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Research Article

UNSTEADY THERMAL CONVECTION IN A ROTATING ANISOTROPIC POROUS LAYER USING THERMAL NON-EQUILIBRIUM MODEL

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ABSTRACT
Stability of rotating anisotropic fluid-saturated porous layer heated from below and cooled from above when the fluid and solid phases are not in local thermal equilibrium is examined analytically. The Darcy model is used for the momentum equation and a two-field model that represents the fluid and solid phase temperature fields separately is used for energy equation. The linear stability analysis is used to obtain the condition for both stationary and oscillatory convection. The effect of thermal non-equilibrium and anisotropy of the porous medium on the onset of both stationary and oscillatory convection is discussed. It is found that inter-phase heat transfer coefficient stabilizes the system. There is a competition between the processes of rotation and thermal diffusion that causes the convection to set in through oscillatory mode rather than stationary. The rotation inhibits the onset of convection in both stationary and oscillatory mode. Besides, the effect of porosity modified conductivity ratio, Darcy-Prandtl number and the ratio of diffusivities on the stability of the system is reported.

Key Words: *Anisotropy, Local Thermal Non-Equilibrium, Convection, Rotation, Taylor Number, Rayleigh Number*

Nomenclature

a	horizontal wavenumber
c	Specific heat
d	Height of the porous layer
Da	Darcy number, $\dfrac{\mu_e K}{\mu_f d^2}$
g	Gravitational acceleration
h	inter phase heat transfer coefficient
H	non-dimensional inter phase heat transfer coefficient, $\dfrac{hd^2}{\varepsilon k_f}$
k	horizontal wave number
k_f, k_s	Thermal conductivity tensor of fluid phase and solid phase respectively $k_{fh}(\mathrm{ii}+\mathrm{jj})+k_{fz}\mathrm{kk}$, $k_{sh}(\mathrm{ii}+\mathrm{jj})+k_{sz}\mathrm{kk}$.
K	Permeability tensor, $K_h(\mathrm{ii}+\mathrm{jj})+K_z\mathrm{kk}$.
$\mathrm{K_x}$	Permeability parameter along x direction
$\mathrm{K_y}$	Permeability parameter along y direction
k	unit vector in the vertical direction,
Pr	Prandtl number, $\dfrac{\mu_e(\rho c)_f}{\rho_0 k_f}$
p	Pressure
$\vec{q}$	Velocity vector, (u,v)

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Ra	Rayleigh number, $\dfrac{\rho_0 g\beta\,(T_l - T_u)Kd}{\varepsilon\mu_f\,\kappa_f}$
T	Temperature
t	Time
(x, y)	Space co-ordinates

Greek Symbols

α	Diffusivity ratio
β	Co-efficient of thermal expansion
γ	Porosity-modified conductivity ratio, $\dfrac{\varepsilon k_f}{(1-\varepsilon)k_s}$
ε	Porosity
κ	Thermal diffusivity
μ_e	Effective viscosity
μ_f	Fluid viscosity
ρ_f	Fluid density
θ	Non-dimensional temperature of the fluid phase
ϕ	Non-dimensional temperature of the solid phase
∇^2	$\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$
μ	Dynamic viscosity
ν	Kinematic viscosity, μ/ρ_0
κ	Thermal diffusivity, $k_f/(\rho_0 c)_f$
ω	Frequency
ξ^{-1}	Anisotropic permeability parameter, $\dfrac{\mathrm{K_z}}{\mathrm{K_h}}$

Subscripts

b	basic state
f	Fluid
l	Lower
Osc	oscillatory
s	solid
St	stationary
u	Upper
*	Non-dimensional
0	references
$'$	perturbed quantity

INTRODUCTION

Thermal convection in fluid saturated porous media is of considerable interest in many of the geophysical and technological problems. There are important applications in geothermal energy utilization, oil reservoir modeling, building thermal insulation, nuclear waste disposals. The problem of convective instability of a horizontal fluid saturated porous layer subject to an adverse temperature gradient has been investigated extensively by several authors in the past. The growing volume of work devoted to this area is well documented by the most recent reviews of Ingham and Pop (1998), Nield and Bejan (2006) and Vafai (2000).

In many engineering applications the porous materials are anisotropic in their mechanical and thermal properties. Anisotropy is generally a consequence of preferential orientation or asymmetric geometry of porous matrix or fibres. An excellent review of research on convective flow through anisotropic porous media has recently been well documented by Mckibbin (1986) and Storesletten (1998).

Castinel and Combarnous (1974) were the first to study the onset the thermal convection in a horizontal porous layer with anisotropic permeability. Epherre (1975) extended the stability analysis to media with anisotropic thermal diffusivity.

In many practical applications, involving hyper-porous materials and also media in which the solid and fluid phases are not in local thermal equilibrium it has been realized that the assumption of local thermal equilibrium (LTE) is inadequate for proper understanding of the heat transfer problems. In such circumstances, the local thermal non-equilibrium (LTNE) effects are taken into consideration. Nield and Bejan (2006) have discussed a two field model for energy equation. Instead of having a single energy equation, which describes the common temperature of the saturated porous media, two equations are used for fluid and solid phase separately. In two-field model, the energy equations are coupled by the terms, which account for the heat lost or gained from the other phase. The review of Kuznetsov (1996) gives detailed information about the works on thermal non-equilibrium effects. Research on thermal non-equilibrium in porous media is provided by Banu and Rees (2002).

The effect of LTNE on the onset of convection in a porous layer has been studied using non-Darcian model for stress free boundaries by Malashetty *et al.,* (2005a) and with an additional effect of anisotropy in permeability as well as thermal diffusivity by Malashetty *et al.,* (2005b). Malashetty *et al.,* (2006) and Shivakumar *et al.,* (2006) have analyzed the LTNE effects on the onset of convection in a porous layer saturated with Oldroyd B fluid. Malashetty *et al.,* (2008, 2009) have studied the onset of double-diffusive convection for the case of both densely packed and sparsely packed porous layer using thermal non-equilibrium model. Malashetty *et al.,* (2009) have studied convective instability by considering Maxwell fluid and couple stress fluid saturated porous layer using thermal non-equilibrium model. Recently, Malashetty *et al.,* (2012) have investigated double-diffusive convection in a viscoelastic fluid-saturated porous layer using thermal non-equilibrium model. Boundary and thermal non-equilibrium effects on the onset of Darcy-Brinkman convection in a porous layer is studied by Shivakumara *et al.,* (2012).

