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Research Article

EFFECT OF MICHELL FUNCTION ON THE THICKNESS OF ANNULAR DISC WITH INTERNAL HEAT GENERATION

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ABSTRACT
The present paper deals with the determination of transient thermal stresses in a thick $(M \neq 0)$ annular disc with internal heat generation. A thick annular disc is considered having zero initial temperature and arbitrary heat supply is applied on the upper and lower surface where as the fixed circular edges are at zero temperature. Here we compute the effects of Michell function of a thick annular disc in terms of stresses along radial direction. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change and stresses have been computed numerically and illustrated graphically.

Key Words: *Thick Annular Disc* $(M \neq 0)$, *Thin Annular Disc*$(M = 0)$, *Internal Heat Generation*

INTRODUCTION

The study of thermal stresses in annular disc is an important problem in engineering. During the second half of the twentieth century, nonisothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure.

Nowacki (1957) has determined the steady state thermal stresses in circular disk subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Shang Sheng Wu (1997) studied the direct thermoelastic problem in an annular fin with its base subjected to a heat flux of a decayed exponential function of time. Gogulwar and Deshmukh (2002) determined the thermal stresses in an annular disc. Kulkarni and Deshmukh (2007) has discussed the quasi-static transient thermal stresses in thick annular disc having zero initial temperature and subjected to arbitrary heat flux on the upper and lower surface where as the fixed circular edges are at zero temperature.

Recently Bhongade and Durge (2013) considered thick annular disc and discuss thermal stresses due to arbitrary heat is applied on the upper surface and heat dissipates by convection from the lower boundary surface into the surrounding at the zero temperature and the circular edges are thermally insulated**,** now here we consider thick $(M \neq 0)$ and thin $(M = 0)$ annular disc with internal heat generation and discussed thermal stresses due to arbitrary heat supply is applied on the upper and lower surface of a thick annular disc and we compute the effects of Michell function in terms of stresses along radial direction. To obtain the temperature distribution, integral transform method is applied. The results are obtained in series form in terms of Bessel's functions and the temperature change; stresses have been computed numerically and illustrated graphically. A mathematical model has been constructed of a thick $(M \neq 0)$ and thin $(M = 0$ annular disc with the help of numerical illustration by considering aluminum (pure) annular disc. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant, gas power plant and the measurement of aerodynamic heating.

Formulation of the Problem

Consider a thick $(M \neq 0)$ annular disc of thickness $2h$ defined by $a \leq r \leq b, -h \leq z \leq h$. The initial temperature in a thick annular disc is zero. The arbitrary heat supply $\pm f(r,t)$ is applied over the upper surface ($z = h$) and the lower surface ($z = -h$) of disc. Assume the circular boundary of a thick annular disc is free from traction. Under these prescribed conditions, the thermal transient temperature and stresses in a thick annular disc with internal heat generation are required to be determined.

The differential equation governing the displacement potential function $\phi(r,z,t)$ is given in (2003) as

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}= K\boldsymbol{\tau} \tag{1}$$

where K is the restraint coefficient and temperature change $\tau = T - T_{i,}$ T_i is initial temperature. Displacement function ϕ is known as Goodier's thermoelastic displacement potential.

Temperature of the disc at time *t* satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{k}= \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

$$T = 0 \ at\ r = a\,, -h \le z \le h \tag{3}$$

$$T = 0 \ at\ r = b\,, -h \le z \le h \tag{4}$$

$$T = \pm f(r,t) \ \text{at}\ z = \pm h\,,\ a \le r \le b \tag{5}$$

$$q(r,z,t) = \delta(r-r_0)\sin(\beta_m z)\,(1-e^{-t}) \tag{6}$$

and the initial condition

$$T = 0 \ at\ t = 0\,,\ a \le r \le b \tag{7}$$

Where α is the thermal diffusivity of the material of the disc, *k* is the thermal conductivity of the material of the disc, q is the internal heat generation and $\delta(r)$ is well known dirac delta function of argument r.

The Michell's function *M* must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{8}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{9}$$

The components of the stresses are represented by the thermoelastic displacement potential ϕ and Michell's function *M* as

$$\sigma_{rr} = 2G\left\{\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{10}$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right]\right\} \tag{11}$$

$$\sigma_{zz} = 2G\left\{\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left[(2-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{12}$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{13}$$

Where *G* and *v* are the shear modulus and Poisson's ratio respectively.

