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Research Article

EXPERIMENTAL ANALYSIS TO DEVELOP GENERAL PROPERTY ON AREA OF POLYGON

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ABSTRACT
Polygon is one of the two dimensional plane figures enclosed by three or more segments of straight lines, in which two adjacent sides meet at vertices. Triangle, quadrilateral, pentagon, hexagon are the specific name of the polygon with respect to its number of sides. In this article, a new property on area of any polygon has been developed and defined with appropriate examples including necessary drawings. This property may be useful for those works related to polygon and also it will be very useful for Research scholars.

Key Words: *Polygon**,** Vertices of Polygon, Triangle, Quadrilateral, Pentagon and Hexagon*

INTRODUCTION
A *Polygon* (Eric Weisstein, 2003) is a closed two dimensional figure formed by connecting three or more straight line segments, where each line segment end connects to only one end of two other line segments. Polygon is one of the most all encompassing shapes in two- dimensional geometry. The sum of the interior angles is equal to 180 degree angle multiplied by number of sides minus two. The sum of the exterior angles is equal to 360 degree angle. From the simple triangle up through square, rectangle, tetragon, pentagon, hexagon, heptagon, octagon, nonagon, dodecagon (Eric Weisstein, 2003) and beyond is called n-gon. Depend upon its interior vertex (Eric Weisstein, 2003) angle, the polygon is divided into various categories viz.,

(i) A *Convex polygon* (Eric Weisstein, 2003) - a polygon with all diagonals in the interior of the polygon. All vertices point outward concave, at least one vertex point inward towards the center of the polygon. A convex polygon is defined as a polygon with all its interior angles less than 180°.

(ii) A *Concave polygon* (Eric Weisstein, 2003) - a non-convex. The concave polygon is a simple polygon having at least one interior angle greater than 180°.

(iii) A *Simple polygon* (Eric Weisstein, 2003) - it does not cross itself.

(iv) A *Complex polygon* (Eric Weisstein, 2003) - its path may self-intersect. It looks as combined polygons.

(v) A *Regular Polygon* (Eric Weisstein, 2003) - is a polygon in which all sides are the same length and same interior vertex angle.

Analysis and Observations
For analysis purpose simple polygons such as triangle, quadrilateral, pentagon and hexagon are taken for experiment. The sides of these primary polygons are divided into 12 equal divisions for easy calculation and observation. A new polygon is formed by are connecting 1/12th of each side by straight lines.

Let it be area of such secondary polygon in drawing as A′and area of primary polygon as A. These values of A and A_1', A_2', A_3'..... A_{11}' are measured and given in table 1, table 2, table 3 and table 4 for triangle, quadrilateral, pentagon and hexagon respectively.

Triangle
Figure 1 is a triangle. Points A, B and C are the vertices of the triangle. In this figure side AB is divided into 12 equal parts as

$$\mathrm{AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = \cdots = A_{11}B} = \frac{\mathrm{AB}}{12}$$

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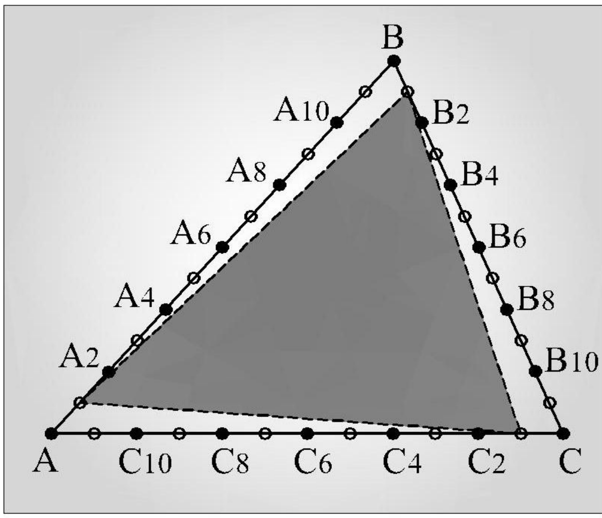
Similarly, the sides BC and CA are divided into 12 equal parts as

$$BB = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = \cdots = B_{11}C = \frac{BC}{12}$$

$$CC_1 = C_1C_2 = C_2CA_3 = C_3C_4 = C_4C_5 = C_5C_6 = \cdots = C_{11}A = \frac{CA}{12}$$

In this figure another triangle connecting A_1, B_1, C_1 and A_1 is drawn. Similarly triangle $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$…, $A_{11}B_{11}C_{11}$ was also drawn.



