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Research Article

BINARY MIXTURE OF ANISOTROPIC DARK ENERGY AND PERFECT FLUID IN BIANCHI TYPE-IX UNIVERSE

***Ghate H. R. and Atish S. Sontakke**
Department of Mathematics, Jijamata Mahavidyalaya, Buldana (India) – 443 001
**Author for Correspondence*

ABSTRACT
The Bianchi type-IX cosmological models with binary mixture of perfect fluid (PF) and anisotropic dark energy (DE) have been studied. In order to obtain a unique solution, it is assumed that the energy conservation equation of the PF and DE vanishes separately together with a special law for the mean Hubble parameter which yields a constant value of the deceleration parameter.To have a general description of an anisotropic DE component in terms of its equation of state (EoS) $\omega^{(de)}$, two skewness parameters (γ, δ) have been introduced. It has been found out that the anisotropic distribution of DE leads to the present accelerated expansion of the universe. The geometrical and physical parameters of the model are studied. The analysis of the model reveals that the present acceleration, isotropy of the universe turns out to be natural consequences of DE.

Key Words: *Anisotropic Dark Energy, Perfect Fluid, Bianchi Type-IX Universe.*

INTRODUCTION
The expansion history of the universe indicates that, the universe is currently experiencing a phase of accelerated expansion. In 1998, two teams studying distant type Ia supernovae (SNeIa) independently presented evidence of expansion (Riess *et al.*,2004; Perlmutter *et al.*,1999; Spergel *et al.*, 2007; Wood-Vasey *et al.*,2007; Davis *et al.*,2007), and confirmed later by cross checks from the cosmic microwave background radiation (Bennett *et al.*,2003; Spergel *et al.*,2003) and large scale structure (Tegmark*et al.*,2004; Abazajian *et al.*,2003, 2004a, 2004b; Hawkins *et al.*,2003). To explain the cosmic positive acceleration, mysterious DE has been proposed. Several DE models are distinguished using variable EoS $p = \omega\rho$ (p is the field pressure and ρ is its energy density) during evolution of the universe. Many cosmologists have studied the cosmological models by considering PF or ordinary matter present in the universe. Thus, the researchers are motivated to consider the cosmological models of the universe filled with some exotic type of matter such as DE along with usual PF. Kremer (2003) has considered the universe containing a binary mixture whose constituents are described by a Van der Walls fluid and a dark energy density. In these studies the authors considered mainly a spatially flat, homogeneous and isotropic universe described by a FRW metric. Khalatnikov and Kamenshchik (2003) and Saha (2005, 2006) have studied Bianchi type-I cosmological model in the presence of perfect fluid and dark energy given by cosmological constant. Adhav *et al.*, (2010, 2011) has studied higher dimensional cosmological models with a binary mixture of perfect fluid and dark energy. Katore *et al.*, (2011a, 2011b, 2011c, 2013) has considered Bianchi type-III, Bianchi type-VI_0, Plane symmetric and Kaluza-Klein cosmological models with a binary mixture of PF and DE. The role of DE in several different cosmological models of universe has been studied recently by Tade and Sambhe (2012), Kumar and Akarsu (2012), Singh and Chaubey (2009).
Bianchi type-IX universe are studied by the number of cosmologists because the solutions of Robertson Walker universe with positive curvature, the de-Sitter universe, the Taub -NUT solutions etc. are of Bianchi type-IX space-times. These models are in general anisotropic and allow not only expansion but also rotation and shear. Waller (1984) has studied dynamical effects of spatially homogeneous electromagnetic fluid on anisotropic Bianchi type-IX models. Bali and Dave (2001) have investigated Bianchi type-IX string cosmological models in General Relativity. Bianchi type-IX stiff fluid tilted

cosmological models with bulk viscosity have been investigated by Bali and Kumawat (2011). Ghate and Sontakke (2013a, 2013b) have studied Bianchi type-IX cosmological models with anisotropic DE and DE model in Brans-Dicke theory of gravitation.

In this paper, the Bianchi type-IX space-timeshas been taken up for the study consisting of a binary mixture of anisotropic DE and PF. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved in Section 3 by using deceleration parameter. The physical and geometrical properties of the model have been discussed in Section 4. In the last Section 5, concluding remarks have been expressed.

Field Equations

Bianchi type-IX metric is considered in the form

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + \left(b^2 \sin^2 y + a^2 \cos^2 y\right) dz^2 - 2a^2 \cos y dx dz \ , \tag{1}$$

Where a, b are scales factors and are functions of cosmic time t.