The study of effect of external rotation on thermal convection has attracted significant experimental and theoretical interest. Because of its general occurrence in geophysical and oceanic flows, it is important to understand how the Coriolis force influences the structure and transport properties of thermal convection. Some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals and rotating machinery. Straughan (2006) has considered a problem of thermal convection in a fluid-saturated porous layer using a global nonlinear stability analysis with a thermal non-equilibrium model. Vadasz (1998) has investigated the Coriolis Effect on gravity driven convection in a rotating porous layer heated from below.

The effect of thermal and mechanical anisotropy on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium is studied by Govender and Vadász (2006). Malashetty *et al.,* (2007, 2010) have studied the effect of rotation on the onset of convection for the case of both densely and sparsely packed porous medium using thermal non-equilibrium model. Linear and non-linear double-diffusive convection for the case of both densely and sparsely packed rotating porous layer using thermal non-equilibrium model is studied by Malashetty *et al.,* (2008, 2009).

The effect of rotation on the onset of thermal convection for the case of both densely and sparsely packed porous layer using thermal non-equilibrium model is studied by Malashetty *et al.,* (2007, 2010). Natural convection in a Nano fluid saturated rotating porous layer using thermal non-equilibrium model is studied by Bhadauria *et al.*, (2011). Bifurcation analysis for thermal convection in a rotating porous layer has been investigated by Shivakumara *et al.*, (2012). The analysis of present paper is carried out with a view to possible application to binary alloy solidification. A possible engineering application of the current study could include the cooling of electronic circuits found in rotating radars. In this paper we extended the work of Govender and Vadasz (2006) for the case of unsteady convection in order to know how the condition for the onset of convection is modified by LTNE, rotation, mechanical and thermal anisotropy in both steady and oscillatory state of convection. Effect of anisotropy parameter, Taylor number and Prandtl numbers on critical Rayleigh number is discussed.

1. Mathematical Formulation

We consider a Boussinesq fluid saturated anisotropic porous layer of depth d, which is heated from below and cooled from above. The lower surface is held at a temperature T_1, while the upper surface is at T_u. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two-field model for temperatures. It is assumed that at the bounding surfaces the solid and fluid phases have identical temperatures. The Darcy model is employed for the momentum equation. The basic governing equations are

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{2}{\varepsilon}\mathbf{\Omega}\times\mathbf{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_f}{\rho_0}\mathbf{g} - \frac{\nu}{K}\mathbf{q}, \tag{2}$$

$$\varepsilon(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \boldsymbol{q}.\nabla T_f = \varepsilon k_{fh}\nabla^2 T_f + h(T_s - T_f), \tag{3}$$

$$(1-\varepsilon)(\rho c)\frac{\partial T}{\partial t} = (1-\varepsilon)k\nabla T - h(T_f - T_s), \tag{4}$$

$$\rho_f = \rho_o[1-\beta(T_f - T_u)]. \tag{5}$$

We eliminate the pressure from the momentum equation and render the resulting equation and the energy equations for fluid phase and solid phase dimensionless by using the following transformations.

$$\begin{gathered}(x,y) = d(x^*,y^*), \quad (u,v,w) = \frac{\varepsilon k_f}{(\rho c)_f d}(u^*,v^*,w^*), \quad p = \frac{k_f \mu}{(\rho c)_f K}p^* \\ T_f = (T_1 - T_u)\theta + T_u, \quad T_s = (T_1 - T_u)\phi + T_u, \quad t = \frac{(\rho c)_f d^2}{k_f}t^*,\end{gathered} \tag{6}$$

to obtain

$$\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}+1\right)\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}(\nabla^2 w) + \frac{1}{\xi}\frac{\partial^2 w}{\partial z^2}\right) + Ta\frac{\partial^2 w}{\partial z^2} = (Ra\nabla_1^2 T_f - \nabla_1^2 w)\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}+1\right) \tag{7}$$

$$\frac{\partial T_f}{\partial t} + (\boldsymbol{q}.\nabla)T_f = \eta_f\nabla_1^2 T_f + \frac{\partial^2 T_f}{\partial z^2} + H(T_s - T_f), \tag{8}$$

$$\alpha\frac{\partial T_s}{\partial t} = \eta_s\nabla^2 T_s + \frac{\partial^2 T_s}{\partial z^2} - \gamma H(T_s - T_f), \tag{9}$$

where,

$$Ra=\frac{\rho_f g\beta(T_1-T_u)K_z d}{\varepsilon\,\mu_f\kappa_f},\;\; \gamma=\frac{\varepsilon\,k_f}{(1-\varepsilon)k_s},\;\; H=\frac{hd^2}{\varepsilon k_{fz}},\; \alpha=\frac{(\rho c)_s}{(\rho c)_f}\frac{k_f}{k_s}=\frac{\kappa_f}{\kappa_s},\; \eta_s=\frac{k_{sh}}{k_{sz}},$$

$$\eta_f=\frac{k_{fh}}{k_{fz}},\; \xi=\frac{\kappa_x}{\kappa_z},\; Ta^{1/2}=\frac{2\Omega dK_z}{\mu_f\varepsilon},\; \mathrm{Pr}_D=\mathrm{Pr}/Da,\; \mathrm{Pr}=\nu/\kappa_f,\;\; Da=K/\varepsilon d^2. \tag{10}$$

(The asterisks have been dropped for simplicity)

1.1 Basic State

The basic state is assumed to be quiescent and is given by

$$u=v=w=0,\qquad T_f=T_{fb}(z),\qquad T_s=T_{sb}(z). \tag{11}$$

The basic state temperatures of fluid phase and solid phase satisfy the equations

$$\frac{d^2T_{fb}}{dz^2}+H(T_{sb}-T_{fb})=0, \tag{12}$$

$$\frac{d^2T_{sb}}{dz^2}-\gamma H(T_{sb}-T_{fb})=0, \tag{13}$$

with boundary conditions

$$T_{fb}=T_{sb}=1\quad at\; z=0,$$
$$T_{fb}=T_{sb}=0\quad at\; z=1, \tag{14}$$

so that the conduction state solutions are given by

$$T_{fb}=T_{sb}=(1-z). \tag{15}$$

1.2 The perturbed state

The basic state is perturbed and the quantities in the perturbed state are given by

$$(u,v,w)=(u',v',w'),\quad T_f=T_{fb}+\theta,\quad T_s=T_{sb}+\phi. \tag{16}$$

Substituting the Equations (16) into Equations (7)-(9) and using the basic state solutions, we obtain the following equations for the perturbed quantities (after neglecting the primes)

$$\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}+1\right)\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}(\nabla^2 w)+\frac{1}{\xi}\frac{\partial^2 w}{\partial z^2}\right)+Ta\frac{\partial^2 w}{\partial z^2}=(Ra\nabla_1^2\theta-\nabla_1^2 w)\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial t}+1\right), \tag{17}$$

$$\frac{\partial\theta}{\partial t}-w=\eta_f\nabla_1^2\theta+\frac{\partial^2\theta}{\partial z^2}+H(\phi-\theta), \tag{18}$$

$$\alpha\frac{\partial\phi}{\partial t}=\eta_s\nabla_1^2\phi+\frac{\partial^2\phi}{\partial z^2}-\gamma H(\phi-\theta). \tag{19}$$

