put, $\cos\theta^\circ=\frac{b}{a}$ in above equation

$$\Rightarrow\left[\frac{b^2}{a^2}(x_p-x_1)^2+(y_p-y_1)^2\right]\times\left[\frac{b^2}{a^2}(x_p-x_2)^2+(y_p-y_2)^2\right]$$
$$=\left[\frac{b^2}{a^2}(x_p-x_3)^2+(y_p-y_3)^2\right]\times\left[\frac{b^2}{a^2}(x_p-x_3)^2+(y_p-y_3)^2\right]\quad ------(17)$$

$$\Rightarrow\left[\frac{b^2(x_p-x_1)^2+a^2(y_p-y_1)^2}{a^2}\right]\times\left[\frac{b^2(x_p-x_2)^2+a^2(y_p-y_2)^2}{a^2}\right]$$
$$=\left[\frac{b^2(x_p-x_3)^2+a^2(y_p-y_3)^2}{a^2}\right]\times\left[\frac{b^2(x_p-x_4)^2+a^2(y_p-y_4)^2}{a^2}\right]\quad ------(18)$$

$$\Rightarrow\left[b^2(x_p-x_1)^2+a^2(y_p-y_1)^2\right]\times\left[b^2(x_p-x_2)^2+a^2(y_p-y_2)^2\right]$$
$$=\left[b^2(x_p-x_3)^2+a^2(y_p-y_3)^2\right]\times\left[b^2(x_p-x_4)^2+a^2(y_p-y_4)^2\right]\quad ------(19)$$

The eqn. (19) is the mathematical expression of the theorem.

Let co ordinates of A $=(x_1,y_1)$

As per eqn of ellipse, $x=\frac{a}{b}\sqrt{b^2-y^2}$

Therefore, $x_1=\frac{a}{b}\sqrt{b^2-{y_1}^2}$

Therefore, coordinates of A $=\left(\frac{a}{b}\sqrt{b^2-{y_1}^2},\ y_1\right)$

Similarly, coordinates of B $=\left(\frac{a}{b}\sqrt{b^2-{y_2}^2},\ y_2\right)$

Similarly, coordinates of C $=\left(\frac{a}{b}\sqrt{b^2-{y_3}^2},\ y_3\right)$

Similarly, coordinates of D $=\left(\frac{a}{b}\sqrt{b^2-{y_4}^2},\ y_4\right)$

Eqn. of line joining A and B is $\Longrightarrow y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$\therefore$ Eqn. of line $\overline{AB} \Longrightarrow y = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$ ---------------- (20)

Eqn. of line joining C and D is $\Longrightarrow y - y_3 = \left(\dfrac{y_4 - y_3}{x_4 - x_3}\right)(x - x_3)$

$\therefore$ Eqn. of line $\overline{CD} \Longrightarrow y = \left(\dfrac{y_4 - y_3}{x_4 - x_3}\right)(x - x_3) + y_3$ ---------------- (21)

Equating, (20) and (21),

$$\left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1 = \left(\frac{y_4 - y_3}{x_4 - x_3}\right)(x - x_3) + y_3$$

$$\frac{(y_2 - y_1)(x - x_1) + y_1(x_2 - x_1)}{x_2 - x_1} = \frac{(y_4 - y_3)(x - x_3) + y_3(x_4 - x_3)}{x_4 - x_3}$$

$$[x(y_2 - y_1) - x_1(y_2 - y_1) + y_1(x_2 - x_1)](x_4 - x_3) = [x(y_4 - y_3) - x_3(y_4 - y_3) + y_3(x_4 - x_3)](x_2 - x_1)$$

$$x(y_2 - y_1)(x_4 - x_3) - x_1(y_2 - y_1)(x_4 - x_3) + y_1(x_2 - x_1)(x_4 - x_3) = x(y_4 - y_3)(x_2 - x_1) - x_3(y_4 - y_3)(x_2 - x_1) + y_3(x_4 - x_3)(x_2 - x_1)$$

$$x(y_2 - y_1)(x_4 - x_3) - x_1(y_2 - y_1)(x_4 - x_3) + y_1(x_2 - x_1)(x_4 - x_3) - x(y_4 - y_3)(x_2 - x_1) + x_3(y_4 - y_3)(x_2 - x_1) - y_3(x_4 - x_3)(x_2 - x_1) = 0$$