The model has one transverse direction x, and two equivalent longitudinal directions y and z.

In natural units ($8\pi G = 1, c = 1$), Einstein's field equations in case of a binary mixture of PF and anisotropic DE components are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -\left({}^{pf}T_{ij} + {}^{de}T_{ij}\right), \tag{2}$$

with

$$\begin{aligned} {}^{pf}T_i^j &= diag\left[-\rho^{(pf)}, p^{(pf)}, p^{(pf)}, p^{(pf)}\right] \\ &= diag\left[-1, \omega^{(pf)}, \omega^{(pf)}, \omega^{(pf)}\right]\rho^{(pf)}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} {}^{de}T_i^j &= diag\left[-\rho^{(de)}, p_x^{(de)}, p_y^{(de)}, p_z^{(de)}\right] \\ &= diag\left[-1, \omega_x^{(de)}, \omega_y^{(de)}, \omega_z^{(de)}\right]\rho^{(de)}, \\ &= diag\left[-1, (\omega^{(de)} + \delta), (\omega^{(de)} + \gamma), (\omega^{(de)} + \gamma)\right]\rho^{(de)}, \end{aligned} \tag{4}$$

where, g_{ij} is the metric potentials with $g_{ij}u^i u^j = 1$; u^i is the flow vector; R_{ij} is the Ricci tensor; R is the Ricci scalar; $\rho^{(pf)}$ and $\rho^{(de)}$ are the energy density of PF and DE components, respectively; $\omega^{(pf)}$ and $\omega^{(de)}$ is the EoS parameter of PF and DE with $\omega^{(pf)} \geq 0$; $\omega_x^{(de)}$, $\omega_y^{(de)}$ and $\omega_z^{(de)}$ are the deviation-free EoS parameter of the DE on the x, y and z axes, respectively. δ and γ are the deviations from the deviation-free EoS parameters of the DE, respectively, on the x, y and z axes. Here ω and γ are not necessarily constants and can be function of the cosmic time t.

Einstein's field equations (2) for metric (1) with the help of equations (3) and (4) can be written as

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} = \rho^{(pf)} + \rho^{(de)} \tag{5}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} = -\omega^{(pf)}\rho^{(pf)} - (\omega^{(de)} + \delta)\rho^{(de)} \tag{6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = -\omega^{(pf)}\rho^{(pf)} - (\omega^{(de)} + \gamma)\rho^{(de)} \ , \tag{7}$$

Where over dot ($\dot{}$) denotes the differentiation with respect to t.

The Bianchi identity is given by

$$G^{ij}_{;j} = {}^{pf}T^{ij}_{;j} + {}^{de}T^{ij}_{;j} = 0. \tag{8}$$

These yields:

$$\dot{\rho}^{(pf)} + \left(1+\omega^{(pf)}\right)\rho^{(pf)}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right) + \dot{\rho}^{(de)} + \left(1+\omega^{(de)}\right)\rho^{(de)}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right) + \rho^{(de)}\left(\delta\frac{\dot{a}}{a}+2\gamma\frac{\dot{b}}{b}\right) = 0. \quad (9)$$

Solution of the Field Equations

The directional Hubble parameters in the direction of x, y, z for the Bianchi type-IX metric (1) are defined as $H_x = \frac{\dot{a}}{a}$ and $H_y = H_z = \frac{\dot{b}}{b}$. (10)

The mean Hubble parameter is given by

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right), \quad (11)$$

Where $V = ab^2$ is the spatial volume of the universe.

The anisotropy parameter of the expansion is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \quad (12)$$

Where $H_i\,(i=1,2,3)$ represents the directional Hubble parameters in the direction of x, y and z axes, respectively.

We have three linearly independent equations (5), (6) and (7) with eight unknowns $\left(a, b, \rho^{(pf)}, \rho^{(de)}, \omega^{(pf)}, \omega^{(de)}, \gamma, \delta\right)$. We need five extra conditions to solve field equations completely. Following Akarsu and Kilinc (2010), we assume that the PF and DE component interacts minimally. Therefore, the energy momentum tensors of these two sources are conserved separately, i.e. the Bianchi identity (8) has been split into two separately additive conserved components.