For traction free surface stress functions

$$\sigma_{rr} = \sigma_{rz} = 0 \ at\ z = h, r = a\ and\ r = b. \tag{14}$$

Equations (1) to (14) constitute mathematical formulation of the problem.

Solution

To obtain the expression for temperature *T* (*r*, *z*, *t*), we introduce the finite Hankel transform over the variable *r* and its inverse transform defined by (1968) as

$$\bar{T}(\beta_m,z,t) = \int_a^b r\,K_0(\beta_m,r)\ T(r,z,t)\,dr \tag{15}$$

$$T(r,z,t) = \sum_{m=1}^{\infty} K_0(\beta_m,r)\ \bar{T}(\beta_m,z,t) \tag{16}$$

where $K_0(\beta_m,r) = \frac{R_0(\beta_m,r)}{\sqrt{N}}$ (17)

$$and\ \ R_0(\beta_m,r) = \left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0(\beta_m b)}\right] \tag{18}$$

The normality constant

$$N = \frac{b^2}{2}\dot{R_0}^2(\beta_m,b) - \frac{a^2}{2}\dot{R_0}^2(\beta_m,a) \tag{19}$$

$\beta_1, \beta_2 \ldots..$ are roots of transcendental equation

$$\frac{J_0(\beta_m a)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m a)}{Y_0(\beta_m b)} = 0 \tag{20}$$

Where $J_n(x)$ is Bessel function of the first kind of order n and $Y_n(x)$ is Bessel function of the second kind of order n.

On applying the finite Hankel transform defined in the Eq. (15), its inverse transform defined in (16) and applying Laplace transform and its inverse by residue method successively to the Eq. (2), one obtains the expression for temperature as

$$T(r,z,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\sqrt{N}} \left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0(\beta_m b)} \right] \times \left\{ \left(\frac{-n\pi\alpha}{2(-1)^n h^2} \right) \left(\sin\left[\frac{n\pi}{2h}(z+h)\right] + \sin\left[\frac{n\pi}{2h}(z-h)\right] \right) g(t) + \left[\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)} \right] \frac{\alpha D_m \sin(\beta_m z)}{k} \right\} \tag{21}$$

Where

$$g(t) = \int_0^t e^{-\alpha\left[\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right](t-u)} \times \left\{ F(\beta_m, u) - \frac{\alpha D_m \sin(\beta_m h)}{k} \left[\frac{1}{2\alpha\beta_m^2} + \frac{e^{-u}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 u}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)} \right] \right\} du$$

and $D_m = \frac{r_0}{\sqrt{N}} \left[\frac{J_0(\beta_m r_0)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r_0)}{Y_0(\beta_m b)} \right]$.

Since initial temperature $T_i = 0, \tau = T - T_i$

$$\tau = T \tag{22}$$

Michell's function *M*

Now suitable form of *M* which satisfy Eq. (8) is given by

$$M = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} K\, F(\beta_m, t) \left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0(\beta_m b)} \right] \times \left[B_{mn} \sin h(\beta_m z) + C_{mn}\, \beta_m z \cos h(\beta_m z) \right] \tag{23}$$

where B_{mn} and C_{mn} are arbitrary functions, which can be determined finally by using condition (14).

Goodiers thermoelastic displacement potential $\phi(r,z,t)$

Assuming the displacement function $\phi(r,z,t)$ which satisfies Eq. (1) as

$$\phi(r,z,t) = K \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\left(\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right)\sqrt{N}} \left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0(\beta_m b)} \right] \times \left\{ \left[\left(\frac{n\pi\alpha}{2(-1)^n h^2} \right) g(t) \left(\sin\left[\frac{n\pi}{2h}(z+h)\right] + \sin\left[\frac{n\pi}{2h}(z-h)\right] \right) \right] - \left(\frac{\alpha D_m n^2\pi^2}{4kh^2\beta_m^2} \right) \left[\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)} \right] \sin(\beta_m z) \right\} \tag{24}$$

