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Research Article

A GOAL PROGRAMMING PROCEDURE FOR FUZZY MULTI OBJECTIVE LINEAR FRACTIONAL PROBLEM IN VAGUE ENVIRONMENT USING TOLERANCE

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ABSTRACT
This paper presents a goal programming (GP) procedure for fuzzy multi objective linear fractional programming (MOLFP) problems under vague environment using tolerance limit.
In the proposed approach, which is motivated by Mohamed (Fuzzy Sets and System 89 (1997) 215), GP model for achievement of the highest membership value of each of fuzzy, goals defined for the fractional objectives is formulated. In the solution process, the method of variable change under tolerance limit of the membership and non membership goal associated with the fuzzy goal of the model is introduced to solve the problem efficiently by using linear goal programming (LGP) methodology. The approach is illustrated by one numerical example.

Key Words: *Multiobjective Linear Fractional Programming, Fuzzy Multiobjective Linear Fractional Programming, Fuzzy Programming, Fuzzy Goal Programming, Linear Goal Programming, Membership Function, Non Membership Function, Tolerance Limit*

INTRODUCTION
During the mid-1960s and early 1970s of the last century fractional programming (FP) was studied extensively (Charnes and Cooper, 1962) in the literature. In contrast to the single objective FP, multiobjective fractional programming (MOFP) has not been discussed that extensively and only a few approaches have appeared in the literature (Craven, 1988; Kornbluth and Steuer, 1981) concerning MOFP. It may be pointed out that in most of the MOFP approaches, the problems are converted into single objective FP problems and then solved employing the method of Charnes and Cooper (Charnes and Cooper, 1962).
To overcome the computational difficulties of using conventional FP approaches to MOFP problems, the theory of fuzzy sets has been introduced in the field of FP. Linguistic variable approach of Zadeh (Zadeh, 1975) to FMOLFP problem has been proposed by Luhandjula in 1984. Luhandjula's approach has been further developed by Dutta *et al.,* (1993) and Dutta (1992). Other approaches in this area have also been investigated (Craven, 1988; Sakawa and Yumine, 1988).
In this article, the GP approach to fuzzy programming problems introduced by Mohamed (1997) is extended to solve FMOLFP problems. In the GP model formulation of the problem, first the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then achievement of the highest membership value (unity) to the extent possible of each of the fuzzy goals is considered.
Present chapter extends the tolerance approach to a special class of fuzzy multi-objective fractional goal programming (FMOFGP) problems in which the fractional objectives essentially have linear terms in the numerator and denominator. In our discussions to follow, we shall refer this class of our approach is that the region of feasible solution in this case is either same or larger than those obtained by other fuzzy goal programming models. This leads to the possibility of arriving at a better solution of problems as Fuzzy Multi-Objective Linear Fractional Goal Programming (FMOLFGP) problems.
The Advantage
1. Problem Formulation
The general format of a classical multiobjective linear fractional programming problem can be stated as

Optimize $\qquad Z_k\,(X) = \frac{c_k\,X+\,\alpha_k}{d_k\,X+\,\beta_k}$, $k = 1, 2\ldots, K$

Subject to $X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, X \ge 0, b \in R^m \right\}$, (2.1)

Where c_k, $d_k \in R^n$; α_k, β_k are constants and $S \neq \phi$.

It is customary to assume that $d_x X + \beta_k > 0, \forall X \in S$.

In MOFP if an imprecise aspiration level is introduced to each of the objective then, these fuzzy objectives are termed as fuzzy goals.

Let g_k is the aspiration level assigned to the *k*th objective $Z_k(X)$. then, fuzzy goals appar as:

(a) $Z_k(X) \gtrsim g_k$ (for maximizing $Z_k(X)$);

(b) $Z_k(X) \lesssim g_k$ (for maximizing $Z_k(X)$);

Where $\gtrsim$ and $\lesssim$ indicate the fuzziness of the aspiration levels, and is to be understood as ‘’essentially more than’’ and “essentially less than” in the sense of Zimmermann (1978).

Hence, the fuzzy linear fractional goal programming can be stated as follows:

Finx X

So as to satisfy $Z_k(X) \gtrsim g_k$, $k = 1, 2 \ldots, k_1$

$$Z_k(X) \lesssim g_k, \; k = k_1 + 1 \ldots, K$$

Subject to $$AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, \tag{2.2}$$

$$X \ge 0.$$

Now, in the field of fuzzy programming, the fuzzy goals are characterized by their associated membership functions. The membership function μ_k for the *k*th fuzzy goal $Z_k \gtrsim g_k$ can be expressed algebraically according to Tiwari *et al*., (1987).

as

$$\mu_k(X) = \begin{cases} 1 & if \quad Z_k(X) \ge g_k \\ \dfrac{Z_k(X) - l_k}{g_k - l_k} & if \quad l_k \le Z_k(X) \le g_k \\ 0 & if \quad Z_K(X) \le l_k \end{cases} \tag{2.3}$$

Where l_k is the lower tolerance limit for the *k*th fuzzy goal.