Hence, the conservation of energy momentum tensor of the DE gives

$${}^{de}T^{ij}_{;j} = \dot{\rho}^{(de)} + \left(1+\omega^{(de)}\right)\rho^{(de)}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right) + \rho^{(de)}\left(\delta\frac{\dot{a}}{a}+2\gamma\frac{\dot{b}}{b}\right) = 0. \quad (13)$$

And the conservation of the energy momentum tensor of the PF component gives

$${}^{pf}T^{ij}_{;j} = \dot{\rho}^{(pf)} + \left(1+\omega^{(pf)}\right)\rho^{(pf)}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right) = 0. \quad (14)$$

One can split up the above conservation of the energy momentum tensor (13) of the DE into two parts:

$${}^{de}T^{ij}_{;j} = {}^{de}T'^{ij}_{;j} + {}^{de}\tau^{ij}_{;j} = 0, \quad (15)$$

where ${}^{de}\tau^{ij}_{;j}$ is the last term of the ${}^{de}T^{ij}_{;j}$ in equation (13) and arise due to the deviation from $\omega^{(de)}$ and is the deviation-free parts of the ${}^{de}T'^{ij}_{;j}$ in equation (13).

Now, we shall make the following strong assumption:

$${}^{de}\tau^{ij}_{;j} = \rho^{(de)}\left(\delta\frac{\dot{a}}{a}+2\gamma\frac{\dot{b}}{b}\right) = 0, \quad (16)$$

which also results in the deviation-free part of the ${}^{de}T^{ij}_{;j}$ to be null i.e.

$${}^{de}T'^{ij}_{;j} = \dot{\rho}^{(de)} + \left(1+\omega^{(de)}\right)\rho^{(de)}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right) = 0. \quad (17)$$

The dynamics of the deviation parameters $\delta(t)$ and $\gamma(t)$ is assumed to be

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$$\delta(t) = n\frac{2}{3}\left[\frac{\dot{b}}{b}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)-\frac{1}{nb^2}+\frac{a^2}{nb^4}\right]\frac{1}{\rho^{(de)}} \tag{18}$$

$$\gamma(t) = -n\frac{1}{3}\left[\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)-\frac{1}{nb^2}+\frac{a^2}{nb^4}\right]\frac{1}{\rho^{(de)}}, \tag{19}$$

where n is a dimensionless constant.
Also, we assume the EoS parameter of the PF to be constant, i.e.

$$\omega^{(pf)} = \frac{p^{(pf)}}{\rho^{(pf)}}. \tag{20}$$

Lastly, for the constant deceleration parameter, we impose a law of variation for the Hubble parameter. According to this law, the mean Hubble parameter for Bianchi type-IX metric is given by

$$H = k(ab^2)^{-m/3}, \tag{21}$$

where $k > 0$ and $m \geq 0$ are constants.
The spatial volume is given by

$$V = A^3 = ab^2 \tag{22}$$

where A is the mean scale factor.
The mean Hubble parameter H for Bianchi type-IX metric is given by

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right). \tag{23}$$

The directional Hubble parameters in the direction of x, y and z axes, respectively, are given as

$$H_x = \frac{\dot{a}}{a} \quad \text{and} \quad H_y = H_z = \frac{\dot{b}}{b}. \tag{24}$$

The volumetric deceleration parameter is

$$q = -\frac{A\ddot{A}}{\dot{A}^2}. \tag{25}$$

On integrating, after equating (21) and (23), we get

$$ab^2 = c_1 e^{3kt}, \qquad \text{for } m = 0 \tag{26}$$

and

$$ab^2 = (mkt + c_2)^{3/m}, \qquad \text{for } m \neq 0 \tag{27}$$

where c_1 and c_2 are positive constants of integration.
Using (21) and (26) for $m = 0$, and with (27) for $m \neq 0$, the mean Hubble parameters are

$$H = k, \qquad \text{for } m = 0 \tag{28}$$

And

$$H = k(mkt + c_2)^{-1}, \qquad \text{for } m \neq 0 \tag{29}$$

Using equations (23), (26) and (27) in (25) we get constant values for the deceleration parameter for the mean scale factor as

$$q = m - 1, \qquad \text{for } m \neq 0 \tag{30}$$

$$q = -1, \qquad \text{for } m = 0 \tag{31}$$

Using (18) and (19), and the mean Hubble parameter (20) in the subtraction of (6) from (7), we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)+3H\frac{d}{dt}\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)=3nH\ . \tag{32}$$

On integrating (32) and then considering (28) and (29), we get

$$\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\lambda e^{-3kt}+nk\quad ,\qquad \text{for } m=0, \tag{33}$$

$$\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\frac{\lambda}{(mkt+c_2)^{3/m}}+\frac{3nk}{(3-m)(mkt+c_2)}\quad , \text{for } m\neq 0, 3, \tag{34}$$

Where λ is the real constant of integration.