Now using Eqs. (21), (23) and (24) in Eqs. (10), (11), (12) and (13), one obtains the expressions for stresses respectively as

$$\frac{\sigma_{rr}}{K} = 2G \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\sqrt{N}} \left\{ \left[-\frac{J_1'(\beta_m r)}{J_0(\beta_m b)} + \frac{Y_1'(\beta_m r)}{Y_0(\beta_m b)} \right] \times \left\{ \frac{\beta_m^2}{\left(\beta_m^2 + \frac{n^2\pi^2}{4h^2}\right)} \left[\left(\frac{n\pi\alpha}{2(-1)^n h^2} \right) \left(\sin\left[\frac{n\pi}{2h}(z+h)\right] + \sin\left[\frac{n\pi}{2h}(z-h)\right] \right) g(t) - \left(\frac{\alpha n^2\pi^2 D_m}{4kh^2\beta_m^2} \right) \left[\frac{1}{2\alpha\beta_m^2} + \frac{e^{-t}}{1-2\alpha\beta_m^2} + \frac{e^{-2\alpha\beta_m^2 t}}{2\alpha\beta_m^2(2\alpha\beta_m^2-1)} \right] \sin(\beta_m z) \right] + F(\beta_m, t)\, \beta_m^3 \left[B_{mn} \cos h(\beta_m z) + C_{mn} \langle \beta_m z \sin h(\beta_m z) + \cosh(\beta_m z) \rangle \right] \right\} + \left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0(\beta_m b)} \right]$$

$$\times\left\{\begin{bmatrix}\left(\frac{n\pi\alpha}{2(-1)^n\,h^2}\right)\left(\sin\left[\frac{n\pi}{2h}\,(z+h)\right]+\sin\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ -\frac{\alpha\,D_m}{k}\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\sin(\beta_m z)\end{bmatrix} \\ +2v\,F(\beta_m,t)\;\beta_m{}^3\cosh(\beta_m z)\right\} \tag{25}$$

$$\frac{\sigma_{\theta\theta}}{K}=2G\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\sqrt{N}}\left[-\frac{J_1(\beta_m r)}{J_0(\beta_m b)}+\frac{Y_1(\beta_m r)}{Y_0(\beta_m b)}\right]$$

$$\times\left\{\frac{-\beta_m}{\left(\beta_m{}^2+\frac{n^2\pi^2}{4h^2}\right)r}\begin{bmatrix}\left(\frac{n\pi\alpha}{2(-1)^n\,h^2}\right)\left(\sin\left[\frac{n\pi}{2h}\,(z+h)\right]+\sin\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ -\left(\frac{\alpha\,n^2\pi^2 D_m}{4\,kh^2\beta_m{}^2}\right)\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\sin(\beta_m z)\end{bmatrix} \\ +F(\beta_m,t)\,\frac{\beta_m{}^2}{r}\left[B_{mn}\cosh(\beta_m z)+C_{mn}\langle\beta_m z\,\sinh(\beta_m z)+\cosh(\beta_m z)\rangle\right]\right\}$$

$$-\left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)}-\frac{Y_0(\beta_m r)}{Y_0(\beta_m b)}\right]$$

$$\times\left\{\begin{bmatrix}\left(\frac{-n\pi\alpha}{2(-1)^n\,h^2}\right)\left(\sin\left[\frac{n\pi}{2h}\,(z+h)\right]+\sin\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ +\frac{\alpha\,D_m}{k}\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\sin(\beta_m z)\end{bmatrix} \\ -2v\;\beta_m{}^3C_{mn}\;F(\beta_m,t)\cosh(\beta_m z)\right\} \tag{26}$$

$$\frac{\sigma_{zz}}{K}=2G\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\left(\beta_m{}^2+\frac{n^2\pi^2}{4h^2}\right)\sqrt{N}}\left[\frac{J_0(\beta_m r)}{J_0(\beta_m b)}-\frac{Y_0(\beta_m r)}{Y_0(\beta_m b)}\right]$$

$$\times\left\{\frac{n^2\pi^2}{4\,h^2}\begin{bmatrix}\left(\frac{-n\pi\alpha}{2(-1)^n\,h^2}\right)\left(\sin\left[\frac{n\pi}{2h}\,(z+h)\right]+\sin\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ +\left(\frac{\alpha\,n^2\pi^2 D_m}{4\,kh^2\beta_m{}^2}\right)\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\sin(\beta_m z)\end{bmatrix}\right.$$

$$-\begin{bmatrix}\left(\frac{-n\pi\alpha}{2(-1)^n\,h^2}\right)\left(\sin\left[\frac{n\pi}{2h}\,(z+h)\right]+\sin\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ +\frac{\alpha D_m}{k}\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\sin(\beta_m z)\end{bmatrix} \tag{27}$$