Since the fluid and solid phases are not in local thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption that the solid and fluid phases have equal temperatures at the bounding surfaces made at the beginning of this section helps in overcoming this difficulty. Accordingly, Equations (17) to (19) are solved for impermeable isothermal boundaries. Hence the boundary conditions are

$$w = 0 \quad \text{at} \quad z = 0,\ 1, \tag{20}$$

$$\theta = \phi = 0 \quad \text{at} \quad z = 0, \quad 1. \tag{21}$$

2. Linear Stability Theory

To study the linear stability theory, we use the linearized version of equations (17)-(19). The principle of exchange of stabilities holds in the presence of anisotropy and non-LTE effects (there is only one destabilizing agency) so that the onset of convection is stationary. We seek the solutions to the linearized equations in the form

$$[w,\theta,\phi] = [W(z),\Theta(z),\Phi(z)]\exp[i(lx+my)+\omega t], \tag{22}$$

Substituting the equations (22) in equations (17) – (19) we obtain the following matrix equation

$$\left[\left(\frac{\omega}{\Pr_D}+1\right)\left(\frac{\omega}{\Pr_D}(D^2-a^2)+\frac{1}{\xi}D^2-a^2\right)+TaD^2\right]W + a^2 Ra\Theta\left(\frac{\omega}{\Pr_D}+1\right) = 0 \tag{23}$$

$$(D^2 - a^2\eta_f - \omega)\Theta + W + H(\Phi - \Theta) = 0 \tag{24}$$

$$(D^2 - a^2\eta_s - \alpha\omega)\Phi - \gamma H(\Phi - \Theta) = 0 \tag{25}$$

where $D = d/dz$ and $a^2 = l^2 + m^2$.

The boundary conditions now become,

$$W = \Theta = \Phi = 0, \text{ at } z = 0,\ 1 \tag{26}$$

We assume the solution to W, Θ and Φ in the form,

$$W = W_0 \sin \pi z,\ \Theta = \Theta_0 \sin \pi z,\ \Phi = \Phi_0 \sin \pi z, \tag{27}$$

which satisfy the boundary conditions (26). Substituting Eqs. (27) into Eqs. (23)- (25) we obtain the following matrix equation

$$\begin{pmatrix} \left(1+\frac{\omega}{\Pr_D}\right)(B_o+\frac{\delta^2\omega}{\Pr_D})+Ta\pi^2 & -\left(1+\frac{\omega}{\Pr_D}\right)Ra\,a^2 & 0 \\ 1 & -(B_1+\omega+H) & H \\ 0 & \gamma H & -(\pi^2+\eta_s a^2+\alpha\omega+\gamma H) \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{28}$$

By setting the determinant of the above matrix to zero we get

$$Ra = \left((B_o + \frac{\delta^2 \omega}{\Pr_D}) + Ta\pi^2 C_o \right)\left((B_1 + \omega + H) - \frac{\gamma H^2}{\pi^2 + \eta_s a^2 + \alpha\omega + H\gamma} \right). \tag{29}$$

The growth rate ω is in general a complex quantity such that $\omega = \omega_r + i\omega_i$. The system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability state $\omega_r = 0$, therefore we now set $\omega = i\omega_i$ in Eq. (29) and clear the complex quantities from the denominator, to obtain

$$Ra = \Delta_1 + i\,\omega_i\,\Delta_2, \tag{30}$$

where

$$\Delta_1 = \frac{1}{a^2}\left(B1 + \frac{H(B2+\alpha^2\omega^2)}{B3+\alpha^2\omega^2} \right)\left(Ta\pi^2 Co + Bo\right) - \left(1 + \frac{\alpha\gamma H^2}{B3+\alpha^2\omega^2} \right)\left(\delta^2 - Ta\pi^2 Co\right)\frac{\omega}{\Pr_D a^2}, \tag{31}$$

$$\Delta_2 = \frac{1}{a^2}\left(B1 + \frac{H(B2+\alpha^2\omega^2)}{B3+\alpha^2\omega^2} \right)\left(\delta^2 - Ta\pi^2 Co\right) + \left(Ta\pi^2 Co + Bo\right)\left(1 + \frac{\alpha\gamma H^2}{B3+\alpha^2\omega^2} \right). \tag{32}$$

i) Stationary Convection

The direct bifurcation (steady state) corresponds to ω_i=0 and steady convection occurs at

$$Ra^{St} = \frac{1}{a^2}\left(\pi^2 + a^2\eta_f + \frac{H((\pi^2 + a^2\eta_s))}{(\pi^2 + a^2\eta_s + \gamma H)} \right)\left(Ta\pi^2 + \pi^2\xi^{-1} + a^2\right) \tag{33}$$

Which coincides with result obtained by Govender and Vadasz (2007).

ii) Oscillatory Convection

The Hopf bifurcation corresponds to et $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$a_1\left(\omega^2\right)^2 + a_2\left(\omega^2\right) + a_3 = 0, \tag{34}$$

where,

$$a_1 = B_o\alpha^2 - a^2 B_1\alpha^2\delta^2 - a^2 H\alpha^2\delta^2,$$

$$a_2 = B_3 B_o + B_o \Pr_D^2 \alpha^2 + \Pr_D^2 \pi^2 Ta\alpha^2 + a^2 B_1 \Pr_D^2 \pi^2 Ta\alpha^2 + a^2 H \Pr_D^2 \pi^2 Ta\alpha^2 + B_o H^2 \alpha\gamma - a^2 B_1 B_3 \delta^2 - a^2 B_2 H\delta^2 - a^2 B_1 \Pr_D^2 \alpha^2\delta^2 - a^2 H \Pr_D^2 \alpha^2\delta^2,$$

$$a_3 = B_3 B_o \Pr_D^2 + B_3 \Pr_D^2 \pi^2 Ta + a^2 B_1 B_3 \Pr_D^2 Ta + a^2 B_2 H \Pr_D^2 \pi^2 Ta + B_o H^2 \Pr_D^2 \alpha\gamma + H^2 \Pr_D^2 \pi^2 Ta\alpha\gamma - a^2 B_1 B_3 \Pr_D^2 \delta^2 - a^2 B_2 H \Pr_D^2 \delta^2.$$