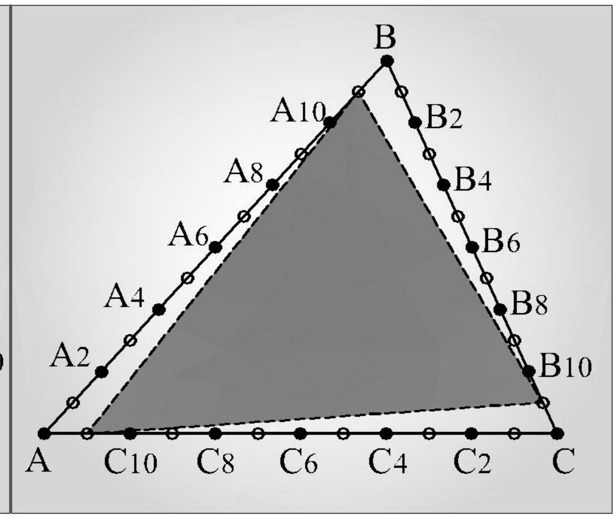


Figure 1: Triangle ABC and triangle $A_1B_1C_1$

Figure 2: Triangle ABC and triangle $A_{11}B_{11}C_{11}$

Let it be A= area of primary triangle ABC, A_1'= area of secondary triangle $A_1B_1C_1$, A_2'= area of secondary triangle $A_2B_2C_2$..., A_{11}'= area of secondary triangle $A_{11}B_{11}C_{11}$.

The area of triangles ABC, $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$… and $A_{11}B_{11}C_{11}$ were measured and tabulated in table 1.

Table 1: Area of inner triangles of triangle

S.No.	Triangle of vertices	Area measured (Sq. units)
1	ABC	72.7916
2	$A_1B_1C_1$	56.1095
3	$A_2B_2C_2$	42.4615
4	$A_3B_3C_3$	31.8459
5	$A_4B_4C_4$	24.2629
6	$A_5B_5C_5$	19.7140
7	$A_6B_6C_6$	18.1979
8	$A_7B_7C_7$	19.7140
9	$A_8B_8C_8$	24.2629
10	$A_9B_9C_9$	31.8459
11	$A_{10}B_{10}C_{10}$	42.4615
12	$A_{11}B_{11}C_{11}$	56.1095

ABC measured from AutoCAD drawing (figure 1 and 2)

These measured areas were analyzed thoroughly and observed the following things (Ref: Figure 2).

(1) The area of $\Delta\, A_1B_1C_1$ = area of $\Delta\, A_{11}B_{11}C_{11}$

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(2) The area of $\Delta\, A_2B_2C_2$ = area of $\Delta\, A_{10}B_{10}C_{10}$
…
…
(5) The area of $\Delta\, A_5B_5C_5$ = area of $\Delta\, A_7B_7C_7$
It is also observed that

(6) $\dfrac{\text{Area of }\Delta\, A_1B_1C_1}{\text{Area of }\Delta\, ABC} = \dfrac{\text{Area of }\Delta\, A_{11}B_{11}C_{11}}{\text{Area of }\Delta\, ABC} = \dfrac{56.1095}{72.7916} = 0.77083$

(7) $\dfrac{\text{Area of }\Delta\, A_2B_2C_2}{\text{Area of }\Delta\, ABC} = \dfrac{\text{Area of }\Delta\, A_{10}B_{10}C_{10}}{\text{Area of }\Delta\, ABC} = \dfrac{42.4615}{72.7916} = 0.58333$
…
…
(10) $\dfrac{\text{Area of }\Delta\, A_6B_6C_6}{\text{Area of }\Delta\, ABC} = \dfrac{18.1979}{72.7916} = 0.25$

These calculated values have are given in table-5

Quadrilateral

Figure 3 is a Quadrilateral. Points A, B, C and D are the vertices of the quadrilateral. In this figure side AB is divided into 12 equal parts as

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = \cdots = A_{11}B = \frac{AB}{12}$$

Similarly sides BC, CD and DA are divided into 12 equal parts as

$$BB = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = \cdots = B_{11}C = \frac{BC}{12}$$

$$CC_1 = C_1C_2 = C_2C_3 = C_3C_4 = C_4C_5 = C_5C_6 = \cdots = C_{11}D = \frac{CD}{12}$$

$$DD_1 = D_1D_2 = D_2D_3 = D_3D_4 = D_4D_5 = D_5D_6 = \cdots = D_{11}A = \frac{DA}{12}$$

In this figure another quadrilateral connecting A_1, B_1, C_1, D_1 and A_1 is drawn. Similarly quadrilateral $A_1B_1C_1D_1$, $A_2B_2C_2D_2$, $A_3B_3C_3D_3$… $A_{11}B_{11}C_{11}D_1$ are also drawn.

