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Research Article

HIGHER DIMENSIONAL VISCOUS COSMOLOGICAL MODEL WITH VARIABLE GRAVITATIONAL AND COSMOLOGICAL CONSTANTS

***Ghate H.R.[1] and Sandhya Mhaske[2]**
[1]*Department of Mathematics, Jijamata Mahavidyalaya, Buldana (India) – 443 001*
[2]*Department of Mathematics, Jijamata Mahavidyalaya, Buldana (India) – 443 001*
**Author for Correspondence*

ABSTRACT
A Homogeneous and isotropic Friedmann-Robertson-Walker (FRW) viscous cosmological model with varying gravitational and cosmological constants in the context of higher dimensional space times is studied. The exact solutions of the Einstein's field equations are obtained which are singularly free and deceleration parameter is in general not a constant, unless we assume perfect fluid with equation of state in the standard cosmologies. The introduction of viscosity is not only free from singularity but also give deceleration parameter a freedom to vary with scale factor.

Key Words: *Cosmological Model, Viscous Fluid, Gravitational Constant, Cosmological Constants*

INTRODUCTION
Nowadays the researchers are much interested in the theory of higher dimensional space-times due to its direct dependence on the strength of gravitational force. The concept of gravitation and electromagnetism could be unified by well known five dimensional theories which were given by Kaluza (1921) and Klein (1926), in the single geometrical structure. Thiry (1918) and Jordan (1959) further generalized the consideration of coefficient of fifth coordinate as constant, which was the idea of Kaluza and Klein. After this Marciano's (1984) theory suggested that strong evidence of higher dimensions may be experimental directions of time variation of fundamental constants. The extension of Kaluza and Klein's formalism to extra dimension is given by number of authors (Witten, 1984; Appelquist *et al.*, 1987) for achieving the unification of all interactions including strong and weak forces. Bianchi type-I cosmological models with viscous fluid in higher dimensional space time is given by Banerjee *et al.*, (1990). Bianchi type-I string cosmological model in higher dimensions is considered by Krori *et al.*, (1994). Also Chatterjee and Bhui (1990), Singh *et al.*, (2004), Rahaman *et al.*, (2003) have investigated on the theories consistis the concept of higher dimensions.
The idea of variable gravitational constant G was first introduced by Dirac (1937), Lau (1985) in the context of general relativity who proposed modifications for linking the variation of G and Λ. Various works have been carried out for a modified general relativity theory with this variation in G. Several cosmological models with Friedmann-Robertson-Walker model (FRW) metric were studied by the number of authors such as Beesham (1986a, b), Berman (1983, 1991a, 1991b), Kalligas *et al.*, (1992), Abdusattar and Vishwakarma (1997) by linking of variation of G & Λ. Recently, FRW cosmological models with variation of G & Λ in the framework of R^2 theory was established by Debnath and Paul (2006). Singh (2006) has established FRW cosmological models with variable G and Λ in general Relativity by using the equation of state $p=(\gamma-1)\rho$, where γ varies continuously as the universe expands. A cosmological constant of the form $\lambda=\beta\frac{\dot{R}}{R}$, where β is the constant has investigated by the authors Al-Rawaf & Taha (1996) and Al-Rawaf (1998).
A large scale distribution of galaxies in our universe shows that the matter distribution is satisfactorily described by perfect fluid. However a realistic treatment of the problem requires the consideration of the material distribution other than perfect fluid, which is supported by the fact when neutrino decoupling occurred, the matter behaved like viscous fluid in early stage of universe. Misner (1967, 1968) has studied

the effect of viscosity on evolution of universe. Number of authors Santos *et al.,* (1985), Coley and Tupper (1983), Roy and Prakash (1977), Goenner and Kowalewski (1989), Padmanabhan and Chitre (1987), Ram and Singh (1998), Bali *et al.,* (1989, 2004, 2005, 2007), have studied the effect of bulk viscosity on the evolution of universe at large and demonstrated that bulk viscosity can lead to inflation like solution. Gron (1990) have studied viscous inflationary universe models.