The solution of Eq.(34) is given by

$$\omega^2 = \frac{1}{2a_1}\left(-a_2 \pm \sqrt{a_2{}^2 - 4a_a a_3}\right). \tag{35}$$

Now Eq. (30) with $\Delta_2 = 0$, gives

$$Ra^{Osc} = \frac{1}{a^2}\left(B_1 + \frac{H(B_2+\alpha^2\omega^2)}{B_3+\alpha^2\omega^2}\right)\left(Ta\pi^2 Co + B_0\right) - \left(1 + \frac{\alpha\gamma H^2}{B_3+\alpha^2\omega^2}\right)\left(\delta^2 - Ta\pi^2 C_0\right)\frac{\omega}{\Pr_D a^2}, \tag{36}$$

with the values of B_0, B_1, B_2 and B_3 given by

$B_0 = \pi^2\xi^{-1} + a^2$, $B_1 = \pi^2 + a^2\eta_f$, $B_2 = (\pi^2 + a^2\eta_s)(\pi^2 + a^2\eta_s + \gamma H)$, $B_3 = (\pi^2 + a^2\eta_s + \gamma H)^2$,

$$C_o = \left(1 + \frac{\omega^2}{\Pr_D}\right)^{-1}.$$

RESULTS AND DISCUSSION

Linear stability analysis of a horizontal fluid-saturated rotating anisotropic porous layer is carried out by considering a thermal non-equilibrium model. The onset thresholds of both marginal and oscillatory convection are derived analytically. The effect of rotation, anisotropy and thermal non-equilibrium on the onset of both oscillatory and steady convection is investigated.

The neutral stability curves in *Ra – a* plane for various parameter values is shown through figures 1(a-h). From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, below which the system is stable and unstable above. The points where the overstable solutions branch off from the stationary convection can be easily identified from these figures. Also we observe that for smaller values of the wavenumber each curve is a margin of the oscillatory instability and at some fixed wavenumber depending on the other parameters the overstability disappears and the curve forms the margin of stationary convection.

The effect of Taylor number *Ta* on the marginal stability curves for the fixed values of anisotropy parameter, inter-phase heat transfer coefficient, porosity modified conductivity ratio, Darcy-Prandtl number and ratio of diffusivities is depicted in figure 1(a). It is noted that there is only stationary convection for Ta =0. For Ta >2 (i.e for Ta = 2.4899, 4, 10, 20, 50,100) we observe that the instability manifests into oscillatory mode. This shows that increase in rotation allows the onset of oscillatory convection. It is also observed from this figure that the minimum of Rayleigh number for both stationary and oscillatory states increases with the Taylor number, indicating that the effect of rotation is to enhance the stability of the system in both stationary and overstable modes.

The effect of the thermal anisotropy parameters η_s and η_f for solid and fluid phases are shown in figure 1(b) and 1(d), respectively for fixed values of other parameters. The effect of increasing these parameters, for fixed value of the other, increases the critical Rayleigh number for the case of both steady and oscillatory state of convection and thus delays the onset of convection. It is to be noted that in this case the onset of convection is through oscillatory state.

In figure 1(c), we display the effect of ratio of diffusivities α on the marginal stability curves. In this case critical Rayleigh number increases with increase in α showing the effect of α is to stabilize the system. In figure 1 (e) we show the effect of mechanical anisotropy parameter $\xi\,(= K_x / K_z)$ on Rayleigh number

Ra for the values of γ =0.5 and η_f =0.5, η_s =1.0, α =0.01, $\Pr_D$ =10 and *Ta* = 25. From this figure it is evident that increase in the value of ξ decreases critical Rayleigh number Ra_c for oscillatory state and thus augments the onset of convection. This may be understood as follows: let us keep the vertical permeability K_z fixed (or the horizontal permeability K_x fixed), and vary the horizontal permeability K_x (or the vertical permeability K_z). Then an increased horizontal permeability reduces the Rayleigh number, indicating that the system becomes unstable.

The variation of marginal curves for different values of Darcy-Prandtl number $\Pr_D$, with all other parameters kept fixed is revealed in figure 1 (f), which indicates that the critical value of oscillatory Rayleigh number increases with the increase in Darcy-Prandtl number. Therefore, the Darcy-Prandtl number makes the system more stable. It is important to note that the points where the overstable solutions bifurcate into the stationary motions are shifted towards a smaller value of the wave number with the increase in Darcy-Prandtl number. Therefore, Darcy-Prandtl number reduces the region oscillatory convection. However, the reverse effect has been observed with Taylor number.

In figure 1(g) the effect of inter-phase heat transfer coefficient *H* on the neutral stability curves is shown for the values of $Ta = 25$, $\xi = 1$, $\gamma = 0.5$, $\Pr_D = 10$, η_f =0.5, η_s =1.0 and $\alpha = 0.01$. It follows that with increase in interphase heat transfer coefficient H the minimum Rayleigh number for oscillatory mode increases with *H,* indicating that the effect of inter-phase heat transfer coefficient is to stabilize the system. It is also observed that in this case the onset of convection is through oscillatory state as in the case of figure 1(b) and (d).

Figure 1(h) indicates the effect of porosity modified conductivity ratio γ on the marginal stability curves when $Ta = 25$, $H = 100$, $\xi = 1$, η_f =0.5, η_s =1.0, $\Pr_D = 10$ and $\alpha = 0.01$. We observe that with the increase in the value of γ the minimum of oscillatory Rayleigh number decreases. Therefore, the effect of γ is to advance the onset of oscillatory convection.

Figures 2(a–g) shows the variation of critical Rayleigh number with inter-phase heat transfer coefficient *H* for different values Ta, ξ, $\eta_f, \eta_s, \gamma, \Pr_D$ and α. These figures indicate that the critical Rayleigh number increases from the LTE value when *H* is small to a non-LTE value when *H* is large. Thus, the inter-phase heat transfer coefficient makes the system more stable for its intermediate values. Figure 2(a) indicates the effect of Taylor number on the critical Rayleigh number for fixed values of other parameters. We observe that the stationary critical Rayleigh number Ra_c increases with increase in Taylor number *Ta*. It is also important to note that as the Taylor number is increased further beyond a critical value the convection is bifurcated into the oscillatory mode. The critical Rayleigh number for both stationary and oscillatory mode is found to increase with the Taylor number. Therefore, the rotation enhances the stability of the system in both stationary and oscillatory modes.