On the other hand, the membership function μ_k for the *k*th fuzzy goal $Z_k(X) \gtrsim g_k$ can be defined as

$$\mu_k(X) = \begin{cases} 1 & if \quad Z_k(X) \le g_k \\ \dfrac{u_k - Z_k(X)}{u_k - g_k} & if \quad g_k \le Z_k(X) \le u_k \\ 0 & if \quad Z_K(X) \ge u_k \end{cases} \tag{2.4}$$

where u_k is the upper tolerance limit.

Now, in a fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible is actually represented by the possible achievement of their respective membership values to the highest degree. Regarding this aspect of fuzzy programming problems, a GP approach seems to be most appropriate for the problem considered in this paper.

1.1 Goal Programming Formulation

In fuzzy programming approaches, the highest degree of membership function is 1. So, as in Mohamed (1997), for the defined membership functions in (2.3) and (2.4), the flexible membership goals with the aspired level I can be presented as:

$$\frac{Z_k(X) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1, \tag{2.5}$$

$$\frac{u_k - Z_k\ (X)}{u_k - g_k} + d_k^- - d_k^+ = 1, \qquad (2.6)$$

where d_k^- (≥ 0) and d_k^+ (≥ 0) with $d_k^-\, d_k^+ = 0$ represent the under and over-deviations, respectively, from the aspired levels.

In conventional GP, the under and/or over-deviational variables are included in the achievement function for minimizing them and that depend upon the type of the objective functions to be optimized.

In this approach, only the under-deviational variable d_k^- is required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value (Dyson, 1980).

Now it can be easily realized that the membership goals in (2.5) and (2.6) are inherently nonlinear in nature and this may create computational difficulties in the solution process. To avoid such problems, a linearization procedure is presented in the following section.

1.1.1 Linearization of Membership Goals

The *k*th membership goal in (2.5) can be presented as

$L_k\ Z_k\ (X) - L_k\ l_k + d_k^- - d_k^- = 1,$ where $L_k = \frac{1}{g_k - l_k}$

Introducing the expression of $Z_k\ (X)$ from (2.1), the above goal can be presented as

$L_k(c_k\ X + x_k) + d_k^-(d_k\ X + \beta_k) - d_k^+(d_k\ X + \beta_k) = L_k^{'}(d_k\ X + \beta_k),$

Where $L_k^{'} = 1 + L_k\ l_k$, or

$$C_k\ X + d_k^-(d_k\ X + \beta_k) - d_k^+(d_k\ X + \beta_k) = G_k, \qquad (2.7)$$

where $C_k = L_k c_k - L_k^{'}\ d_k,$ $G_k = L_k^{'}\ \beta_k - L_k x_k,$

Similar goal expressions for the membership goal in (2.6) can also be obtained.

However, for model simplification, the expression in (2.7) can be considered as a general from of goal expression for any type of the stated membership goals.

Now, using the method of variable change as presented by Kornbluth and Steuer (1981), the goal expressioin in (2.7) can be linearized as follows:

Letting $D_k^- = d_k^-\ (d_k\ X + \beta_k)$ and$D_k^+ = d_k^+\ (d_k\ X + \beta_k)$, the linear from of the expression in (2.7) is obtained as

$$C_k\ X + D_k^- - D_k^+ = G_k \qquad (2.8)$$

with $D_k^-, D_k^+ \geq 0$ and $D_k^-\ D_k^+ = 0$ since $d_k^-, d_k^+ \geq 0$ and $d_k\ X + \beta_k > 0$.

Now, in making decision, minimization of d_k^- means minimization of $D_k^-/d_k\ X + \beta_k$, which is also a non-linear one.

It may be noted that when a membership goal is fully achieved, $d_k^- = 0$ and when its achievement is zero, $d_k^- = 1$ are found in the solution.

So, involvement of $d_k^- \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\frac{D_k^-}{d_k\ X + \beta_k} \leq 1,$$

i.e.,

$$-d_k\ X + D_k^- \leq \beta_k.$$

Here, on the basis of the previous discussion, it may be pointed out that any such constraint corresponding to d_k^+ do not arise in the model formulation.