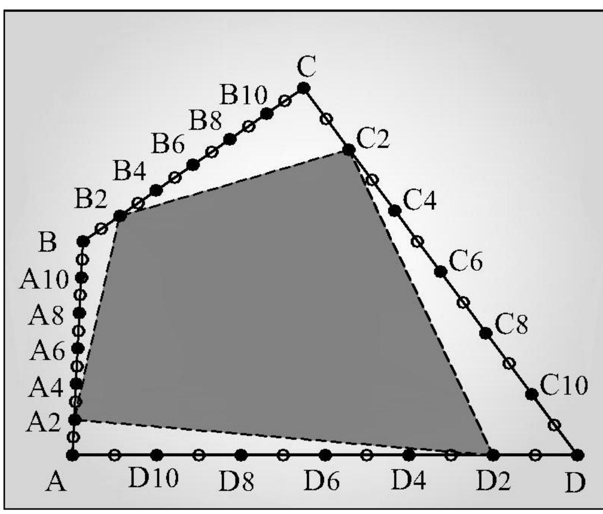

Figure 3: Quadrilateral ABCD and $A_2B_2C_2D_2$

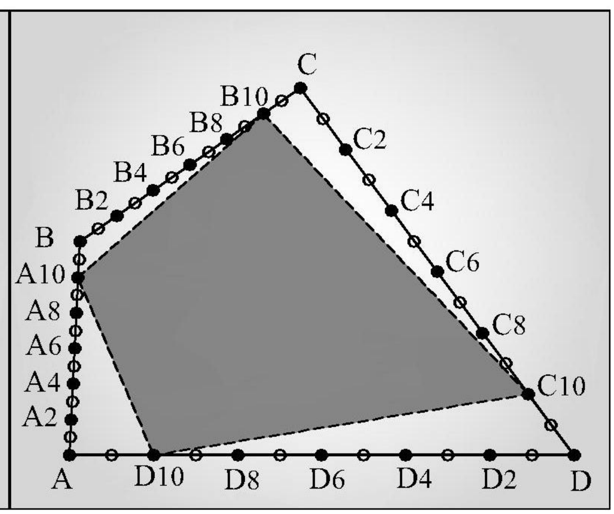

Figure 4: Quadrilateral ABCD and $A_{10}B_{10}C_{10}D_{10}$

The area of quadrilateral ABCD, $A_1B_1C_1D_1$, $A_2B_2C_2D_2$, $A_3B_3C_3D_3$…and $A_{11}B_{11}C_{11}D_{11}$ were measured and tabulated in table-2.

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These measured areas were analyzed thoroughly and observed the following things.

(11) The area of quadrilateral $A_1B_1C_1D_1$ = area of quadrilateral $A_{11}B_{11}C_{11}D_{11}$

(12) The area of quadrilateral $A_2B_2C_2D_2$ = area of quadrilateral $A_{10}B_{10}C_{10}D_{10}$ (Ref: Figure 4)

…

…

(15) The area of quadrilateral $A_5B_5C_5D_5$ = area of quadrilateral $A_7B_7C_7D_7$

Table-2: Area of inner quadrilaterals of quadrilateral

S.No.	Quadrilateral of vertices	Area measured (Sq. units)
1	ABCD	90.8135
2	$A_1B_1C_1D_1$	76.9392
3	$A_2B_2C_2D_2$	65.5875
4	$A_3B_3C_3D_3$	56.7584
5	$A_4B_4C_4D_4$	50.7584
6	$A_5B_5C_5D_5$	46.6681
7	$A_6B_6C_6D_6$	45.4068
8	$A_7B_7C_7D_7$	46.6681
9	$A_8B_8C_8D_8$	50.7584
10	$A_9B_9C_9D_9$	56.7584
11	$A_{10}B_{10}C_{10}D_{10}$	65.5875
12	$A_{11}B_{11}C_{11}D_{11}$	76.9392