$$x(y_2 - y_1)(x_4 - x_3) - x(y_4 - y_3)(x_2 - x_1) = x_1(y_2 - y_1)(x_4 - x_3) - y_1(x_2 - x_1)(x_4 - x_3) - x_3(y_4 - y_3)(x_2 - x_1) + y_3(x_4 - x_3)(x_2 - x_1)$$

$$x[(y_2 - y_1)(x_4 - x_3) - (y_4 - y_3)(x_2 - x_1)] = x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[y_3(x_4 - x_3) - y_1(x_4 - x_3) - x_3(y_4 - y_3)]$$

$$x[(y_2 - y_1)(x_4 - x_3) - (y_4 - y_3)(x_2 - x_1)] = x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[y_3(x_4 - x_3) - y_1(x_4 - x_3) - x_3(y_4 - y_3)]$$

$$x = \frac{x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[y_3(x_4 - x_3) - y_1(x_4 - x_3) - x_3(y_4 - y_3)]}{[(y_2 - y_1)(x_4 - x_3) - (x_2 - x_1)(y_4 - y_3)]} \quad ----(22)$$

$\therefore$ Eqn. (20) $\Longrightarrow y = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$

$$x - x_1 = (y - y_1)\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$$

$$x = (y - y_1)\left(\frac{x_2 - x_1}{y_2 - y_1}\right) + x_1 \quad ----------------------(23)$$

$\therefore$ Eqn. (21) $\Longrightarrow y = \left(\dfrac{y_4 - y_3}{x_4 - x_3}\right)(x - x_3) + y_3$

$$x - x_3 = (y - y_3)\left(\frac{x_4 - x_3}{y_4 - y_3}\right)$$

$$x = (y - y_3)\left(\frac{x_4 - x_3}{y_4 - y_3}\right) + x_3 \quad ----------------------(24)$$

Equating, (20) and (21),

$$(y - y_1)\left(\frac{x_2 - x_1}{y_2 - y_1}\right) + x_1 = (y - y_3)\left(\frac{x_4 - x_3}{y_4 - y_3}\right) + x_3$$

$$\frac{(y - y_1)(x_2 - x_1) + x_1(y_2 - y_1)}{(y_2 - y_1)} = \frac{(y - y_3)(x_4 - x_3) + x_3(y_4 - y_3)}{(y_4 - y_3)}$$

$$(y - y_1)(x_2 - x_1)(y_4 - y_3) + x_1(y_2 - y_1)(y_4 - y_3) = (y - y_3)(x_4 - x_3)(y_2 - y_1) + x_3(y_4 - y_3)(y_2 - y_1)$$

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$$y(x_2 - x_1)(y_4 - y_3) - y_1(x_2 - x_1)(y_4 - y_3) + x_1(y_2 - y_1)(y_4 - y_3) = y(x_4 - x_3)(y_2 - y_1) - y_3(x_4 - x_3)(y_2 - y_1) + x_3(y_4 - y_3)(y_2 - y_1)$$

$$y(x_2 - x_1)(y_4 - y_3) - y_1(x_2 - x_1)(y_4 - y_3) + x_1(y_2 - y_1)(y_4 - y_3) - y(x_4 - x_3)(y_2 - y_1) + y_3(x_4 - x_3)(y_2 - y_1) - x_3(y_4 - y_3)(y_2 - y_1) = 0$$

$$y(x_2 - x_1)(y_4 - y_3) - y(x_4 - x_3)(y_2 - y_1) = y_1(x_2 - x_1)(y_4 - y_3) - x_1(y_2 - y_1)(y_4 - y_3) - y_3(x_4 - x_3)(y_2 - y_1) + x_3(y_4 - y_3)(y_2 - y_1)$$

$$y = \frac{y_1(x_2 - x_1)(y_4 - y_3) + (y_2 - y_1)[x_3(y_4 - y_3) - x_1(y_4 - y_3) - y_3(x_4 - x_3)]}{[(x_2 - x_1)(y_4 - y_3) - (y_2 - y_1)(x_4 - x_3)]} \quad --- (25)$$

Therefore, the coordinates of intersection point P are derived as follows:

$$x_p = \frac{x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[(x_4 - x_3)(y_3 - y_1) - x_3(y_4 - y_3)]}{(x_4 - x_3)(y_2 - y_1) - (x_2 - x_1)(y_4 - y_3)} \quad ------ (26)$$

$$y_p = \frac{y_1(y_4 - y_3)(x_2 - x_1) + (y_2 - y_1)[(y_4 - y_3)(x_3 - x_1) - y_3(x_4 - x_3)]}{[(x_2 - x_1)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_1)]} \quad ------- (27)$$

RESULTS AND DISCUSSION

Referring fig. 3, In a circle, if AB and CD are the two chords intersecting at point P, the theorem of*Chords intersecting in a circle* (Krishnan Ganesh, Online) is $\overline{PA} \times \overline{PB} = \overline{PC} \times \overline{PD}$. Now, the newly developed theorem is the extended form of thetheorem for the circle, since circle is the particular case of the ellipse. Now this theorem also can be proven with the following example.