Now, if the most widely used and simplest version of GP (i.e., *minsum* GP) is introduced to formulate the model of the problem under consideration, then the GP model formulation becomes:

Find *X* so as to

Minimize $Z = \sum_{k=1}^{K} w_k^-\ D_k^-$

and satisfy $\quad C_k\, X + D_k^- - D_k^- = G_k$

subject to $\quad AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \qquad (2.9)$

and

$$-d_k\, X + D_k^- \leq \beta_k,$$
$$X \geq 0,$$
$$D_k^-, D_k^- \geq 0, \quad k = 1, 2\ldots, K,$$

where Z represents the fuzzy achievement function consisting of the weighted under-deviational variables, where the numerical weights $w_k^-\ (\geq 0), k = 1, 2, \ldots, K$ represent the relative importance of achieving the aspired of the respective fuzzy goals subject to the constraints set in the decision situation. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed (1997) and Kornbluth and Steuer (1981) can be used to assign the values to $w_k^-\ (k = 1, 2, \ldots, K)$. In the present formulation, w_k^- is determined as

$$w_k^- = \begin{cases} \frac{1}{g_k - l_k} \text{ for the defined } \mu_k \text{ in } (2.3) \\ \frac{1}{u_k - g_k} \quad \text{ for the defined } \mu_k \text{ in} (2.4) \end{cases} \qquad (2.10),$$

can then be used to solve the problem in (2.9).

2.22 Goal programming formulation for non membership function

In fuzzy programming approaches, the highest degree of non membership function is 1 so we defined goal programming corresponding to non membership function is as follows.

$$\frac{g_k - Z_k\ (X)}{g_k - l_k 1} + d_{k1}^- - d_{k1}^+ = 1, \qquad (2.11)$$

$$\frac{Z_k\ (X) - g_k\ (X)}{u_{k1} - g_k} + d_{k1}^- - d_{k1}^+ = 1, \qquad (2.12)$$

where $d_{k1}^-\ (\geq 0)$ and $d_{k1}^+\ (\geq 0)$ with $d_{k1}^-\, d_{k1}^+ = 0$ represent the under and over-deviations, respectively, from the aspired levels.

Linearization of non membership goal

The *k*th non membership goal in (2.5) can be presented as

$L_{k1}\, g_k - L_{k1}\, Z_k\, (X) + d_{k1}^- - d_{k1}^- = 1,$ where $L_{k1} = \frac{1}{g_k - l_{k1}}$

Introducing the expression of $Z_k\ (X)$ from (2.1), the above goal can be presented as

$L_{k1}\ g_k\, (d_k\, X + \beta_k) - l_{k1}^-(c_k\, X + \alpha_k) + d_{k1}^-(d_k\, X + \beta_k) - d_{k1}^+(d_k\, X + \beta_k) = (d_k\, X + \beta_k),$

Where $\quad L_k'' = 1 + L_{k1}\, l_{k1},$

or

$$C_{k1}\, X + d_{k1}^-(d_k\, X + \beta_k) - d_{k1}^+(d_k\, X + \beta_k) = G_{k1}, \qquad (2.13)$$

where $\quad C_{k1} = -L''_k c_k - d_k\, (1 - L''_k)$,

$$G_{k1} = \left(1 - L_k''\right)\beta_k + L_{k1}\alpha_k,$$

Similar goal expressions for the non membership goal in can also be obtained.

However, for model simplification, the expression in (2.13) can be considered as a general from of goal expression for any type of the stated non membership goals.

Now, using the method of variable change as presented by Kornbluth and Steuer (1981), the goal expressioin in (2.13) can be linearized as follows:

Letting $D_{k1}^{-} = d_{k1}^{-}(d_k X + \beta_k)$ and$D_{k1}^{+} = d_{k1}^{+}(d_k X + \beta_k)$, the linear from of the expression in (2.13) is obtained as

$$C_{k1} X + D_{k1}^{-} - D_{k1}^{+} = G_{k1}$$

with $D_{k1}^{-}, D_{k1}^{+} \geq 0$ and $D_{k1}^{-} D_{k1}^{+} = 0$ since $d_{k1}^{-}, d_{k1}^{+} \geq 0$ and $d_k X + \beta_k > 0$.

Now, in making decision, minimization of d_{k1}^{-} means minimization of $D_{k1}^{-}/d_k X + \beta_k$, which is also a non-linear one.

It may be noted that when a non membership goal is fully achieved, $d_{k1}^{-} = 0$ and when its achievement is zero, $d_{k1}^{-} = 1$ are found in the solution.