ABCD measured from AutoCAD drawing (figure 3 and 4)

and also observed that

(16) $\dfrac{\text{Area of quadrilateral } A_1B_1C_1D_1}{\text{Area of quadrilateral ABCD}} = \dfrac{\text{Area of quadrilateral } A_{11}B_{11}C_{11}D_{11}}{\text{Area of quadrilateral ABCD}} = \dfrac{76.9392}{90.8135}$
$= 0.84722$

(17) $\dfrac{\text{Area of quadrilateral } A_2B_2C_2D_2}{\text{Area of quadrilateral ABCD}} = \dfrac{\text{Area of quadrilateral } A_{10}B_{10}C_{10}D_{10}}{\text{Area of quadrilateral ABCD}} = \dfrac{65.5875}{90.8135}$
$= 0.72222$

…

…

(20) $\dfrac{\text{Area of quadrilateral } A_6B_6C_6D_6}{\text{Area of quadrilateral ABCD}} = \dfrac{45.4068}{90.8135} = 0.5$

The calculated values have are given in table-5

Pentagon

Figure 5 is a Pentagon. Points A, B, C, D and E are the vertices of the pentagon. In this figure side AB is divided into 12 equal parts as

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = \cdots = A_{11}B = \frac{AB}{12}$$

Similarly side BC, CD, DE and EA are divided into 12 equal parts as

$$BB = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = \cdots = B_{11}C = \frac{BC}{12}$$

$$CC_1 = C_1C_2 = C_2C_3 = C_3C_4 = C_4C_5 = C_5C_6 = \cdots = C_{11}D = \frac{CD}{12}$$

$$DD_1 = D_1D_2 = D_2D_3 = D_3D_4 = D_4D_5 = D_5D_6 = \cdots = D_{11}E = \frac{DE}{12}$$

$$EE_1 = E_1E_2 = E_2E_3 = E_3E_4 = E_4E_5 = E_5E_6 = \cdots = E_{11}A = \frac{EA}{12}$$

In this figure another pentagon connecting A_1, B_1, C_1, D_1 E_1 and A_1 is also drawn. Similarly pentagon $A_1B_1C_1D_1E_1$, $A_2B_2C_2D_2E_2$, $A_3B_3C_3D_3E_3$… $A_{11}B_{11}C_{11}D_{11}E_{11}$ are also drawn. Therefore,