Model for m = 0 (q = -1):

On integrating (33) and using (26) we get,

$$a=K_c^{2/3}c_1^{1/3}e^{kt-\frac{2\lambda}{9k}e^{-3kt}+\frac{2}{3}nkt}, \tag{35}$$

$$b=K_c^{-1/3}c_1^{1/3}e^{kt+\frac{\lambda}{9k}e^{-3kt}-\frac{1}{3}nkt}, \tag{36}$$

where K_c is a positive constant of integration.

The directional Hubble parameters on the x, y, and z are respectively, given by

$$H_x=k+\frac{2\lambda}{3}e^{-3kt}+\frac{2}{3}nk\ , \tag{37}$$

$$H_y=H_z=k-\frac{\lambda}{3}e^{-3kt}-\frac{1}{3}nk\ . \tag{38}$$

The spatial volume is given by

$$V=c_1e^{3kt}\ . \tag{39}$$

By using (37), (38) and (28) in (12), we get

$$\Delta=\frac{2}{9}\frac{(\lambda e^{-3kt}+nk)^2}{k^2}\quad . \tag{40}$$

The expansion scalar Θ is found to be

$$\Theta=3k\ . \tag{41}$$

The shear scalar σ^2, defined by $\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}\approx\frac{3}{2}\Delta H^2$, is found as

$$\sigma^2=\frac{1}{3}(\lambda e^{-3kt}+nk)^2\quad . \tag{42}$$

Using (35) and (36) in (14), the energy density of the PF is found as

$$\rho^{(pf)}(t)=\rho_0^{(pf)}e^{-3k(1+\omega^{(pf)})t}. \tag{43}$$

The energy density of DE is found by using the scale factors and energy density of the PF (43) in equation (5) as

$$\rho^{(de)}=3k^2\left(1-\frac{1}{2}\Delta(t)+\frac{1}{3}L_1(t)+\frac{1}{3}L_2(t)\right)-\rho^{(pf)}, \tag{44}$$

where

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$$L_1(t) = \frac{1}{k^2 (c_1 K_c^{-1})^{2/3}} e^{-2kt - \frac{2\lambda}{9k} e^{-3kt} + \frac{4}{3} nkt} \text{ and}$$

$$L_2(t) = \frac{-3}{4k^2 (c_1 K_c^{-4})^{2/3}} e^{-2kt - \frac{8\lambda}{9k} e^{-3kt} + \frac{8}{3} nkt}.$$

Now using (35), (36) and (44) in equation (17), the deviation-free part of the anisotropic EoS parameter may be obtained as

$$\omega^{(de)} = \frac{\left\{ \begin{array}{l} 3\omega^{(pf)} \rho^{(pf)} + \lambda^2 e^{-6kt} - k^2 (n^2 - 9) + L_1(t) \left(k^2 + \frac{2}{3} \lambda k e^{-3kt} + \frac{4}{3} nk^2 \right) \\ + L_2(t) \left(k^2 + \frac{8}{3} \lambda k e^{-3kt} + \frac{8}{3} nk^2 \right) \end{array} \right\}}{3\rho^{(pf)} + \lambda^2 e^{-6kt} + 2nk\lambda e^{-3kt} + k^2 (n^2 - 9) - 3k^2 L_1(t) - 3k^2 L_2(t)}. \quad (45)$$

The deviation parameters δ and γ can be obtained by using equations (35), (36) and (44) in equations (18) and (19) as

$$\delta(t) = \frac{2nk\lambda e^{-3kt} - 2nk^2 (n+3) + 2k^2 L_1(t) + \frac{8}{3} k^2 L_2(t)}{3\rho^{(pf)} + \lambda^2 e^{-6kt} + 2nk\lambda e^{-3kt} + k^2 (n^2 - 9) - 3k^2 L_1(t) - 3k^2 L_2(t)}, \quad (46)$$

and

$$\gamma(t) = \frac{2nk\lambda e^{-3kt} + nk^2 (2n+3) - k^2 L_1(t) - \frac{4}{3} k^2 L_2(t)}{3\rho^{(pf)} + \lambda^2 e^{-6kt} + 2nk\lambda e^{-3kt} + k^2 (n^2 - 9) - 3k^2 L_1(t) - 3k^2 L_2(t)}. \quad (47)$$

Physical Behavior of the Model

In this model, $dH/dt = 0 \Rightarrow q = -1$. This shows that the rate of expansion of the universe is faster. Thus, this model may represent the inflationary era in the early universe and very late times of the universe.