So, involvement of $d_{k1}^{-} \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\frac{D_{k1}^{-}}{d_k X + \beta_k} \leq 1,$$

i.e.,

$$-d_k X + D_{k1}^{-} \leq \beta_k.$$

Here, on the basis of the previous discussion, it may be pointed out that any such constraint corresponding to d_{k1}^{+} do not arise in the model formulation.

Now, if the most widely used and simplest version of GP (i.e., *minsum* GP) is introduced to formulate the model of the problem under consideration, then the GP model formulation becomes:

Find X so as to

Minimize $\quad Z = \sum_{k1=1}^{K1} w_{k1}^{-} D_{k1}^{-}$

and satisfy $\quad C_k X + D_{k1}^{-} - D_{k1}^{-} = G_{k1}$

subject to $\quad AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \qquad (2.14)$

and

$$-d_k X + D_{k1}^{-} \leq \beta_k,$$
$$X \geq 0,$$
$$D_{k1}^{-}, D_{k1}^{-} \geq 0, \; k = 1, 2 \ldots, K,$$

where Z represents the fuzzy achievement function consisting of the weighted under-deviational variables, where the numerical weights $w_{k1}^{-} (\geq 0), k = 1, 2, \ldots, K$ represent the relative importance of achieving the aspired of the respective fuzzy goals subject to the constraints set in the decision situation. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed (1997) can be used to assign the values to w_{k1}^{-} $(k = 1, 2, \ldots, K)$. In the present formulation, w_k^{-} is determined as

$$w_{k1}^{-} = \begin{cases} \frac{1}{g_k - l_{k1}} \text{ for the defined } \mu_{k1} \text{ in} \\ \frac{1}{u_k 1 - g_k} \quad \text{for the defined } \mu_{k1} \text{ in} \end{cases} \qquad (2.15),$$

The *minsum* GP method can then be used to solve the problem in (2.14).

Numerical Example

Example 1: The following numerical example studied by Luhandjula (1984) is considered to illustrate the above approach:

Maximize $\quad Z_1 = \frac{x_1 - 4}{-x_2 + 3}$

And Maximize $Z_2 = \frac{-x_1+4}{x_2+1}$

Subject to $-x_1 + 3x_2 \leq 0,$

$$x_1 \leq 6, \quad (3.1)$$

$$x_1, x_2 \geq 0.$$

Let the fuzzy aspiration levels of the two objectives be (2, 4) respectively. Then the problem can be designed as

Find X (x_1, x_2) so as to satisfy the following fuzzy goals:

$$Z_1 = \frac{x_1-4}{-x_2+3} \gtrsim 2,$$

$$Z_2 = \frac{-x_1+4}{x_2+1} \gtrsim 4,$$

Subject to the given set of constraints in (3.1).

Now, let the tolerance limits of the two fuzzy objective goals are (– 1, – 2) respectively.

The membership functions of the goals are obtained as

$$\mu_1 = \frac{(x_1-4)/(-x_2+3)+1}{3} \quad (3.2)$$

and

$$\mu_2 = \frac{(-x_1+4)/(x_2+3)+1}{6} \quad (3.3)$$

Then the membership goals can be expressed as

$$\mu_1 = \frac{(x_1-4)/(-x_2+3)+1}{3} + d_1^- - d_1^+ = 1, \quad (3.4)$$

$$\mu_2 = \frac{(-x_1+4)/(x_2+3)+1}{6} + d_1^- - d_2^- = 1, \quad (3.5)$$

where d_i^-, $d_i^+ \geq 0$ with $d_i^- \, d_i^+ = 0$, $i = 1, 2$.

Following the procedure, the membership goals are restated as:

$$x_1 + 2x_2 + D_1^- - D_1^+ = 10 \quad (3.6)$$

and

$$x_2 + 4x_2 + D_2^- - D_2^+ = 0, \quad (3.7)$$

where

$$D_1^- = 3d_1^- \left(-x_2+3\right), \; D_1^+ = 3d_1^+ \left(-x_2+3\right),$$

$$D_2^- = 6d_1^- \left(x_2+1\right), \; D_2^+ = 6d_2^+ \left(-x_2+1\right)$$

Now, the restrictions $d_1^- \leq 1$ and $d_2^- \leq 1$ give

$$3x_2 + D_1^- \leq 9, \quad (3.8)$$

$$-6x_2 + D_2^- \leq 6. \quad (3.9)$$

Thus the equivalent GP formulation is obtained as

Find X $(x_1, \, x_2)$ so as to

Minimize $(\frac{1}{3}D_1^- + \frac{1}{6}D_2^-)$

And satisfy $x_1 + 2x_2 + D_1^- - D_1^+ = 10$

$-x_1 - 4x_2 + D_2^- - D_2^+ = 0$

subject to $3x_2 + D_1^- \leq 9,$

$-6x_2 + D_2^- \leq 6,$

$-x_1 + 3x_2 \leq 6.$

$x_1 \leq 6,$ $\quad x_1, x_2 \geq 0,$

D_i^- , $D_i^- \geq 0,$ $\quad i = 1,2.$

The problem is solved by using *minsum* GP method and the optimal solution obtained is