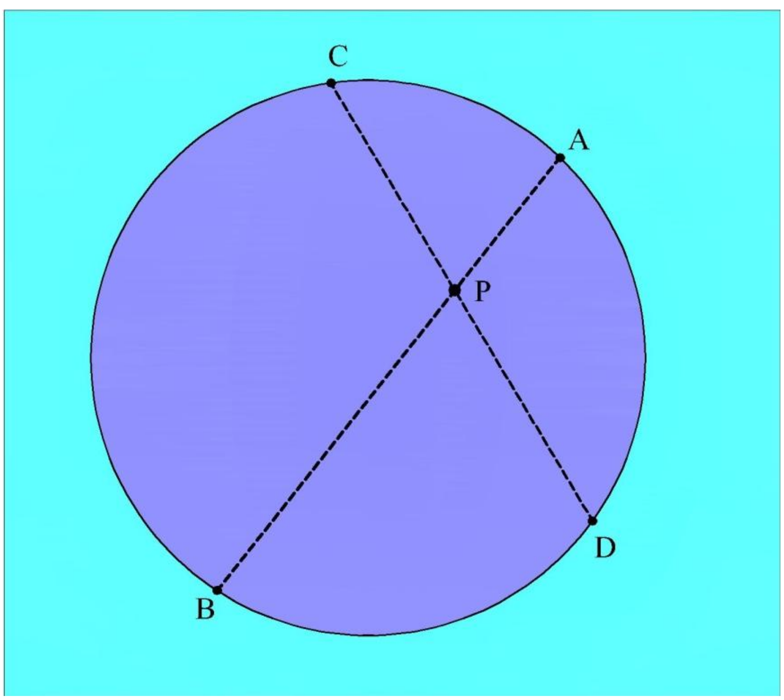

Figure 3: Two chords AB and CD intersecting at point P in a circle

Proof with Example

An ellipse of semi major axis (a) = 4 units and semi minor axis (b) = 3 units is drawn. There two chords $\overline{AB}$ and $\overline{CD}$ and these two chords are intersecting at point P. The coordinates of the extremities A, B, C and D of these two chords and intersection point P are given in the table 1.

Table 1: The co-ordinates ofthe extremities of two chords and intersection point

S. No	Point	x	y
1	A	$x_1 = 2.3342$	$y_1 = 2.4362$
2	B	$x_2 = -1.7517$	$y_2 = -2.6970$
3	C	$x_3 = 1.0422$	$y_3 = 2.8964$
4	D	$x_4 = 3.2384$	$y_4 = -1.7610$
5	P	$x_p = 1.6591$	$y_p = 1.5881$

These values are substituted in the theorem (eqn. 19).

$$\left[b^2(x_p - x_1)^2 + a^2(y_p - y_1)^2\right] \times \left[b^2(x_p - x_2)^2 + a^2(y_p - y_2)^2\right] = \left[b^2(x_p - x_3)^2 + a^2(y_p - y_3)^2\right] \times \left[b^2(x_p - x_4)^2 + a^2(y_p - y_4)^2\right]$$

LHS of the therem $\Longrightarrow \left[b^2(x_p - x_1)^2 + a^2(y_p - y_1)^2\right] \times \left[b^2(x_p - x_2)^2 + a^2(y_p - y_2)^2\right]$

$x_1 = 2.3342, \quad y_1 = 2.4362, \quad x_2 = -1.7517, \quad y_2 = -2.6970$

Substituting the above values in eqn. 26, we get

$$x_p = \frac{x_1(y_2 - y_1)(x_4 - x_3) + (x_2 - x_1)[(x_4 - x_3)(y_3 - y_1) - x_3(y_4 - y_3)]}{(x_4 - x_3)(y_2 - y_1) - (x_2 - x_1)(y_4 - y_3)}$$

$x_p = 1.6591$ units

Substituting the values of x_1, x_2, y_1 *and* y_2 in eqn. 27, we get

$$y_p = \frac{y_1(y_4 - y_3)(x_2 - x_1) + (y_2 - y_1)[(y_4 - y_3)(x_3 - x_1) - y_3(x_4 - x_3)]}{[(x_2 - x_1)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_1)]}$$

$y_p = 1.5881$units.