$$\left.+F(\beta_m,t)\,\beta_m{}^3\left[-B_{mn}\cosh(\beta_m z)+C_{mn}\left\langle\begin{matrix}-\beta_m z\,\sinh(\beta_m z)\\+(1-2v)\cosh(\beta_m z)\end{matrix}\right\rangle\right]\right\}$$

$$\frac{\sigma_{rz}}{K}=2G\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\beta_m}{\sqrt{N}}\left[\frac{J_1(\beta_m r)}{J_0(\beta_m b)}-\frac{Y_1(\beta_m r)}{Y_0(\beta_m b)}\right]$$

$$\times\left\{\frac{1}{\left(\beta_m{}^2+\frac{n^2\pi^2}{4h^2}\right)}\begin{bmatrix}\left(\frac{\alpha\,n^2\pi^2}{4(-1)^n\,h^3}\right)\left(\cos\left[\frac{n\pi}{2h}\,(z+h)\right]+\cos\left[\frac{n\pi}{2h}\,(z-h)\right]\right)g(t)\\ -\left(\frac{\alpha\,n^2\pi^2 D_m}{4\,kh^2\beta_m}\right)\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\cos(\beta_m z)\end{bmatrix}\right.$$

$$\left.-F(\beta_m,t)\beta_m{}^2\left[B_{mn}\sinh(\beta_m z)+C_{mn}\left\langle\begin{matrix}\beta_m z\cosh(\beta_m z)\\+2(1+v)\sinh(\beta_m z)\end{matrix}\right\rangle\right]\right\} \tag{28}$$

Determination of unknown arbitrary functions B_{mn} *and* C_{mn}

In order to satisfy condition (14), solving Eqs. (25) and (28) for B_{mn} and C_{mn} one obtain,

$$B_{mn}=\left(\frac{\alpha\,n^2\pi^2}{4\,h^2\beta_m{}^2}\right)\frac{1}{(2v+1)}\frac{1}{\left(\beta_m{}^2+\frac{n^2\pi^2}{4h^2}\right)}\frac{1}{F(\beta_m,t)\;\sinh(\beta_m z)\cosh(\beta_m z)}$$

$$\times\left\{\begin{matrix}[\cosh(\beta_m h)+\;\beta_m h\;\sinh(\beta_m h)\,](1+(-1)^n)\frac{g(t)}{h(-1)^n}\\ -[\cosh^2(\beta_m h)+2(1+v)\,\sin(\beta_m h)\;\sinh(\beta_m h)\,]\frac{D_m}{k\,\beta_m}\end{matrix}\right\}$$

$$\times\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right] \quad (29)$$

$$C_{mn}=\left(\frac{-\alpha\, n^2\pi^2}{4\,h^2\beta_m{}^2}\right)\frac{1}{(2v+1)}\frac{1}{\left(\beta_m{}^2+\frac{n^2\pi^2}{4h^2}\right)}\frac{1}{F(\beta_m,t)\ \ \sin h(\beta_m z)\cos h(\beta_m z)}$$
$$\times\left\{(1+(-1)^n)\frac{g(t)}{h(-1)^n}+\frac{D_m}{k\,\beta_m}[sin(\beta_m h)\ sin\,h(\beta_m h)\ -cosh(\beta_m h)]\right.$$
$$\left.\times\left[\frac{1}{2\alpha\beta_m{}^2}+\frac{e^{-t}}{1-2\alpha\beta_m{}^2}+\frac{e^{-2\alpha\beta_m{}^2 t}}{2\alpha\beta_m{}^2(2\alpha\beta_m{}^2-1)}\right]\right\} \quad (30)$$

Setting

Special Case and Numerical Calculations

$f(r,t)=\delta(r-r_0)t,$ (31)

where $\delta(r)$ is well known dirac delta function of argument r.

Applying finite Hankel transform as defined in Eq.(15) to the Eq.(31), one obtains

$F(\,\beta_m,t)=\frac{r_0}{\sqrt{N}}\left[\frac{J_0(\beta_m\, r_0\,)}{J_0^{'}(\beta_m b)}-\frac{Y_0(\beta_m\, r_0\,)}{Y_0^{'}(\beta_m b)}\right]t$

$a=1m, b=2m,$

$h=0.3m, r_0=1.5m$ and $t=2\ sec.$

Material Properties

The numerical calculation has been carried out for aluminum (pure) annular disc with the material properties defined as

Thermal diffusivity α = 84.18× 10^{-6} m^2s^{-1},

Specific heat c_ρ = 896 J/kg,

Thermal conductivity k = 204.2 W/m K,

Shear modulus $G=25.5\ G$ pa,

Poisson ratio $\vartheta=0.281$.