$x_1 = 6,$ $\quad x_2 = 2,$

$Z_1 = 2,$ $\quad Z_2 = -\frac{2}{3}$

and the membership values achieved are

$$\mu_1 = 1, \qquad \mu_2 = 0.222.$$

By this model solution
Using Non Membership Function

Maximize $\quad Z_1 = \frac{x_1 - 4}{-x_2 + 3}$

And Maximize $\quad Z_2 = \frac{-x_1 + 4}{x_2 + 1}$

Subject to

$$-x_1 + 3x_2 \le 0,$$
$$x_1 \le 6, \tag{4.1}$$
$$x_1, x_2 \ge 0.$$

Let the fuzzy aspiration levels of the two objectives be (2, 4) respectively. Then the problem can be designed as

Find X (x_1, x_2) so as to satisfy the following fuzzy goals:

$$Z_1 = \frac{x_1 - 4}{-x_2 + 3} \gtrsim 2,$$
$$Z_2 = \frac{-x_1 + 4}{x_2 + 1} \gtrsim 4,$$

Subject to the given set of constraints in (4.1).

Now, let the tolerance limits of the two fuzzy objective goals be (– 2, – 1), respectively.

The non membership functions of the goals are obtained as

$$\nu_1 = \frac{2 - (x_1 - 4)/(-x_2 + 3)}{1} \tag{4.2}$$

and

$$\nu_2 = \frac{4 - (-x_1 + 4)/(x_2 + 3)}{1} \tag{4.3}$$

Then the non membership goals can be expressed as

$$\nu_1 = \frac{2 - (x_1 - 4)/(-x_2 + 3)}{1} + d_1^{'-} - d_1^{'+} = 1, \tag{4.4}$$
$$\nu_2 = \frac{4 - (-x_1 + 4)/(x_2 + 3)}{1} + d_2^{;-} - d_2^{'+} = 1, \tag{4.5}$$

where $d_i^{'-}$, $d_i^{'+} \ge 0$ with $d_i^{'-}\, d_i^{'+} = 0, \;\; i = 1, 2.$

Following the procedure, the non membership goals are restated as:

$$x_1 + x_2 - D_1^{'-} + D_1^{'+} = 7 \tag{4.6}$$

and

$$x_1 + 3x_2 + D_2^{'-} - D_2^{'+} = 1 \tag{4.7}$$

where

$$D_1^{'-} = d_1^{'-}\left(-x_2 + 3\right),\;\; D_1^{+} = d_1^{'+}\left(-x_2 + 3\right),$$
$$D_2^{'-} = d_1^{'-}\,(x_2 + 1),\;\; D_2^{'+} = d_2^{'+}\,(x_2 + 1)$$

Now, the restrictions $d_1^{'-} \le 1$ and $d_2^{'-} \le 1$ give

$$x_2 + D_1^{'-} \le 3, \tag{4.8}$$
$$-x_2 + D_2^{'-} \le 1. \tag{4.9}$$

Thus the equivalent GP formulation is obtained as

Find X $(x_1,\, x_2)$ so as to

Minimize $\quad (D_1^{'-} + D_2^{'-})$

And satisfy

$$x_1 + x_2 - D_1^{'-} + D_1^{'+} = 7$$
$$x_1 + 3x_2 + D_2^{'-} - D_2^{'+} = 1$$

subject to $x_2 + D_1^{\prime -} \le 3,$

$$-x_2 + D_2^{\prime -} \le 1.$$

$$-x_1 + 3x_2 \le 0$$

$$x_1 \le 6, \qquad x_1, x_2 \ge 0,$$

$$D_i^{\prime -},\ D_i^{\prime -} \ge 0, \qquad i = 1, 2.$$

The problem is solved by using *minsum* GP method and the optimal solution obtained is

$$x_1 = 5.25, \qquad x_2 = 1.75,$$

$$Z_1 = 1, \qquad Z_2 = -.454$$

The result shows that the optimal solution from this method is good as compare to the previous problem. But it may be noted that the second objective goal is overly satisfied here, i.e., its value is now satisfactorily lower than the given upper limit. The happening of this situation is actually exhibited by the over-deviation value of the associated non membership goal.

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