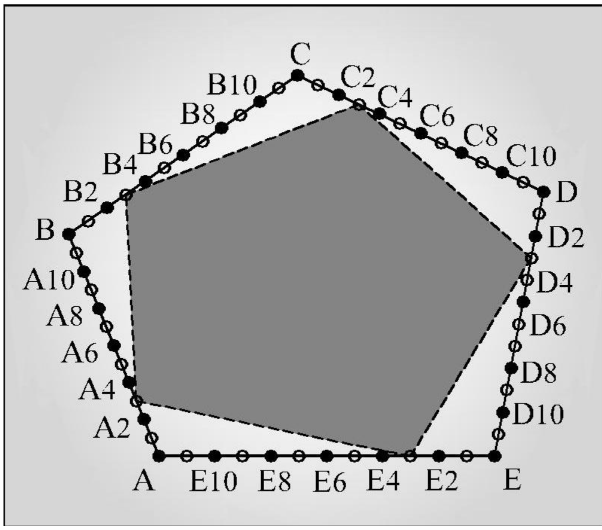

Figure 5: Pentagon ABCDE and $A_3B_3C_3D_3E_3$

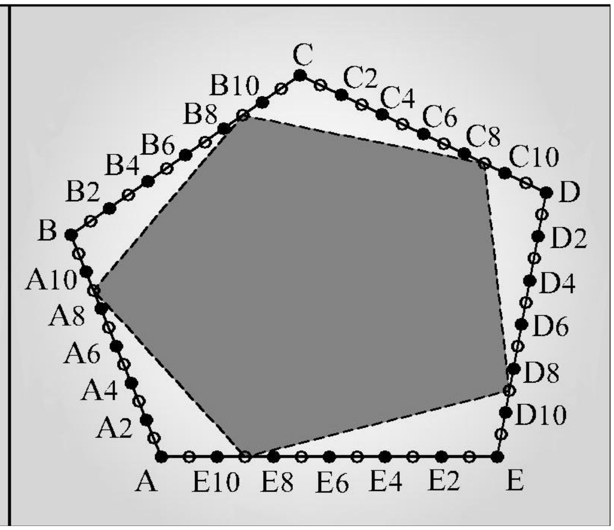

Figure 6: Pentagon ABCDE and $A_9B_9C_9D_9E_9$

The area of pentagon ABCDE, $A_1B_1C_1D_1E_1$, $A_2B_2C_2D_2E_2$, $A_3B_3C_3D_3E_3$… $A_{11}B_{11}C_{11}D_{11}E_{11}$ were measured and tabulated in table 3.

Table 3: Area of inner pentagons of pentagon

S.No.	Pentagon of vertices	Area measured (Sq. units)
1	ABCDE	96.4113
2	$A_1B_1C_1D_1E_1$	86.2222
3	$A_2B_2C_2D_2E_2$	77.8856
4	$A_3B_3C_3D_3E_3$	71.4016
5	$A_4B_4C_4D_4E_4$	66.7702
6	$A_5B_5C_5D_5E_5$	63.9913
7	$A_6B_6C_6D_6E_6$	63.0650
8	$A_7B_7C_7D_7E_7$	63.9913
9	$A_8B_8C_8D_8E_8$	66.7702
10	$A_9B_9C_9D_9E_9$	71.4016
11	$A_{10}B_{10}C_{10}D_{10}E_{10}$	77.8856
12	$A_{11}B_{11}C_{11}D_{11}E_{11}$	86.2222

ABCDE measured from AutoCAD drawing (figure 5 and 6)

(21) The area of pentagon $A_1B_1C_1D_1E_1$ = area of pentagon $A_{11}B_{11}C_{11}D_{11}E_{11}$
(22) The area of pentagon $A_2B_2C_2D_2E_2$ = area of pentagon $A_{10}B_{10}C_{10}D_{10}E_{10}$ (Ref: Figure 4)
…
…
(25) The area of pentagon $A_5B_5C_5D_5E_5$ = area of pentagon $A_7B_7C_7D_7E_7$
And also observed that

(26) $$\frac{\text{Area of pentagon } A_1B_1C_1D_1E_1}{\text{Area of pentagon ABCDE}} = \frac{\text{Area of pentagon } A_{11}B_{11}C_{11}D_{11}E_{11}}{\text{Area of pentagon ABCDE}} = \frac{86.2222}{96.4113}$$
$= 0.89432$
…

…

$$(27)\ \frac{\text{Area of pentagon } A_4B_4C_4D_4E_4}{\text{Area of pentagon ABCDE}} = \frac{\text{Area of pentagon } A_8B_8C_8D_8E_8}{\text{Area of pentagon ABCDE}} = \frac{66.7702}{96.4113}$$
$$= 0..69256$$

…

$$(30)\ \frac{\text{Area of pentagon } A_6B_6C_6D_6E_6}{\text{Area of pentagon ABCDE}} = \frac{63.0650}{96.4113} = 0.65412$$

The calculated values are shown in table 5

Hexagon

Figure 7 is a Hexagon. Points A, B, C, D, E and F are the vertices of the hexagon. In this figure side AB is divided into 12 equal parts as

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = \cdots = A_{11}B = \frac{AB}{12}$$

Similarly sides BC, CD, DE, EF and FA are also divided into 12 equal parts and side FA is divided into 12 equal parts as

$$BB = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = \cdots = B_{11}C = \frac{BC}{12}$$
$$CC_1 = C_1C_2 = C_2C_3 = C_3C_4 = C_4C_5 = C_5C_6 = \cdots = C_{11}D = \frac{CD}{12}$$
$$DD_1 = D_1D_2 = D_2D_3 = D_3D_4 = D_4D_5 = D_5D_6 = \cdots = D_{11}E = \frac{DE}{12}$$
$$EE_1 = E_1E_2 = E_2E_3 = E_3E_4 = E_4E_5 = E_5E_6 = \cdots = E_{11}F = \frac{EF}{12}$$
$$FF_1 = F_1F_2 = F_2F_3 = F_3F_4 = F_4F_5 = F_5F_6 = \cdots = F_{11}A = \frac{FA}{12}$$

In this figure another hexagon connecting A_1, B_1, C_1, D_1, E_1, F_1 and A_1 is also drawn.

