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Research Article

SOME COMMON FIXED POINT THEOREMS IN FUZZY 2-METRIC SPACES UNDER STRICT CONTRACTIVE CONDITIONS FOR MAPPINGS SATISFYING NEW PROPERTY

***Amita Joshi**
Department of Mathematics, IPS Academy, ISLE, Indore, India
**Author for Correpondence*

ABSTRACT

In this paper, we define a new property that generalize the concept of non-compatible mappings and give some common fixed point theorems in fuzzy 2-metric space under strict contractive conditions for mappings satisfying new property. We extend result of Sharma and Bamboria for fuzzy 2-metric spaces.

Key Words: *Common Fixed Point, Fuzzy 2-Metric Spaces, Weakly Compatible*

INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh (1965). Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and its applications. Especially, Deng (1982), Erceg (1979), kaleva and Seikkala (1984), Kramosil and Michalek (1975) have introduced the concept of fuzzy metric spaces in different ways.

There are many view, points of the notion of a metric space in fuzzy topology, we can divide them into two groups.

The first group is formed by those results in which a fuzzy metric on a set X is treated as a map $d:XxX \rightarrow R^+$ where $X \subset I^x$ (Erceg, 1979) or X = the totality of all fuzzy points of a set (Bose and Sahani, 1987) and Hu (1985) satisfying some collection of axioms or that are analogous of the ordinary metric axioms. Thus in such an approach numerical distances are set up between fuzzy objects.

We keep in the second group results in which the distance between objects is fuzzy, the objects themselves may be fuzzy or not.

Many authors have studied common fixed point theorems in fuzzy metric spaces. The most interesting references in this direction are Cho (1997), George and veeramani (1994), Grabiec (1988), Kaleva (1985), Kramosil and Mickalek (1975), Mishra *et al.,* (1995), Sharma (2002), Sharma (2002), Sharma and Bhagwan (2002), Sharma and Deshpande (2002), Sharma and Deshpande (2002), Sharma and Deshpande (2003) and fuzzy mappings (Bose and Sahani, 1987; Butnariu, 1982; Chang, 1985; Chang *et al.,* 1997; Heilpern, 1981; Lee *et al.,* 1966; Sharma, 2002). Gähler in a series of papers Gahler (1963), Gahler (1964), Gahler (1969) investigated 2-metric spaces.

It is to be remarked that Sharma *et al.,* (1976) studied for the first time contraction type mappings in 2-metric spaces.

The aim of this paper is to define a new property that generalize the concept of non-compatible mappings and give some common fixed point theorems in fuzzy 2-metric space under strict contractive conditions. We extend results of Sharma and Bamboria (2006) for fuzzy 2-metric spaces.

Preliminaries

Definition 1 A binary operation $* :[0,1]\times[0,1] \times[0,1] \rightarrow[0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1 \leq a_2 *b_2 *c_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in $[0,1]$.

Definition 2 The 3-tuple (X, M,*) is called a fuzzy 2-metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in
$X^3 \times [0,\infty)$ satisfying the following conditions: for all $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$.

(FM-1) $M(x, y, z, 0) = 0$,

(FM-2) $M(x, y, z, t) = 1$, $t > 0$ and when at least two of the three points are equal,

(FM-3) $M(x,y,z,t) = M(x,z,y,t) = M(y,z,x,t)$,
(Symmetry about three variables)
(FM-4) $M(x,y,z,t_1 + t_2 + t_3) \geq M(x,y,u,t_1) * M(x,u,z,t_2) * M(u,y,z,t_3)$ (This corresponds to tetrahedron inequality in 2-metric space)
The function value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.
(FM-5) $M(x,y,z, .):[0,1) \rightarrow [0,1]$ is left continuous.
Example 1 Let (X, d) be a 2-metric space. Define a*b = ab (or a*b = min {a, b}) and for all $x, y \in X$ and $t > 0$,