The spatial volume is finite at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. The directional Hubble parameters are finite at $t = 0$ and $t = \infty$. They deviate from the mean Hubble parameter due to λ. While λ is supporting (opposing) the expansion on the *x*axis, it opposes (supports) the expansion on the *y*&*z*-axes. The expansion scalar $\theta = 3H = 3k$, is constant throughout the evolution of the universe.

The energy density of the PF $\rho^{(pf)}$ decreases exponentially and converges to zero since $\omega^{(pf)} \geq 0$. The energy density of the DE component changes slightly at early times and converges to a non-zero value as t increases. Thus, the ratio $\rho^{(de)} / (\rho^{(pf)} + \rho^{(de)})$ converges to 1 as t increases, *i.e.* the DE dominates the PF in the inflationary era. The EoS parameter of the DE $\omega^{(de)}$ begins in phantom region $\omega^{(de)} < -1$ and tends to -1 by exhibiting various patterns as t increases. As t increases, the anisotropy of the expansion (Δ) decreases exponentially to null. Thus the space approaches to isotropy in this model. The deviation parameter $\delta = 0$ throughout and γ is finite at $t = 0$.

Here the model isotropized for large values of t, provided that $n = 0$, otherwise, it is anisotropic. Also, the anisotropy of the DE isotropizes for large values of t.

Model for m ≠ 0 (q ≠ -1)

The solutions in this section are valid for all possible values of m except for $m=3$ and $m=0$.

From (27) one can see that the initial time of the universe is $t_* = \frac{-c_2}{mkt}$ for $m \neq 0$. For brevity of the equation, we may redefine the cosmic time as

$$t' = mkt + c_2, \tag{48}$$

and by doing that the initial time of the universe has also been set to $t' = 0$.

Thus, we may rewrite the metric (1) as

$$ds^2 = \begin{Bmatrix} -(mk)^{-2} dt'^2 + a(t')^2 dx^2 + b(t')^2 dy^2 + \left(b(t')^2 \sin^2 y + a(t')^2 \cos^2 y\right) dz^2 \\ -2a(t')^2 \cos y dx dz \end{Bmatrix} \tag{49}$$

Using (34), we can obtain the ratio of scale factors $\frac{a}{b}$ and then using (27), we obtain the exact expression for the scale factors as

$$a(t') = K_c^{2/3} (t')^{-\frac{1}{m} + \frac{2n}{m(3-m)}} e^{\frac{2\lambda}{3(m-3)k}(t')^{1-\frac{3}{m}}}, \tag{50}$$

$$b(t') = K_c^{-1/3} (t')^{\frac{1}{m} - \frac{n}{m(3-m)}} e^{\frac{-\lambda}{3(m-3)k}(t')^{1-\frac{3}{m}}}, \tag{51}$$

where K_c is a positive constant of integration.

The spatial volume of the universe is given by

$$V = (t')^{\frac{3}{m}}. \tag{52}$$

The directional Hubble parameters on the x, y, and z are respectively, given by

$$H_x = k(t')^{-1} + \frac{2\lambda}{3}(t')^{\frac{-3}{m}} - \frac{2nk}{(m-3)}(t')^{-1}, \tag{53}$$

$$H_y = H_z = k(t')^{-1} - \frac{\lambda}{3}(t')^{\frac{-3}{m}} + \frac{nk}{(m-3)}(t')^{-1}. \tag{54}$$

By using (29), (53) and (54) in (12), we get

$$\Delta = \frac{2}{9}\left[\frac{\lambda}{k}(t')^{1-\frac{3}{m}} - \frac{3n}{(m-3)}\right]^2 . \tag{55}$$

The expansion scalar Θ is found to be

$$\Theta = 3k(t')^{-1}. \tag{56}$$

The shear scalar σ^2, defined by $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \approx \frac{3}{2}\Delta H^2$, is found as

$$\sigma^2 = \frac{1}{3}k^2 (t')^{-2}\left[\frac{\lambda}{k}(t')^{1-\frac{3}{m}} - \frac{3n}{(m-3)}\right]^2. \tag{57}$$