In the present paper, we have considered a higher dimensional homogeneous and isotropic Friedmann-Robertson-Walker (FRW) viscous cosmological model with varying gravitational and cosmological constants. The work of Singh *et al.,* (2011) is extended to five dimensions.

2. Field Equations

The higher dimensional Friedmann-Robertson-Walker metric has been considered in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta_1^{\,2} + \sin^2\theta_1 d\theta_2^{\,2} + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2)\right], \tag{1}$$

where $R(t)$ is the scale factor and $k = -1, 0, +1$ is the curvature parameter for open, flat and closed universe respectively.

The Einstein's field equations with time varying cosmological and gravitational constants are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G(t)T_{ij} - \lambda(t)g_{ij}\ , \tag{2}$$

where R_{ij} is a Ricci tensor, G (t) and $\lambda(t)$ being the variable gravitational and cosmological constants.

The energy momentum tensor due to bulk viscous fluid is written in the form

$$T_{ij} = (\bar{p} + \rho)u_i u_j - \bar{p}\, g_{ij}\ , \tag{3}$$

together with

$$\bar{p} = p - 3\xi\frac{\dot{R}}{R} = p - 3\xi H\ ,$$

where u^i is the five velocity vector of the distribution, ρ is energy density, $\bar{p}$ is effective pressure of field and ξ is coefficient of bulk viscosity that determines the magnitude of viscous stress relative to the expansion, $H = \frac{\dot{R}}{R}$ is the Hubble parameter.

The Einstein's field equations (2) with the metric (1) and the energy momentum tensor (3) can be written as

$$6\frac{\ddot{R}}{R} = -8\pi G(t)\left[\rho + 2(p - 3\xi H\ \right] + \lambda(t) \tag{4}$$

$$6\frac{\dot{R}^2}{R^2} + 6\frac{k}{R^2} = 8\pi G(t)\left[\rho + \frac{\lambda(t)}{8\pi G(t)}\right]\ . \tag{5}$$

Eliminating $\ddot{R}$ from equations (4) and (5) we get

$$\dot{\rho} + (4p + \rho)H - 12\,\xi\, H^2 = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \tag{6}$$

The energy momentum conservation $T_{;j}^{\ ij} = 0$ yields

$$\dot{\rho} + 4\left(\bar{p} + \rho\right)\frac{\dot{R}}{R} = 0\,. \tag{7}$$

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Using (7), equation (6) reduces to

$$\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G} = 3\rho H \,. \tag{8}$$

In terms of Hubble parameter $H = \frac{\dot{R}}{R}$, equations (4) and (5) can be written as

$$\dot{H} + H^2 = -\frac{4\pi G}{3}\left[\rho + 2(p - 3\xi H)\right] + \frac{\Lambda(t)}{6} \tag{9}$$

$$H^2 = \frac{4\pi G}{3}\left[\rho + \frac{\Lambda(t)}{8\pi G}\right] - \frac{k}{R^2} \,. \tag{10}$$

3. Solution of the Field Equations

To solve the field equations (9) and (10), the energy density ρ and pressure p of the perfect fluid are related by the equation of the state given by

$$\bar{p} = (\gamma - 1)\rho \,, \quad 1 \le \gamma \le 2 \tag{11}$$

where γ is adiabatic parameter.

In most of investigations in cosmology, the viscosity is assumed to be simple power law function of the energy density i.e.

$$\xi = \xi_0 \rho^n \,, \tag{12}$$

where ξ_0 and n are constants $(\xi_0 \ge 0)$ and the variable cosmological constant Λ is

$$\Lambda = 3\beta H^2 \,. \tag{13}$$

Using (11) , equation (9) transforms to

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(2\gamma - 1)\rho + 8\pi G \xi H + \frac{\Lambda(t)}{6} \,. \tag{14}$$

Eliminating ρ from equations (10) and (14) we get

$$\dot{H} + 2\gamma H^2 + (2\gamma - 1)\frac{k}{R^2} = 8\pi G \xi H + \frac{\gamma}{3}\Lambda(t) \,. \tag{15}$$