Figure 2(b) shows the effect of thermal anisotropy parameter η_s of the solid phase on Ra_c for fixed values of other parameters. It is found that the critical Rayleigh number for both stationary and oscillatory state increases with increase in η_f. It is also observed that for large H the critical Rayleigh number attained common limit depending on the value of η_s.However, for small values of *H*, Ra_c is found to be independent of η_s.

The effect of thermal anisotropy parameter η_f of the fluid phase on Ra_c is shown in figure 2(c) for fixed values of other parameters. The effect is found similer to that of η_s.The effect of mechanical anisotropy parameter ξ on the critical Rayleigh number is shown in figure 2 (d) for fixed values of other parameters. We observe that the onset of convection is through oscillatory state. The critical Rayleigh number Ra_c for both stationary and oscillatory state decreases with an increase in the value of ξ . Further the value of Ra_c for both stationary and oscillatory state increases slowly with *H* reaches a maximum value and for large *H*, Ra_c ultimately approaches to an asymptotic value depending on the value of ξ.

The variation of oscillatory critical Rayleigh number with *H* for different values of porosity modified conductivity ratio γ is depicted in figure 2(e) for fixed values of other parameters. We observe from this figure that for $H \to 0$, Ra_c^{osc} is independent of γ and is close to that of the LTE case, since for very small values of *H*, there is no significant transfer of heat between the phases and the onset criterion is not affected by the properties of the solid phase. On the other hand, for large values of *H*, though the stability criterion is independent of *H*, the condition for the onset of convection is based on the mean properties of the medium and therefore, the critical Rayleigh number is function of γ . This figure also indicates that for moderate and large values of *H*, oscillatory critical Rayleigh number decreases with the increasing values of γ . Therefore, the effect of porosity modified conductivity ratio is to inhibit the stabilizing effect of both rotation and inter-phase heat transfer coefficient. It is important to note that for sufficiently large values of γ (≥ 10), the critical Rayleigh number of oscillatory convection becomes independent of *H*.

In figure 2(f) the variation of Ra_c^{osc} with *H* for different values of diffusivities ratio α is indicated for fixed values of other parameters. We observe that for small values of *H* the diffusivity ratio does not affect the stability criterion. While for large values of *H* the effect of α on Ra_c^{osc} is significant. This figure indicates that for moderate and large values of *H* the oscillatory critical Rayleigh number increases with increasing α . As α increases, the contribution of heat conduction from the solid phase becomes negligible, and therefore the critical Rayleigh number for oscillatory mode increases towards a constant value. Thus, the ratio of diffusivities reinforces the stabilizing effect of rotation and inter-phase heat transfer coefficient towards the overstable mode.

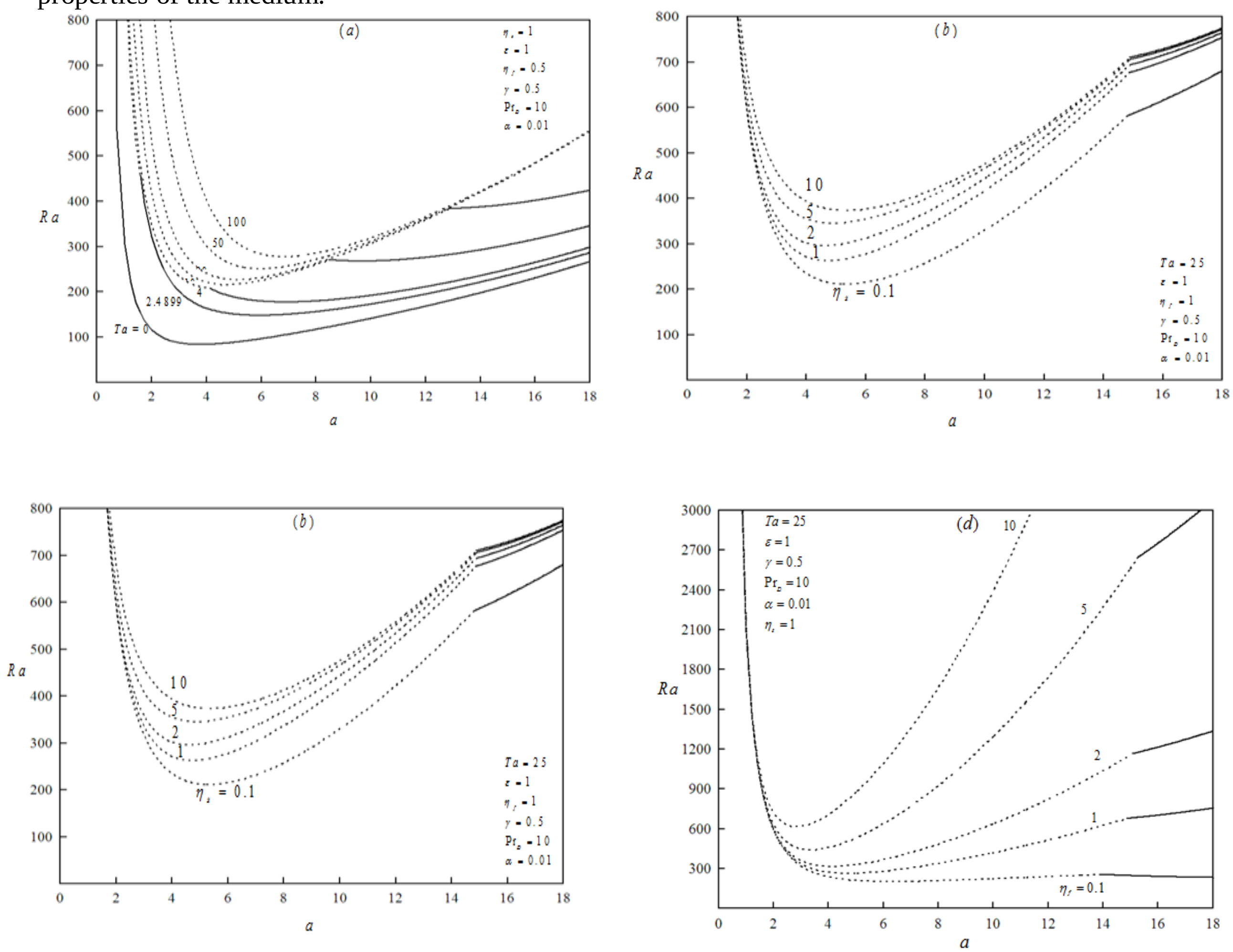
Figure 2(g) displays the variation of oscillatory critical Rayleigh number with *H* for different values of Darcy-Prandtl number $\Pr_D$ for fixed values of other parameters. This figure reveals that Ra_c^{osc} increases with the increasing values of $\Pr_D$, indicating that the Darcy-Prandtl number enhances the stability of rotating porous layer towards the overstable mode.