Similarly pentagon $A_1B_1C_1D_1E_1F_1$, $A_2B_2C_2D_2E_2F_2$, $A_3B_3C_3D_3E_3F_3$… $A_{11}B_{11}C_{11}D_{11}E_{11}F_{11}$ are also drawn.

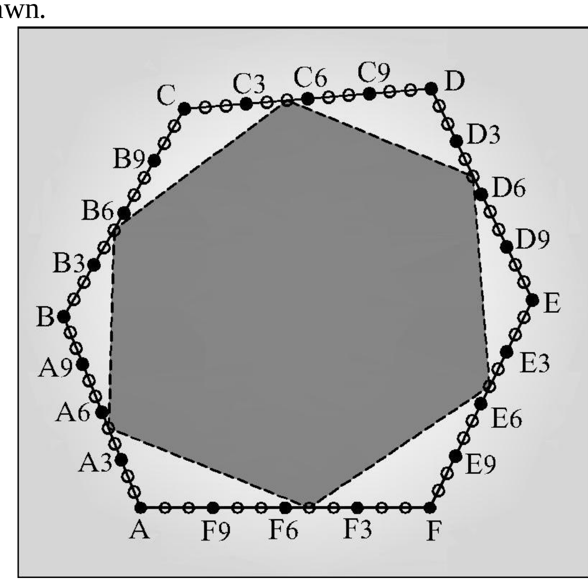

Figure 7: Hexagon ABCDEF and $A_5B_5C_5D_5E_5F_5$

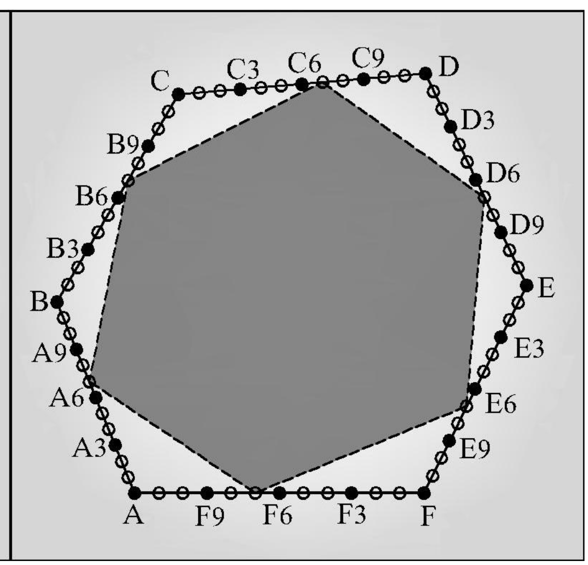

Figure 8: Hexagon ABCDEF and $A_7B_7C_7D_7E_7F_7$

The area of hexagon ABCDEF, $A_1B_1C_1D_1E_1F_1$, $A_2B_2C_2D_2E_2F_2$, $A_3B_3C_3D_3E_3F_3\ldots$ $A_{11}B_{11}C_{11}D_{11}E_{11}F_{11}$ are measured and tabulated in table 4.

(31) The area of hexagon $A_1B_1C_1D_1E_1F_1$ = area of hexagon $A_{11}B_{11}C_{11}D_{11}E_{11}F_{11}$

(32) The area of hexagon $A_2B_2C_2D_2E_2F_2$ = area of hexagon $A_{10}B_{10}C_{10}D_{10}E_{10}F_{10}$ (Ref: Figure 8)

…

…

(35) The area of hexagon $A_5B_5C_5D_5E_5F_5$ = area of hexagon $A_7B_7C_7D_7E_7F_7$

Table 4: Area of inner hexagons of hexagon

S.No.	Hexagon of vertices	Area measured (Sq. units)
1	ABCDEF	147.9697
2	$A_1B_1C_1D_1E_1F_1$	136.6803
3	$A_2B_2C_2D_2E_2F_2$	127.4435
4	$A_3B_3C_3D_3E_3F_3$	120.2593
5	$A_4B_4C_4D_4E_4F_4$	115.1278
6	$A_5B_5C_5D_5E_5F_5$	112.0488
7	$A_6B_6C_6D_6E_6F_6$	111.0225
8	$A_7B_7C_7D_7E_7F_7$	112.0488
9	$A_8B_8C_8D_8E_8F_8$	115.1278
10	$A_9B_9C_9D_9E_9F_9$	120.2593
11	$A_{10}B_{10}C_{10}D_{10}E_{10}F_{10}$	127.4435
12	$A_{11}B_{11}C_{11}D_{11}E_{11}F_{11}$	136.6803