In particular, when the universe is flat (i.e. $k = 0$), put $k = 0$ in equation (15)

$$\dot{H} + 2\gamma H^2 = 8\pi G \xi H + \frac{\gamma}{3}\Lambda(t) \,. \tag{16}$$

Transforming equation (16) to the form

$$H' + 2\gamma\frac{H}{R} = \frac{8\pi\xi G}{R} + \frac{\gamma}{3HR}\Lambda(t) \,, \tag{17}$$

where a prime (‘) denotes the differentiation w. r. t. scale factor $R(t)$.

Using (12) and (13) in equation there in (17) yields

$$H' + (2 - \beta)\gamma\frac{H}{R} = \frac{8\pi G \xi_0 \rho^n}{R} \,. \tag{18}$$

From equations (10) and (13) we get,

$$8\pi G\rho = 3(2 - \beta)H^2 \,. \tag{19}$$

Also, from equations (11)and (7) we get

$$\rho = AR^{-4\gamma} \,, \tag{20}$$

where, A is constant.

Using (19) and (20) in equation (18), it leads to

$$H' + (2-\beta)\gamma \frac{H}{R} = 3(2-\beta) H^2 \xi_0 A^{n-1} R^{4\gamma - 4n\gamma - 1} \tag{21}$$

i.e. $H' + a\dfrac{H}{R} = bH^2 R^{4\gamma-4n\gamma-1}$,

where $a = (2-\beta)\gamma$ and $b = 3(2-\beta)\xi_0 A^{n-1}$.

Integrating (21) w. r. t. R, the solution is given by

$$H = \left\{ \frac{b}{4n\gamma - 4\gamma + a} R^{4\gamma - 4n\gamma} + CR^a \right\}^{-1}, \tag{22}$$

where C is a constant of integration.

As $\xi_0 \geq 0$ and $\beta \leq 1$, we conclude that $a \geq 0$ and $b \geq 0$.

Using $H = \dfrac{\dot{R}}{R}$ in equation (22) and integrating w.r.t. R we get,

$$CR^a + \frac{abR^{4\gamma-4n\gamma}}{4\gamma(1-n)(4n\gamma - 4\gamma + a)} = a(t - t_0), \tag{23}$$

where t_0 is an arbitrary constant of integration.

Using suitable transformation of coordinates, equation (23) can be written as

$$cR^a + DR^{4\gamma-4n\gamma} = at, \tag{24}$$

where

$$D = \frac{ab}{4\gamma(1-n)(4n\gamma-4\gamma+a)}.$$

From equation (24), we conclude that as $t \to \alpha$, $R \to \alpha$. Thus $c \geq 0$, the model has no singularity in the future. However, it is to be noted that such a non-singularity behavior is exhibited in absence of viscosity $b = 0$, $\xi = 0$, the model becomes singular.

In general the scale factor $R(t)$ cannot be expressed as a function of time. However the physical and dynamical parameter can possibly expressed as functions of scale factor which are calculated as follows:

The energy density of the fluid is

$$\rho = \frac{3(2-\beta)}{8\pi G}\left[\frac{b}{4n\gamma-4\gamma+a} R^{4\gamma-4n\gamma} + cR^a\right]^{-2}. \tag{25}$$

The coefficient of bulk viscosity ξ which determines the magnitude of viscous stress relative to expansion is

$$\xi = \xi_0 \left\{ \frac{3(2-\beta)}{8\pi G}\left[\frac{b}{4n\gamma-4\gamma+a} R^{4\gamma-4n\gamma} + cR^a\right]^{-2} \right\}^n. \tag{26}$$

The variable cosmological constant Λ is

$$\Lambda = 3\beta \left[\frac{b}{4n\gamma-4\gamma+a} R^{4\gamma-4n\gamma} + cR^a\right]^{-2} \tag{27}$$

and the variable gravitational constant G is

$$G = \frac{3(2-\beta)}{8\pi A}\left[\frac{b}{4n\gamma-4\gamma+a}R^{4\gamma-4n\gamma}+cR^{a}\right]^{-2}R^{4\gamma} . \tag{28}$$