The variation of critical wave number with inter-phase heat transfer coefficient *H* is shown in figures 3(a-g) for different parameter values. We observe from these figures that the oscillatory critical wavenumber decreases monotonically from the LTE value when *H* is small to a non-LTE value when *H* is large, while the stationary critical wave number increases with *H* to its maximum value and then decreases back with further increase in *H*. The effect of Taylor number on critical wave number is displayed in figure 3(a). This figure indicates that the critical wave number increases with the increasing *Ta* in both stationary and overstable modes. We found that the critical wave number for the stationary mode approaches to that of LTE case when $H \to 0$ and $H \to \infty$. This is quite obvious as the corresponding physical problems are equivalent. As $H \to 0$, the solid phase ceases to affect the thermal field of the fluid, which is free to act independently, while as $H \to \infty$ the solid phase and fluid phase have identical temperatures and may be treated as a single phase. At intermediate values of *H* we observe that the critical wave number for stationary mode attains a maximum value and returns back to the LTE value. This is in agreement with the result reported in Rees paper (2002) in the absence of rotation. However, the oscillatory critical wavenumber a_c^{Osc} decreases monotonically with *H* for intermediate values of *H*.

The effect of thermal anisotropy parameter η_s of the solid phase on Ra_c is shown in figure 3(b) for fixed values of other parameters. We found that an increase in the value of η_s increases the stationary critical Rayleigh number Ra_c and decreases oscillatory critical Rayleigh number indicating that the effect of increasing the thermal anisotropy parameter of solid phase is to delay the onset of convection for the steady state where as for the oscillatory state the effect is to advance the oscillatory convection. Figure 3 (c) shows the effect of thermal anisotropy parameter η_f of the fluid phase on Ra_c for fixed values of other parameters. Its effect is found to be similar to that of η_s .

Figure 3(d) indicates the effect of mechanical anisotropy parameter ξ on the critical wave number for fixed values of other parameters. We observe that critical wave number for oscillatory state decreases with increase in ξ where as for stationary state the critical wave number increases with increase in ξ indicating the effect of ξ is to stabilize stationary state of convection and destabilize the oscillatory state of convection.

Figure 3(e) indicates the variation of critical wavenumber for both oscillatory and stationary mode with *H* for different values of γ . For stationary as stated earlier for very small values of *H* the solid phase does not affect the onset criterion, and therefore a_c^{Osc} becomes independent of γ for small *H*. On the other hand for large values of *H*, a_c^{Osc} is a function of γ , since the stability criterion depends on the mean properties of the medium.



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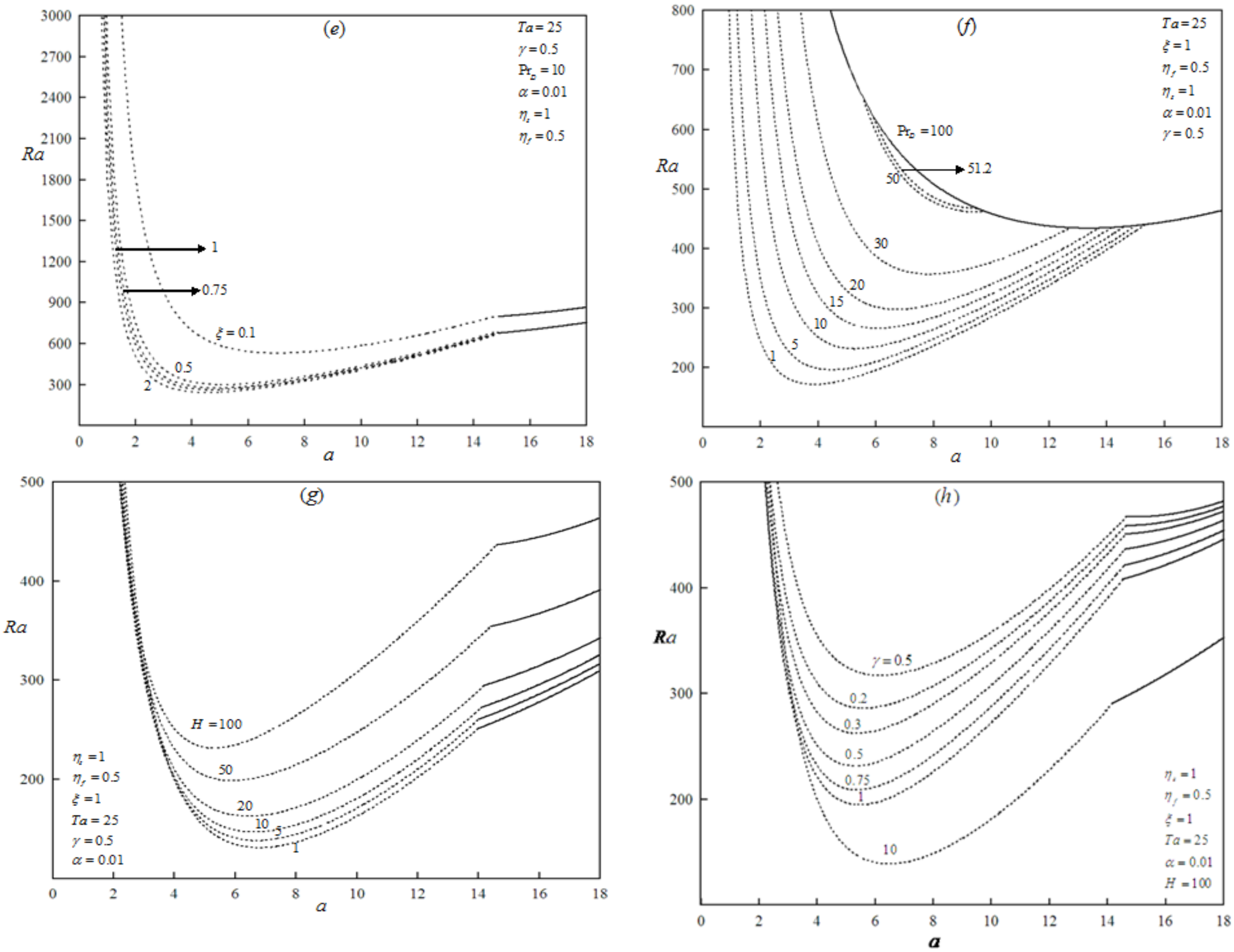
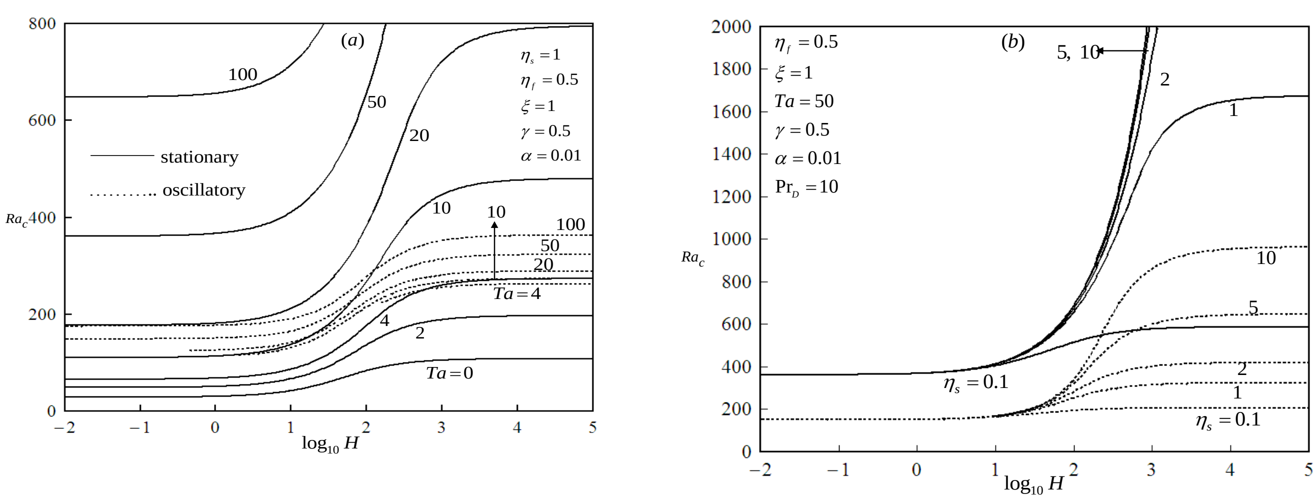