ABCDEF measured from AutoCAD drawing (figure 7 and 8)

and also observed that

(36) $$\frac{\text{Area of hexagon } A_1B_1C_1D_1E_1F_1}{\text{Area of hexagon ABCDEF}} = \frac{\text{Area of hexagon } A_{11}B_{11}C_{11}D_{11}E_{11}F_{11}}{\text{Area of hexagon ABCDEF}} = \frac{136.6803}{147.9697} = 0.9237$$

(37) $$\frac{\text{Area of hexagon } A_2B_2C_2D_2E_2F_2}{\text{Area of hexagon ABCDEF}} = \frac{\text{Area of hexagon } A_{10}B_{10}C_{10}D_{10}E_{10}F_{10}}{\text{Area of hexagon ABCDEF}} = \frac{127.4435}{147.9697} = 0.86128$$

…

…

(39) $$\frac{\text{Area of hexagon } A_5B_5C_5D_5E_5F_5}{\text{Area of hexagon ABCDEF}} = \frac{\text{Area of hexagon } A_7B_7C_7D_7E_7F_7}{\text{Area of hexagon ABCDEF}} = \frac{112.0488}{147.9697} = 0.75724$$

(40) $$\frac{\text{Area of hexagon } A_6B_6C_6D_6E_6F_6}{\text{Area of pentagon ABCDEF}} = \frac{111.0225}{147.9697} = 0.75031$$

The calculated values are shown in table 5.

The values obtained from the drawing for various geometrical shapes of polygon were analyzed by the author and developed a formula based on the results of A'/A obtained from the experimental drawings defined above as follows:

$$\frac{A'}{A} = 1 - kf(1 - f)$$

where, A = Area of prime polygon

A' = Area of partial polygon by connecting secondary poygon

$$k = 4\sin^2\left(\frac{180°}{n}\right)$$

$$f = \frac{1}{12}, \frac{2}{12}, \frac{3}{12} \dots, \frac{11}{12} \text{ and } \frac{12}{12}$$

The values of A'/A calculated from above formula are also tabulated in table-5 for comparison. It is also proved that the formula is in order.

Table 5: Consolidated values of area of inner polygons in polygon shown in figure 1, 2, 3, 4, 5, 6, 7 and 8

S.No	Name of Polygon	No. of sides of Polygon (n)	d = Part of m no of equal division	Value obtained from drawing		Value calculated by formula		
				A'	A'/A	k = 4sin²(180/n)	f = d/12	A'/A = 1-kf(1-f)
1	Triangle	3	0	72.7916	1.0	3.0	0.0	1.0
2	Triangle	3	1	56.1095	0.77082	3.0	0.08333	0.77083
3	Triangle	3	2	42.4615	0.58333	3.0	0.16667	0.58333
4	Triangle	3	3	31.8459	0.43749	3.0	0.25	0.4375
5	Triangle	3	4	24.2629	0.33332	3.0	0.33333	0.33333
6	Triangle	3	5	19.7140	0.27083	3.0	0.41667	0.27083
7	Triangle	3	6	18.1979	0.25	3.0	0.5	0.25
8	Quadrilateral	4	0	90.8135	1.0	2.0	0.0	1.0
9	Quadrilateral	4	1	76.9392	0.84722	2.0	0.08333	0.84722
10	Quadrilateral	4	2	65.5875	0.72222	2.0	0.16667	0.72222
11	Quadrilateral	4	3	56.7584	0.62500	2.0	0.25	0.625
12	Quadrilateral	4	4	50.4519	0.55556	2.0	0.33333	0.55556
13	Quadrilateral	4	5	46.6681	0.51389	2.0	0.41667	0.51389
14	Quadrilateral	4	6	45.4068	0.5	2.0	0.5	0.5
15	Pentagon	5	0	96.4113	1.0	1.38197	0.0	1.0
16	Pentagon	5	1	86.2222	0.89432	1.38197	0.08333	0.89443
17	Pentagon	5	2	77.8856	0.80785	1.38197	0.16667	0.80806
18	Pentagon	5	3	71.4016	0.74059	1.38197	0.25	0.74088
19	Pentagon	5	4	66.7702	0.69256	1.38197	0.33333	0.69290
20	Pentagon	5	5	63.9913	0.66373	1.38197	0.41667	0.66411
21	Pentagon	5	6	63.0650	0.65412	1.38197	0.5	0.65451
22	Hexagon	6	0	147.9697	1.0	1.0	0.0	1.0
23	Hexagon	6	1	136.6803	0.92370	1.0	0.08333	0.92361
24	Hexagon	6	2	127.4435	0.86128	1.0	0.16667	0.86111
25	Hexagon	6	3	120.2593	0.81273	1.0	0.25	0.8125
26	Hexagon	6	4	115.1278	0.77805	1.0	0.33333	0.77778
27	Hexagon	6	5	112.0488	0.75724	1.0	0.41667	0.75694
28	Hexagon	6	6	111.0225	0.75031	1.0	0.5	0.75

