International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2013 Vol. 3 (3) July-September, pp.81-84/Dubey and Pratima
Research Article

COSMOLOGICAL MODEL WITH EINSTEIN DE SITTER UNIVERSE

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ABSTRACT
A de Sitter Universe is a Cosmological solution of Einstein's field equation of general Relativity which is named after Willem de Sitter. Also we have discussed in mathematics and physics, a de Sitter space or space time, of a sphere in ordinary Euclidean space and Penrose Diagram of De Sitter Space. Also we have discuss Pure de sitter space is the unique vacuum solution to the Einstein equation with maximal symmetry and constant positive curvature.

Key Words: *De Sitter Space, Albert Einstein, Willen se Sitter Cosmological Model*

INTRODUCTION
DE-Sitter Universe
A de sitter universe is a cosmological solution of Einstein's field equation of general-Relativity. It models the universe as spatially flat and neglects ordinary mates, so the dynamics of the universe are dominated by the cosmological constant, thought to correspond to dark energy in our universe or the inflation field in the early universe. According to the models of inflation and current observations of the accelerating universe the concordance models of physical cosmology are converging on a consistent model where our universe was best described as a de sitter universe at about a time t=10^{-33} seconds after the fiducial Big Bank singularity, and far into the future.

DE -Sitter Space
In mathematics and physics, a de sitter space is the along in Minkowski space, or space-time, of a sphere in ordinary Euclidean space. The n-dimensional de sitter space denoted ds_n , is the Loventzain manifold an along of an n- sphere it is maximally symmetric, the constant positive curvature, and is simply connected far n at least 3.

In the language of general relativity, de sitter space is the maximally vacuum solution of Einstein's field equation with a positive cosmological constantΛ. When n=4 (3 space dimensions plus time) it is a cosmological model for the physical universe.

Mathematical Expression
A de-sitter universe has no ordinary matter constant but with a positive cosmological constant which sets the expansion rate H. A large cosmological constant leads to a large expansion rate

$H\propto \sqrt{\Lambda}$ 1

Where the constant of proportionality depends on co mentions. The cosmological constant is Λ.H is common to describe a patch of this solution as an expanding universe of the FLRW from where the scale factor is given by

$a(t) = e^{Ht}$. 2

Where the constant H is the Hubble expansion rate and t is time. As is all FLW spaces, a(t), the scale factor. De-sitter space can be defined as a sub manifold of a Minkowski space of one dimension. Take Minkowski space $R^{1,n}$ with the standard metric.

$ds^2 = -dx_0{}^2 + \sum_{i=0}^{n} dx_i{}^2$...................................... 3

De sitter space is the sub manifold described by the hyperboloid of one sheet

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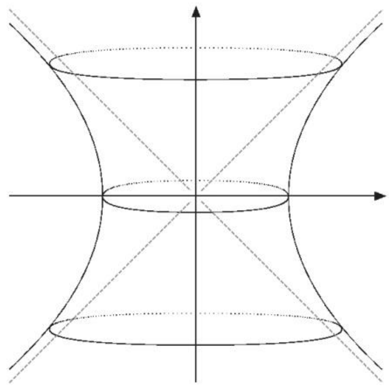
$-x_0{}^2 + \sum_{i=0}^{n} x_i{}^2 = \alpha^2$.............................. 4

Where α is some positive constant with dimension of length. The metric on de sitter space is the metric induced from the ambient Minkowski metric. The induced metric is non degenerate and has Lorentzian signature.

Classical Geometry of de Sitter Space

Holography in de Sitter Space



$X^0 X^i$, $i = 1, \ldots, D$

De Sitter space realized as a hyperboloid in flat Minkowki space

Pure de sitter space is the unique vacuum solution to the Einstein equation with maximal symmetry and constant positive curvature. In D= n+1 space time dimensions, it is locally characterized by

$R_{\mu\nu} = \frac{D-1}{R^2} g_{\mu\nu}$................................. 5

Where R is the radius of curvature of de sitter space, and by the vanishing of the Wey l tensor. The cosmological constant Λ is given as a fuction of R by

$\Lambda = \frac{(D-1)(D-2)}{2R^2}$.............................. 6

It is convenient to think of de sitter space as a hyper surface embedded in D+1-dimensional flat Minkowski space. The embedding equation is

$-X_0{}^2 + X_1{}^2 - - - - - - - \mp -X_p{}^2 = R^2$.................................. 7

And the resulting time likes hyperboloid. The embedding equation makes manifest the O(1,D) isometric group of de sitter space. The de sitter metric is the induced metric from the flat Minkowski metric on the embedding space. In this way several coordinates system can be obtained.