Figure 1: Neutral stability curves for different values of a) Ta, b) η_s, c) α, d) η_f, e) ξ, f) Pr_D, g) H, h) γ



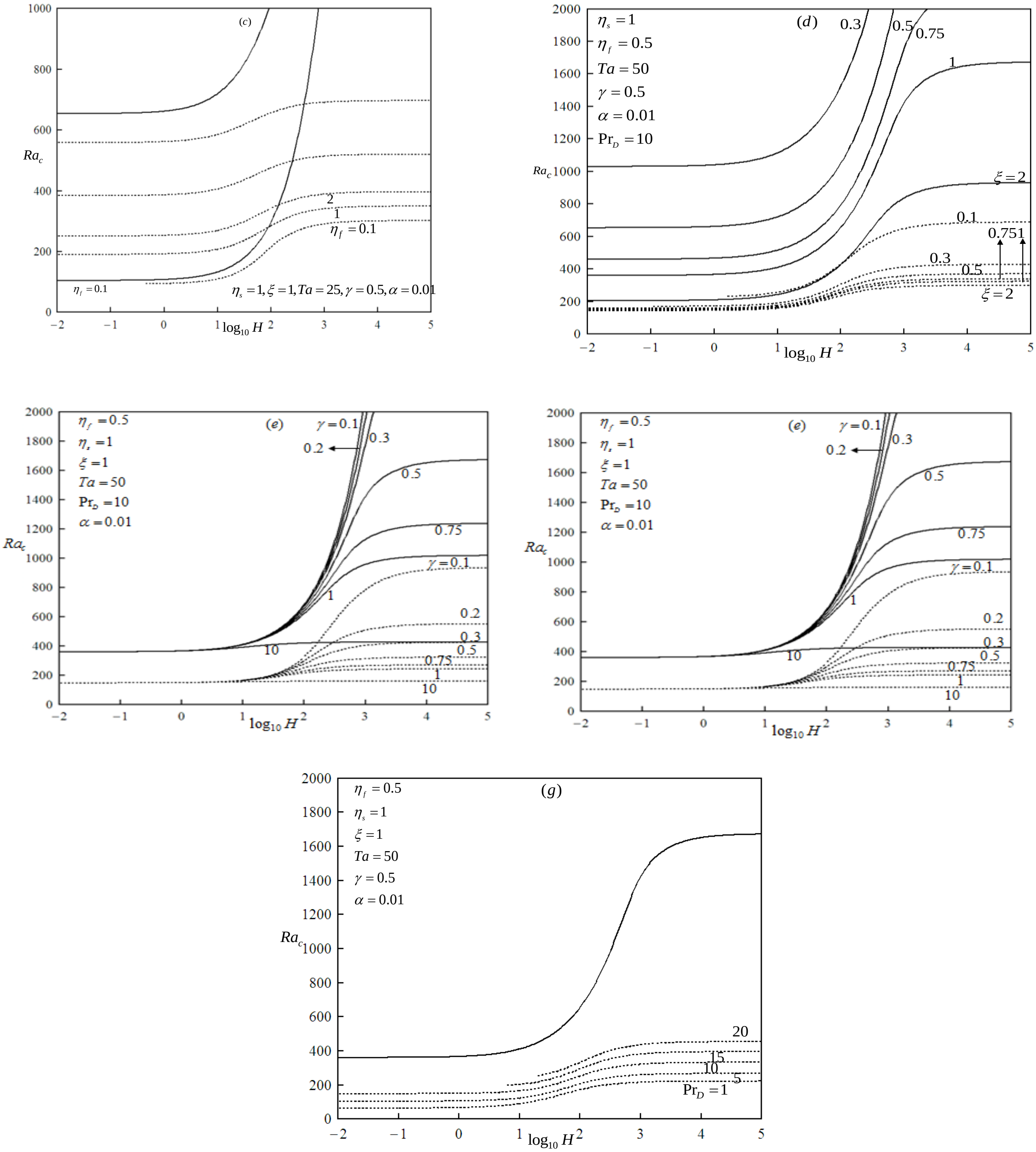


Figure 2: Variation of critical Rayleigh number with interphase heat transfer coefficient H for different values of a) Ta, b) η_s, c) η_f, d) ξ, e) γ, f) α, g) $\Pr_D$