The deceleration parameter $q = \dfrac{-\ddot{R}R}{R^2}$ is obtained as

$$q = \left[\frac{B(4\gamma-4\gamma n-1)+c(a-1)R^{a-4\gamma+4n\gamma}}{B+cR^{a-4\gamma+4n\gamma}}\right], \tag{29}$$

where $B = \dfrac{3(2-\beta)\xi_0 A^{n-1}}{(4n\gamma-4\gamma+a)}$. It is to be noted that q is a function of the scale factor $R(t)$. In absence of viscosity $B = 0\ (\xi_0 = 0)$, q becomes a constant $(a-1)$ corresponding to the perfect fluid model discussed Vishwakarma (2000) and $q \gtreqless 0$ according as $a \gtreqless 1$.

Next we discuss the following cases.
Case I : $C > 0$, $a > 1$, $B > 0$, $4\gamma(1-n) > 1$

It follows from (29) that $q \gtreqless 0$ according as

$$R(t) \gtreqless \left[\frac{B(4\gamma-4n\gamma-1)}{c(a-1)}\right]^{1/(a-4\gamma+4n\gamma)} \tag{30}$$

Hence the expansion is accelerated in the early phase and decelerated in the later phase of evolution. The constant parameter $3B = \dfrac{\Lambda}{H^2}$ has a upper limit $B < 2 - \dfrac{1}{\gamma}$.

It follows that the density parameter $\Omega_\Lambda < 0.5$ for radiation dominated universe and for dust universe $\Omega_\Lambda < 0.33$. Here the density parameter is defined as $\Omega_\Lambda < \dfrac{\Lambda}{3H^2}$.

Case II: $C > 0$, $a < 1$
The deceleration parameter q in this case derived from (28) is given by

$$q = -\left[\frac{B(4\gamma-4n\gamma-1)+c|a-1|}{B+cR^{a-4\gamma-4n\gamma}}\right]$$
$$= -\left[\frac{c|a-1|R^{a-4\gamma-4n\gamma}+B(4\gamma-4n\gamma)+1}{B+cR^{a-4\gamma-4n\gamma}}\right] \tag{31}$$

where $1-a = -|a-1|$

In this case, the expansion of the universe is accelerated throughout the evolution.
However the acceleration converges to $|a-1|$ at the later stage of evolution.

The parameter B is found within the limits $2-\frac{1}{\gamma}<B\le 1$.

It indicates that for radiation universe $1.5\Omega_\Lambda \le 3$ for dense universe $1.5<\Omega_\Lambda \le 3$ and for dust universe $1.5<-\Omega_\Lambda \le 3$.

Case III: $C=0$

The scale factor $R(t)$ can be written as

$$R(t)\propto t^{\frac{1}{4\lambda(1-n)}} \tag{32}$$

and $q=4\gamma-4n\gamma-1$

This shows that deceleration parameter is constant. The constant deceleration parameter models have been discussed by Berman (1983, 1991a, 1991b), Berman and Som (1990).

The model becomes those discussed by Arbab (1997, 2003).

The physical parameters are

$$\rho(t)\sim t^{1/1-n}$$

$$G(t)\sim t^{\frac{2n-1}{1-n}}$$

$$\xi(t)\sim t^{1/1-n}$$

CONCLUSION

In this paper we have obtained a variety of exact solutions to the field equations for higher dimensional viscous cosmological models with variable gravitational and cosmological constants. We have derived the forms of G, ρ, Λ, ξ as function of the scale factor $R(t)$. The introduction of viscosity not only frees from singularity but also give the deceleration parameter a freedom to vary with the scale factor. Thus a viscous cosmological fluid gives a more general situation in the early universe. Viscous cosmological model with variable gravitational and cosmological constant has been investigated by Singh (2011) whose work has extended and studied in five dimensions. An attempt has been made to retain Singh (2011) form of the various quantities.

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