Frequently used are the so called global coordinates, in terms of which the metric takes the from

$ds^2 = -dT^2 + R^2 cosh^2 T / R d\Omega^2{}_n$............................... 8

The time coordinates T takes value $-\infty < T < \infty$. In these coordinates, which cover all of the hyperboloid the de sitter space starts out as an infinitely large n-sphere at T= $-\infty$. Subsequently, it shrinks and reaches its minimal radius R at T=0, after which it re-expands. Note that the global time coordinates T does not define a Killing vector. The appearance of a cosmological horizon is not manifest in these coordinates. Before discussing other useful coordinates system for de-sitter space.

The Debate with Willem de Sitter

The road to cosmology passed the Netherlands. The initial investigation about different effects of the universe can be inferred in a few letters between Einstein and Hendark A Lorentz. They discussed in particular about the relativity of rotation and about the relationships among fixed stars, centrifugal forces and coriolis forces.

However, it was Willem de sitter who gave important contribution the cosmological consequence of General Relativity.

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Einstein and de sitter met in Leiden in the fall of 1916. They held a fruitful correspondence focused in particular on the boundary condition at spatial infinity. It was during such a debate that both Einstein and de Sitter formulated their own mathematical expressions of the metric of the universe, i.e. the first two relativistic cosmological models.
According Machian view ‘’in a consistent theory of Relativity” – Einstein wrote there can be no inertia relativity to ‘space ‘but only inertia of masses relatively to one another. This statement required that at very large distance from all masses the sources of influence a mass test: intrtia of this body had to be zero. Such a condition was represented by a space- time that at infinity was pseudo – Euclidean.
In the absolute reference frame of Newtonian theory this condition could be expressed in relativistic notation, by requiring that at infinity the $g_{\mu\nu}$'s assumed the diagonal values of Minkowski flat space-time

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix} \ldots\ldots\ldots\ldots \quad 9$$

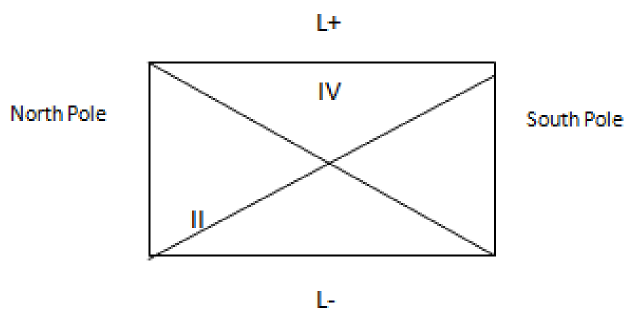
The presence of some material sources did not influence this value, except for the g_{44} term, which became in this case:

$$g_{44} = 1 + \frac{2\emptyset}{c^2} \ldots\ldots\ldots\ldots \quad 10$$

The gravitational potential Ø was produced by these sources, and could be calculated by Poisson's equation.

Penrose Diagram of De Sitter Space

Penrose Diagram



To elucidate the causal structure of de Sitter space. Through the coordinate transformation

$$\cosh {}^{T}/_{R} = \frac{1}{\cos {}^{\tau}/_{R}} \ldots\ldots\ldots\ldots 12$$

Put the metric (8) in the form

$$ds^2 = \frac{1}{cos^{2\tau/R}}(-d\tau^2 + R^2 R d\Omega^2{}_n) \ldots\ldots\ldots\ldots \quad 13$$

The range of the new time coordinate τ is ${}^{\pi}/_{2} < {\tau}/_{R} < {}^{\pi}/_{2}$. Without affecting the causal structure, we can perform a conformal transformation, bringing the metric to the form

$$d\hat{s}^2 = cos^2\, {}^{\tau}/_{R}\, ds^2 = -d\tau^2 + R^2 d\Omega^2{}_n \ldots\ldots\ldots\ldots 14$$

This can also be written as

$$d\hat{s}^2 = -d\tau^2 + R^2(d\theta^2 + sin^2\theta d\Omega^2{}_{n-1}) \ldots\ldots\ldots\ldots \quad 15$$

Where θ is a poler angle with range $0 \leq \theta \leq \pi$. where every point is a n-1 dimensional sphere with radius Rs in θ.

ACKNOWLEDGMENT
The authors would like to thank the external reviewers for the suggestion and it was helpful during revision of the manuscript.

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