$$M(x, y, a, t) = \frac{t}{t + d(x, y, a)} \qquad (1.a)$$

Then (X, M,*) is a fuzzy 2-metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.
Remark 1 Since * is continuous, it follows from (FM-4) that the limit of the sequence in FM-space is uniquely determined.
Let (X, M,*) is a fuzzy metric space with the following condition:
(FM-6) $\lim_{t\to\infty} M(x,y,a,t) = 1$ for all $x,y,a \in X$.
Definition 3 Let (X, M,*) is a fuzzy 2-metric space:
A sequence $\{x_n\}$ in fuzzy 2-metric space X is said to be convergent to a point $x \in X$, if
$\lim_{n\to\infty} M(x_n, x,a, t) = 1$
For all $a \in X$ and $t > 0$.
Definition 4 A pair of mappings A and S is called weakly compatible in fuzzy 2-metric space if they commute at coincidence points.
Lemma 1 For all $x, y \in X$, M(x, y, z) is nondecreasing.
Lemma 2: If, for all $x, y, a \in X$, $t > 0$ and for a number $q \in (0, 1)$,

$$M(x, y, a, qt) \geq M(x, y, a, t),$$

then x = y.

Lemma 3: If for all $x, y \in X$ $t > 0$ and for a number $k \in (0, 1)$

$$M(x, y, kt) \geq M(x, y, t),$$

then x = y.

Definition 5 Let S and T be two self mappings of a fuzzy 2-metric space (X, M, *). We say that S and T satisfy the property (S-B) if there exists a sequence $\{x_n\}$ in X such that
$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.
Example 2 Let $X = [0, +\infty[$. Define $S, T: X \to X$ by
$Tx = x/2$ and $Sx = 3x/2$, $\forall x \in X$.
Consider the sequence $x_n = 1/n$. Clearly $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = 0$.
Then S and T satisfy (S-B).
Example 3 Let $X = [1, +\infty[$. Define $S, T: X \to X$ by
$Tx = x + 1/2$ and $Sx = 2x + 1/2$, $\forall x \in X$.
Suppose property (S-B) holds; then there exists in X a sequence $\{x_n\}$ satisfying
$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.
Therefore
$\lim_{n\to\infty} x_n = z - 1/2$ and $\lim_{n\to\infty} x_n = (2z-1)/4$.
Then $z = 1/2$, which is a contradiction since $1/2 \notin X$. Hence, S and T do not satisfy (S-B).
Remark 2 It is clear from the definition of Mishra et al. and Sharma and Deshpande that two selfmappings S and T of a fuzzy 2-metric space

(X, M, *) will be non-compatible if there exists at least one sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X,$$

But for all $a \in X$, $\lim_{n\to\infty} M(STx_n, TSx_n, a, t)$ is either not equal to 1 or non-existent. Therefore two non-compatible self-mappings of a fuzzy metric space (X, M, *) satisfy the property (S-B).

RESULTS

Theorem 1 Let (X, M, *) be a fuzzy 2-metric space with $t * t \geq t$ for some $t \in [0, 1]$ and the condition (FM-6). Let A, B and S be self mappings of X into itself such that

(1.1) $AX \subset SX$ and $BX \subset SX$,

(1.2) (A, S) or (B, S) satisfies the property (S-B),

(1.3) there exists a number $k \in (0, 1)$ such that

$M(Ax, By, a, kt) > M(Ax, Sx, a, t) * M(Sx, By, a, t)$

For all $x, y, a \in X$ and $Ax \neq By$

(1.4) (A, S) and (B, S) are weakly compatible,

(1.5) one of AX, BX or SX is a closed subset of X.

Then A, B and S have a unique common fixed point in X.

Proof Suppose that (B, S) satisfies the property (S-B). Then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sx_n = z \text{ for some } z \in X.$$

Since $BX \subset SX$, there exists in X a sequence $\{y_n\}$ such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = z$. Let us show that $\lim_{n\to\infty} Ay_n = z$. Indeed, in view of (1.3), we have

$M(Ay_n, Bx_n, a, kt) > M(Ay_n, Sy_n, a, t) * M(Sy_n, Bx_n, a, t)$

$> M(Ay_n, Bx_n, a, t) * M(By_n, Bx_n, a, t)$

$> M(Ay_n, Bx_n, a)$

$M(Ay_n, Bx_n, a, kt) > M(Ay_n, Bx_n, a, t)$

Therefore by lemma 1.3, we deduce that $\lim_{n\to\infty} Ay_n = z$.