Substituting the values of x_1, x_2, y_1, y_2, x_p and y_p in eqn.19, we get

$$\text{LHS} = [3^2(1.6591 - 2.3342)^2 + 4^2(1.5881 - 2.4362)^2] \times [3^2(1.6591 + 1.7517)^2 + 4^2(1.5881 + 2.6970)^2]$$

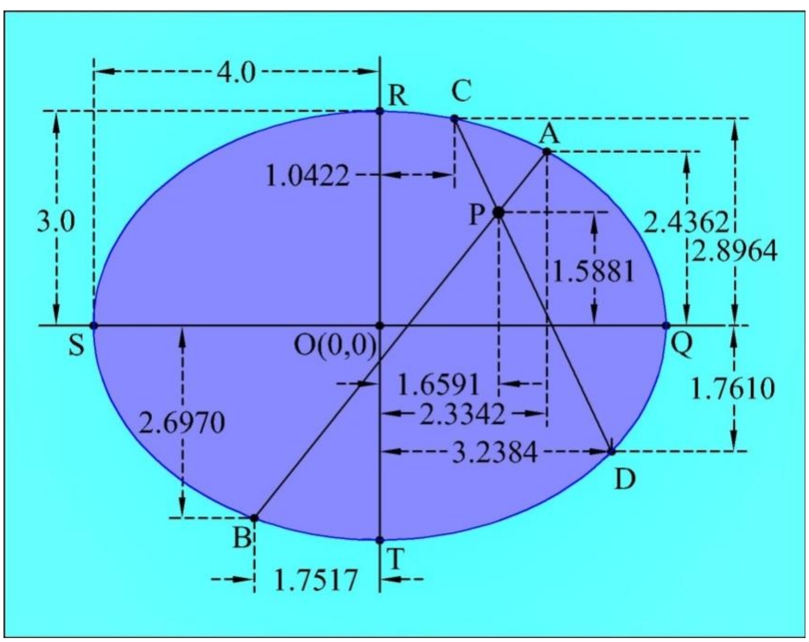

Figure 4: Example-1 for an ellipse with two chords

Intersecting at Point Pincluding Dimensions

Therefore, $\text{LHS} = [9(-0.6751)^2 + 16(-0.8481)^2] \times [9(3.4108)^2 + 16(4.2851)^2]$

Therefore, $\text{LHS} = (4.1018 + 11.5084) \times (104.7020 + 293.7933)$

Therefore, $\text{LHS} = 15.6102 \times 398.4953$

Therefore, $\text{LHS} = 6220.59 \text{ sq. units}$ $\qquad ------------------(28)$

RHS of the theorem $= \left[b^2(x_p - x_3)^2 + a^2(y_p - y_3)^2\right] \times \left[b^2(x_p - x_4)^2 + a^2(y_p - y_4)^2\right]$

Substituting $x_3 = 1.0422$, $y_3 = 2.8964$, $x_4 = 3.2384$,
$y_4 = -1.7610$, $x_p = 1.6591$ and $y_p = 1.5881$ in above eqn.

$\text{RHS} = [3^2(1.6591 - 1.0422)^2 + 4^2(1.5881 - 2.8964)^2] \times [3^2(1.6591 - 3.2384)^2 + 4^2(1.5881 + 1.7610)^2]$

Therefore, $\text{RHS} = [9(0.6169)^2 + 16(-1.3083)^2] \times [9(-1.5793)^2 + 16(3.3491)^2]$

Therefore, $\text{RHS} = [3.4251 + 27.3864] \times [22.4477 + 179.4635]$

Therefore, $\text{RHS} = 30.8115 \times 201.9112$

Therefore, $\text{RHS} = 6221.19 \text{ sq. units}$ $\qquad ------------------(29)$

Comparing eqns. 28 and 29, both values are more or less same.The difference is very small and it may be due to 5th decimal of the inputs values of the coordinates. However, from (28) and (29), LHS = RHS and hence the theorem is proven as correct.

Conclusion

In this article the author has developed a new theorem for a property of intersection of two chords of an ellipse and derived necessary equations with analytic geometry and defined very clearly along with relevant drawings and proved with appropriate examples. It will be very useful for those doing research or further study in the geometry especially in ellipse.

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