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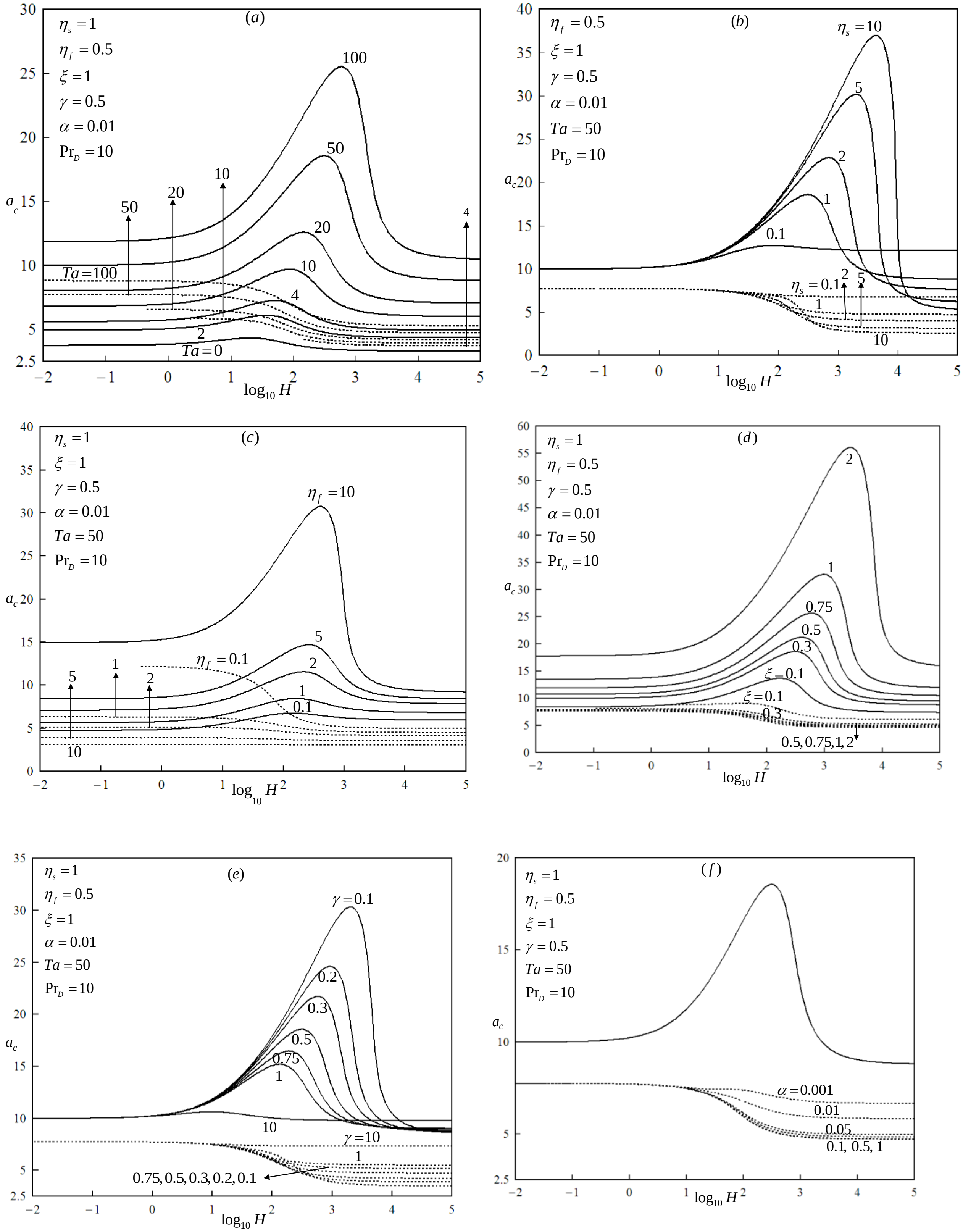

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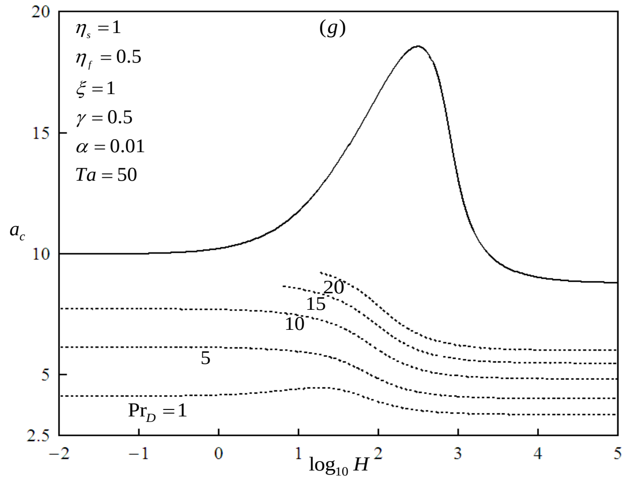

Figure 3: Variation of critical wave number with interphase heat transfer coefficient H for different values of a) Ta, b) η_s **, c)** η_f **, d)** ξ **, e)** γ **, f)** α **, g)** Pr_D

We observe from this figure that for intermediate values of *H* the critical wavenumber for the overstable mode decreases with the decreasing values of γ. For oscillatory state the critical wave number increases with increase in γ indicating the effect of γ is to delay the oscillatory convection.

The variation of oscillatory critical wavenumber with *H* for different values of diffusivity ratio α is shown in figure 3(f). The effect similar to that of conductivity ratio γ for oscillatory state is observed in this case also. In figure 3(g) we display the effect of Darcy-Prandtl number on the oscillatory critical wavenumber for fixed values of other parameters. This figure indicates that for small values of Pr_D, the critical wavenumber a_c^{Osc} increases with *H*, attains a maximum value and then decreases when *H* is increased further. Similer effect is observed for stationary mode. It is observed that the critical wave number for the oscillatory mode increases with increasing Darcy-Prandtl number.

Conclusion

The stability of a rotating anisotropic rotating fluid saturated porous layer heated from below and cooled from above when the solid and fluid phases are not in local thermal equilibrium is examined analytically. We found that there is competition between the processes of rotation and thermal diffusion that causes the convective instability to set in as an oscillatory mode rather than stationary. It is reported that the inter-phase heat transfer coefficient stabilizes the system towards both stationary and oscillatory modes. The rotation reinforces the stabilizing effect of inter-phase heat transfer coefficient.

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