Suppose SX is a closed subset of X. Then $z = Su$ for some $u \in X$. Subsequently, we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sx_n = Su$$

By (1.3), we have

$M(Au, Bx_n, a, kt) > M(Au, Su, a, t) * M(Su, Bx_n, a, t)$

Letting $n\to\infty$, we obtain

$M(Au, Su, a, kt) > M(Au, Su, a, t) * M(Su, Su, a, t) > M(Au, Su, a, t) * 1M(Au, Su, a, kt) > M(Au, Su, a, t)$

Therefore by lemma3, we have $Au = Su$.

The weak compatibility of A and S implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$, whenever $Az = Sz$.

On the other hand, since $AX \subset SX$, there exists a point $v \in X$ such that $Au = Sv$. We claim that $Sv = Bv$. Using (1.3), we have

$M(Au, Bv, a, kt) > M(Au, Su, a, t) * M(Su, Bv, a, t) > M(Au, Au, a, t) * M(Au, Bv, a, t) > 1 * M(Au, Bv, a, t)$

$M(Au, Bv, a, kt) > M(Au, Bv, a, t)$

Therefore by lemma 3, we have $Au = Bv$.

Thus $Au = Su = Sv = Bv$. The weak compatibility of B and S implies

$BSv = SBv$ and then $BBv = BSv = SBv = SSv$.

Let us show that Au is a common fixed point of A, B and S. In view of (1.3), it follows that

$M(AAu, Bv, a, kt) > M(AAu, SAu, a, t) * M(SAu, Bv, a, t) > M(AAu, AAu, a, t) * M(AAu, Au, a, t)$

$> 1 * M(AAu, Au, a, t)$

$M(AAu, Au, a, kt) > M(AAu, Au, a, t).$

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Therefore by lemma 3**,** we have AAu = Au = SAu and Au is a common fixed point of A and S. Similarly, we prove that Bv is a common fixed point of B and S.
Since Au = Bv, we conclude that Au is a common fixed point of A, B and S.
If Au = Bu = Su = u and Av = Bv = Sv = v, then by (1.3), we have

$$M (Au, Bv, a, kt) > M (Au, Su, a, t) * M (Su, Bv, a, t)$$

$M (u, v, a, kt) > M (u, u, a, t) * M (u, v, a, t)$
$M (u, v, a, kt) > 1 * M (u, v, a, t)$
$M (u, v, a, kt) > M (u, v, a, t).$
By lemma 3, we have u = v and the common fixed point is unique. This completes the proof of the theorem.

Theorem 2 Let (X, M, *) be a fuzzy 2-metric space with $t * t \geq t$ for some $t \in [0, 1]$ and the condition (FM-6), Let A, B, S and T be self-mappings of X into itself such that
(2.1) $AX \subset TX$ and $BX \subset SX$,
(2.2) (A, S) or (B, T) satisfies the property (S-B),
(2.3) there exists a number $k \in (0, 1)$, such that
$[1 + pM (Sx, Ty, a, kt)] * M (Ax, By, a, kt)$
$\geq p [M (Ax, Sx, a, kt) * M (By, Ty,a, kt) + M (Ax, Ty,a, kt)$
$* M (By, Sx, a, kt)] + M (Sx, Ty, a, t) * M (Ax, Sx, a, t)$
$* M (By, Ty, a, t) * M (By, Sx, a, t) * M (Ax, Ty, a, (2 - \alpha) t)$
For all $x, y,a \in X$, $p \geq 0$ and $\alpha \in (0, 2)$.
(2.4) The pairs (A, S) and (B, T) are weakly compatible,
(2.5) One of AX, BX, SX or TX is a closed subset of X.
Then A, B, S and T have a unique common fixed point in X.