Roots of Transcendental Equation

The $\beta_1=3.120,\ \beta_2=6.2734,\ \beta_3=9.4182,\ \beta_4=12.5614,\ \beta_5=15.7040$ are the roots of transcendental equation $\frac{J_0(\beta_m a)}{J_0(\beta_m b)}-\frac{Y_0(\beta_m a)}{Y_0(\beta_m b)}$. The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

DISCUSSION

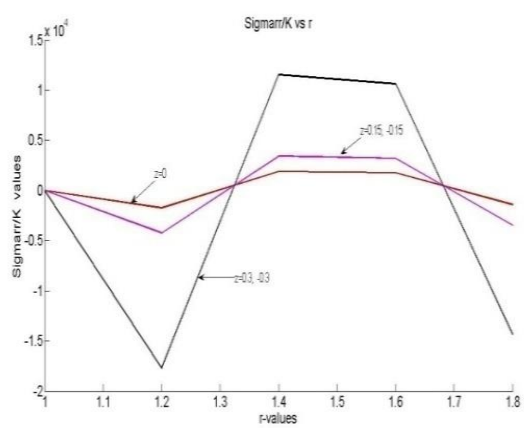

Figure 1: Radial stress function $\frac{\sigma_{rr}}{K}$ for thick annular disc ($M\neq 0$).

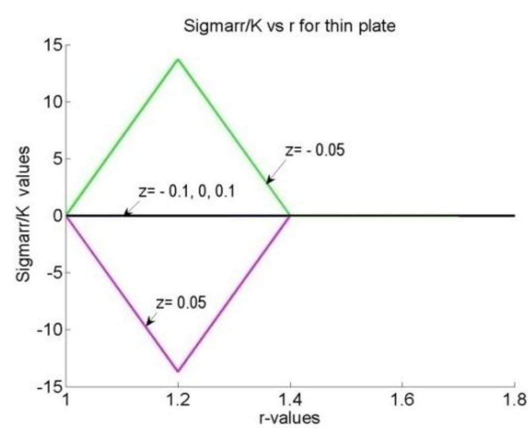

Figure 2: Radial stress function $\frac{\sigma_{rr}}{K}$ for thin annular disc (M = 0).

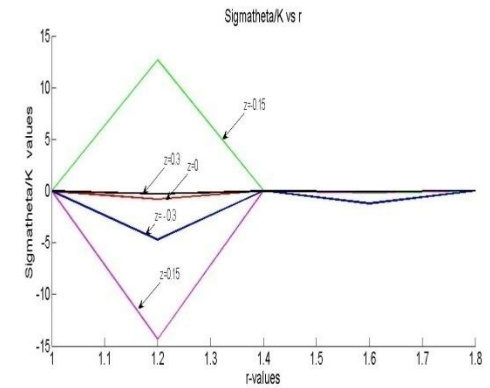

Figure 3: Angular stress function $\frac{\sigma_{\theta\theta}}{K}$ for thick annular disc ($M \neq 0$).

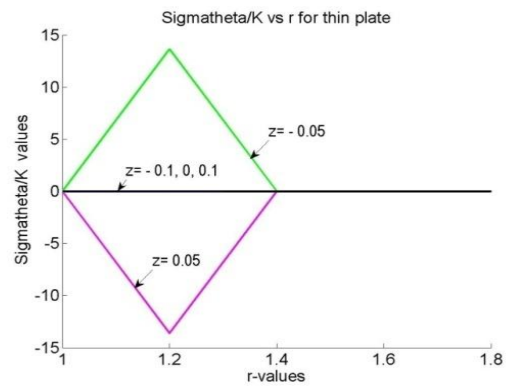

Figure 4: Angular stress function $\frac{\sigma_{\theta\theta}}{K}$ for thin annular disc ($M = 0$).

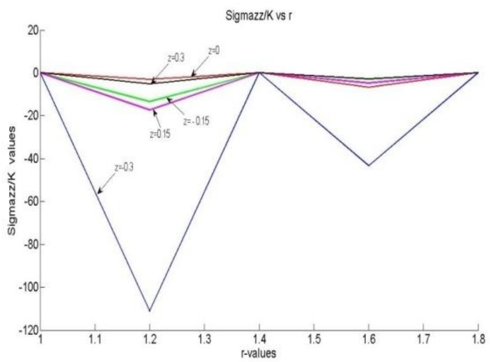

Figure 5: Axial stress function $\frac{\sigma_{zz}}{K}$ for thick annular disc ($M \neq 0$).