New Property Developed on Area of Polygon

The values shown in the table-5 are examined thoroughly and observed that the result A'/A obtained from the experimental drawings and calculated from formula are found same. Therefore, the author has developed a common property on area of polygon as given below.

If A, B, C, D, …, Z are the vertices of a N-gon and side AB is equally divided into 'm' parts such that AA_1, AA_2, …, AA_{m-1} and $A_{m-1}B$, side BC is equally divided into 'm' parts such that BB_1, BB_2, …, BB_{m-1} and $B_{m-1}C$, similarly side ZA is equally divided into 'm' parts such that ZZ_1, ZZ_2, …, ZZ_{m-1} and $Z_{m-1}A$ .Therefore,

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = \cdots = A_{m-1}B = \frac{AB}{m}$$

$$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = \cdots = B_{m-1}C = \frac{BC}{m}$$

...

...

...

$$YY_1 = Y_1Y_2 = Y_2Y_3 = Y_3Y_4 = Y_4Y_5 = Y_5Y_6 = \cdots = Y_{m-1}Z = \frac{YZ}{m}$$

$$ZZ_1 = Z_1Z_2 = Z_2F_3 = Z_3Z_4 = Z_4Z_5 = Z_5Z_6 = \cdots = Z_{m-1}A = \frac{ZA}{m}$$

and the following statement

(i) $\frac{\text{Area } A_1B_1C_1D_1 \ldots Z_1A_1}{\text{Area ABCDEF} \ldots Z} = \frac{\text{Area } A_{m-1}B_{m-1}C_{m-1}D_{m-1} \ldots Z_{m-1}A_{m-1}}{\text{Area ABCDEF} \ldots Z} = 1 - kf_1(1 - f_1)$

(ii) $\frac{\text{Area } A_2B_2C_2D_2 \ldots Z_2A_2}{\text{Area ABCDEF} \ldots Z} = \frac{\text{Area } A_{m-2}B_{m-2}C_{m-2}D_{m-2} \ldots Z_{m-2}A_{m-2}}{\text{Area ABCDEF} \ldots Z} = 1 - kf_2(1 - f_2)$

…
…
…

$$\frac{\text{Area } A_{m/2}B_{m/2}C_{m/2} \ldots Z_{m/2}A_{m/2}}{\text{Area ABCDEF} \ldots Z} = 1 - kf_{m/2}\left(1 - f_{m/2}\right)$$

is always true.

Where,

m = Number of parts equally divided of each side of the polygon.

$f_1 = \frac{1}{m}, f_2 = \frac{2}{m}, f_3 = \frac{3}{m}, f_4 = \frac{4}{m} \ldots., f_{m/2} = \frac{1}{2}$

$k = 4\sin^2\left(\frac{180°}{n}\right)$

$n =$ Number of sides of the polygon

Note: This formula is applicable for either concave or convex polygon but not for complex polygon since, the complex polygon can be treated as combination of some other polygons.

CONCLUSION

In this article, the experimental analysis has been carried out with necessary drawings on area of polygon obtained by connecting relevant points which are obtained by equally dividing the each side of the polygon. Finally a general property has been developed based on the data obtained from the experiment. This property may be very useful for mechanical *fabrication* (John Sinclair, 2003) works. It may also be useful for the researchers who are interested in area of polygon and geometry.

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