Proof Suppose that (B, T) satisfies the property (S-B). Then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.
Since $BX \subset SX$, there exists in X a sequence $\{y_n\}$ such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = z$. Let us show that $\lim_{n\to\infty} Ay_n = z$. Indeed, in view of (2.3) for $\alpha = 1 – q$, $q \in (0, 1)$, we have
$[1 + pM (Sy_n, Tx_n, a, kt)] * M (Ay_n, Bx_n, a, kt) \geq p [M (Ay_n, Sy_n, a, kt) * M (Bx_n, Tx_n, a, kt) + M(Ay Ay_n, Tx_n,a, kt) * M (Bx_n, Sy_n, a, kt)] + M (Syn, Tx_n, a, t) * M (Ay_n, Sy_n, a, t) * M (Bx_n, Tx_n, a, t) * M (Bx_n, Sy_n, a, t) * M (Ayn, Tx_n, a, (2 - \alpha) t)$
$M (Ay_n, Bx_n, a, kt) + p [M (Sy_n, Tx_n, a, kt) * M (Ay_n, Bx_n, a, kt)] \geq p [M(Ay_n, Sy_n,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Ay_n, Tx_n,a, kt)$
$* M (Bx_n, Sy_n, a, kt)] + M (Syn, Tx_n, a, t) * M (Ay_n, Sy_n, a, t) * M (Bx_n, Tx_n, a, t) * M (Bx_n, Sy_n, a, t) * M (Ayn, Tx_n, a, (1 + q)t)$
$M (Ay_n, Bx_n, a, kt) + p [M (Bx_n, Tx_n, a, kt) * M (Ay_n, Bx_n, a, kt)] \geq p [M(Ay_n, Bx_n,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Ay_n, Tx_n,a, kt) * M (Bx_n, Bx_n, a, kt)] + M (Bxn, Tx_n, a, t) * M (Ay_n, Bx_n, a, t) * M (Bx_n, Tx_n, a, t) * M (Bx_n, Bx_n, a, t) *M (Ayn, Tx_n, B_{xn}, t) *M (Ayn, Bx_n, a, qt/2) * M (Bx_n, Tx_n, a, qt/2)$
Thus, it follows that
$M(Ay_n, Bx_n,a, kt) \geq M(Bx_n, T_xn,a, t)*M(Ay_n,Bx_n,a,qt/2)*M(Bx_n,Tx_n,a,qt/2)$
Since the t-norm * is continuous and $M (x, y, a,\cdot)$ is continuous, letting
$q \to 1$, we have
$M (Ay_n, Bx_n, a, kt) \geq M (Bx_n, Tx_n, a, t) * M (Ay_n, Bx_n, a, t/2)$
It follows that
$\lim_{n\to\infty} M (Ay_n, Bx_n, a, kt) \geq \lim_{n\to\infty} M (Ay_n, Bx_n,a, t)$
and we deduce that $\lim_{n\to\infty} Ay_n = z$.
Suppose SX is a closed subset of X. Then z = Su for some $u \in X$. Subsequently, we have
$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = Su.$
By (2.3) with $\alpha =1$, we have

$[1 + pM (Su, Tx_n, a, kt)] * M (Au, Bx_n, a, kt) \geq p [M (Au, Su,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Au, Tx_n,a, kt) * M (Bx_n, Su, a, kt)] + M (Su, Tx_n, a, t) * M (Au, Su, a, t) * M (Bx_n, Tx_n, a, t) * M (Bx_n, Su, a, t) * M (Au, Tx_n, a, t)$

$M (Au, Bx_n, a, kt) + p [M (Su, Tx_n, a, kt)] * M (Au, Bx_n, a, kt)] \geq p [M (Au, Su,a,kt) * M(Bx_n, Tx_n,a, kt) + M(Au, Tx_n,a, kt) * M (Bx_n, Su, a, kt)] + M (Su, Tx_n, a, t) * M (Au, Su, a, t) * M (Bx_n, Tx_n, a, t) * M (Bx_n, Su, a, t) * M (Au, Tx_n, a, t)$

Taking the $\lim_{n\to\infty}$, we have

$M(Au, Su,a, kt) \geq p[(Au, Su,a, kt) * M(Su, Su,a,kt)] + M(Su, Su,a,t) * M (Au, Su, a, t) * M (Su, Su, a, t) * M (Su, Su, a, t) * M (Au, Su, a, t)$

This gives

$M (Au, Su, a, kt) \geq M (Au, Su, a, t)$

Therefore by lemma3, we have Au = Su. The weak compatibility of A and S implies that ASu = SAu and then AAu = ASu = SAu = SSu. On the other hand, since $AX \subset TX$, there exists a point $v \in X$ such that Au = Tv. We claim that Tv = Bv using (2.3) with $\alpha = 1$, we have

$[1 + pM (Su, Tv, a, kt)] * M (Au, Bv, a, kt) \geq p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt) * M (Bv, Su, a, kt)] + M (Su, Tv, a, t) * M (Au, Su, a, t) * M (Bv, Tv, a, t) * M (Bv, Su, a, t) * M (Au, Tv, a, t)$

$M (Au, Bv, a, kt) + p [M (Su, Tv, a, kt) * M (Au, Bv, a, kt)] \geq p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt) * M (Bv, Su, a, kt)] + M (Su, Tv, a, t) * M (Au, Su, a, t) * M (Bv, Tv, a, t) * M (Bv, Su, a, t) * M (Au, Tv, a, t)$

Thus it follows that

$M (Au, Bv, a, kt) \geq M (Au, Bv, a, t)$

Therefore by lemma3, we have Au = Bv.