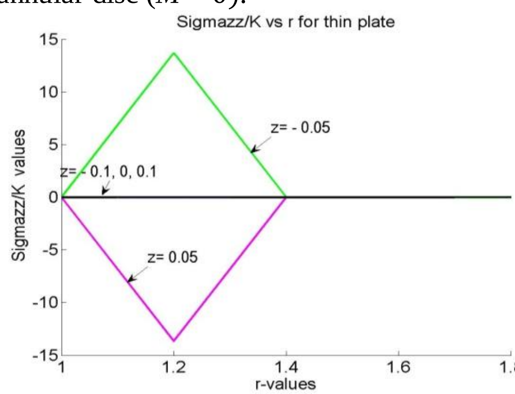

Figure 6: Axial stress function $\frac{\sigma_{zz}}{K}$ for thin annular disc ($M = 0$).

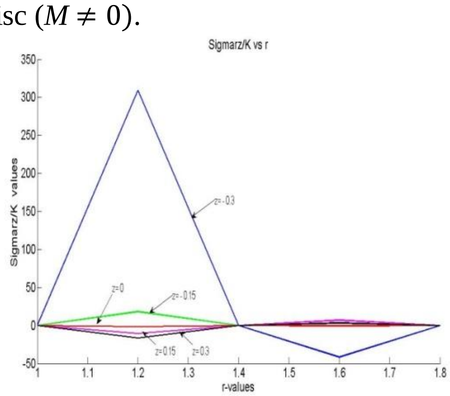

Figure 7: Stress function $\frac{\sigma_{rz}}{K}$ for thick annular disc ($M \neq 0$).

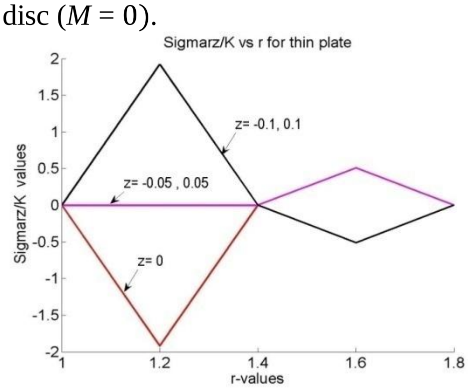

Figure 8: Stress function $\frac{\sigma_{rz}}{K}$ for thin annular disc ($M = 0$).

In this paper a thick ($M \neq 0$) and thin ($M = 0$) annular discs are considered and the expressions for temperature and stresses are determined due to internal heat generation within it and we compute the effects of Michell function on the thickness of annular disc in terms of stresses along radial direction by substituting $M = 0$ in Eqs. (25), (26), (27) and (28) and we compare the results for $M = 0$ and $M \neq 0$. As a special case mathematical model is constructed for considering aluminum (pure) annular disc with the material properties specified above.

From figure 1 and 2, it is observed that due to Michell function the thick annular disc sustain the radial stresses as compared to thin annular disc along radial direction.
From figure 3 and 4, it is observed that due to Michell function the angular stress function $\frac{\sigma_{\theta\theta}}{K}$ in a thick annular disc is comparatively less than thin annular disc along radial direction.
From figure 5 and 6, it is observed that due to Michell function the axial stress function $\frac{\sigma_{zz}}{K}$ are compressive in nature along radial direction in a thick annular disc whereas the axial stress function is tensile for lower half of a thin annular disc and it is compressive in nature for upper half of a thin annular disc along radial direction.
From figure 7 and 8, it is observed that due to Michell function the stress function $\frac{\sigma_{rz}}{K}$ in a thick annular disc is comparatively large than thin annular disc along radial direction.

CONCLUSION

We can conclude that due to Michell function the radial stress function $\frac{\sigma_{rr}}{K}$, angular stress function $\frac{\sigma_{\theta\theta}}{K}$, the axial stress function $\frac{\sigma_{zz}}{K}$ and the stress function $\frac{\sigma_{rz}}{K}$ vary with the thickness of a annular disc along radial direction.
The results obtained here are useful in engineering problems particularly in the determination of state of stress in a annular disc and base of furnace of boiler of a thermal power plant and gas power plant and the measurement of aerodynamic heating.

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