Thus Au = Su = Tv = Bv. The weak compatibility of B and T implies that BTv = TBv and TTv = TBv = BTv = BBv. Let us show that Au is a common fixed point of A, B, S and T. In view of (2.3) with $\alpha = 1$, we have

$[1 + pM (SAu, Tv, a, kt)] * M (AAu, Bv, a, kt) \geq p[M(AAu, SAu,a, kt) * M(Bv, Tv,a, kt) + M(AAu, Tv,a, kt) * M (Bv, SAu, a, kt)] + M (SAu, Tv, a, t) * M (AAu, SAu,a, t) * M (Bv, Tv, a, t) * M (Bv, SAu, a, t) * M (AAu, Tv, a, t)$

$M(AAu, Bv,a, kt) + p[M(SAu, Tv,a, kt) * M(AAu, Bv,a, kt)] \geq p[M(AAu, SAu,a, kt) * M(Bv, Tv,a, kt) + M (AAu, Tv,a, kt) * M (Bv, SAu, a, kt)] + M (SAu, Tv, a, t) * M (AAu, SAu, a, t) * M (Bv, Tv, a, t) * M (Bv, SAu, a, t) * M (AAu, Tv, a, t)$

$M(AAu, Au,a, kt) + p[M(AAu, Au,a, kt) * M(AAu, Au,a, kt)] \geq p[M(AAu, AAu,a, kt) * M(Au, Au,a, kt) + M(AAu, Au,a, kt) * M(Au, AAu,a, kt)] + M(AAu, Au,a, t) * M(AAu, AAu,a, t) * M (Au, Au, a, t) * M (Au, AAu, a, t) * M (AAu, Au, a, t)$

Thus, it follows that

$M (AAu, Au, a kt) \geq M (AAu, Au, a, t)$

Therefore by lemma 3, we have Au = AAu = SAu and Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and T. Since Au = Bv, we conclude that Au is a common fixed point of A, B, S and T.

If Au = Bu = Su = Tu = u and Av = Bv = Sv = Tv = v, then by (2.3) with $\alpha = 1$, we have

$[1 + pM (Su, Tv, a, kt)] * M (Au, Bv, a, kt) \geq p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt) (Bv, Su, a, kt)] + M (Su, Tv, a, t) * M (Au, Su, a, t) *M (Bv, Tv, a, t) * M (Bv, Su, a, t) * M (Au, Tv, a, t)$

$M (u, v, a, kt) + p [M (u, v, a, kt) * M (u, v, a, kt)] \geq p [M (u, u, a, kt) * M(v, v,a, kt) + M(u, v,a, kt) * M (v, u, a, kt)] + M (u, v, a, t) * M (u, u, a, t) * M (v, v, a, t) * M (v, u, a, t) * M (u, v, a, t)$

This gives

$M (u, v, a, kt) \geq M (u, v, a, t)$

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By lemma 3, we have u = v and the common fixed point is a unique. This completes the proof of the theorem.

If we put p = 0, we get the following result:

Corollary 1 Let (X, M, *) be a fuzzy 2-metric space with $t * t \geq t$ for some $t \in [0, 1]$ and the condition (FM-6), Let A, B, S and T be self-mappings of X into itself such that

(1.1) $AX \subset TX$ and $BX \subset SX$,

(1.2) (A, S) or (B, T) satisfies the property (S-B),

(1.3) there exists a number $k \in (0, 1)$, such that

$M(Ax, By, a, kt) \geq M(Sx, Ty, a, t) * M(Ax, Sx, a, t) * M(By, Ty, a, t) * M(By, Sx, a, t) * M(Ax, Ty, a, (2 - \alpha) t)$

For all $x, y, a \in X$ and $\alpha \in (0, 2)$.

(1.4) (A, S) and (B, T) are weakly compatible,

(1.5) One of AX, BX, SX or TX is a closed subset of X.

Then A, B, S and T have a unique common fixed point in X.

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