Using (50) and (51) in (14), the energy density of the PF is found as

$$\rho^{(pf)}(t') = \rho_0^{(pf)} (t')^{\frac{-3}{m}(1+\omega^{(pf)})}. \tag{58}$$

The energy density of DE is found by using the scale factors and energy density of the PF (58) in equation (5) as

$$\rho^{(de)}(t') = 3k^2 (t')^{-2}\left(1 - \frac{1}{2}\Delta(t') + \frac{1}{3}L_1(t') + \frac{1}{3}L_2(t')\right) - \rho^{(pf)}(t'), \tag{59}$$

where

$$L_1(t') = \frac{1}{k^2} K_c^{2/3} (t')^{-2\left(\frac{1}{m}+\frac{n}{m(m-3)}\right)+2} e^{\frac{2\lambda}{3k(m-3)}(t')^{1-\frac{3}{m}}} \text{ and}$$

$$L_2(t') = \frac{-1}{4k^2} K_c^{8/3} (t')^{-2\left(\frac{1}{m}+\frac{4n}{m(m-3)}\right)+2} e^{\frac{8\lambda}{3k(m-3)}(t')^{1-\frac{3}{m}}}$$

Now using (50), (51) and (59) in equation (17), the deviation-free part of the anisotropic EoS parameter may be obtained as

$$\omega^{(de)}(t') = \frac{\left\{\begin{array}{l} \omega^{(pf)}(t')\rho^{(pf)}(t') - \dfrac{k^2(2m-3)(m^2-n^2-6m+9)}{(m-3)^2}(t')^{-2} + \dfrac{1}{3}\lambda^2 (t')^{-\frac{6}{m}} \\ -\dfrac{2\lambda mnk}{3(m-3)}(t')^{-1-\frac{3}{m}} + L_1(t')\left(\dfrac{k^2(m-2n-3)}{3(m-3)}(t')^{-2} + \dfrac{2\lambda k}{9}(t')^{-1-\frac{3}{m}}\right) \\ -L_2(t')\left(\dfrac{k^2(-m+8n-3)}{3(m-3)}(t')^{-2} - \dfrac{8\lambda k}{9}(t')^{-1-\frac{3}{m}}\right) \end{array}\right\}}{3k^2(t')^{-2}\left(1-\frac{1}{2}\Delta(t') + \frac{1}{3}L_1(t') + \frac{1}{3}L_2(t')\right) - \rho^{(pf)}(t')}. \tag{60}$$

The deviation parameters δ and γ can be obtained by using equations (50), (51) and (59) in equations (18) and (19) as

$$\delta(t') = -\frac{\frac{2}{3}k^2(t')^{-2}\left\{\frac{n\lambda}{k}(t')^{1-\frac{3}{m}} - \frac{3n(m+n-3)}{(m-3)} + L_1(t') + L_2(t')\right\}}{3k^2(t')^{-2}\left(1-\frac{1}{2}\Delta(t') + \frac{1}{3}L_1(t') + \frac{1}{3}L_2(t')\right) - \rho^{(pf)}(t')}, \tag{61}$$

and

$$\delta(t') = -\frac{k^2(t')^{-2}\left\{\frac{2n\lambda}{3k}(t')^{1-\frac{3}{m}} + \frac{n(m-2n-3)}{(m-3)} - \frac{1}{3}L_1(t') + \frac{4}{3}L_2(t')\right\}}{3k^2(t')^{-2}\left(1-\frac{1}{2}\Delta(t') + \frac{1}{3}L_1(t') + \frac{1}{3}L_2(t')\right) - \rho^{(pf)}(t')}. \tag{62}$$

Physical Behavior of the Model

The mean Hubble parameter H is infinitely large at $t'=0$ and null at $t'=\infty$. For $0 \le m < 1$ or $q<0$ indicates that the universe is accelerating. For $m>1$, the universe is decelerating. In particular, for $m=1$ we get $q=0$ indicating that the universe is expanding with constant velocity. The volume of the universe expands indefinitely for all values of m. The anisotropy of the expansion ($\Delta(t')$) diverges as $t' \to 0$, converges to constant as $t' \to \infty$ for $m<3$ and vice versa for $m>3$. Here one can observe that $\Delta(t')$,

$L_1(t')$, $L_2(t')$, λ and $\rho^{(pf)}$ lowers the value of the energy density of DE $\rho^{(de)}$. Also, one can set the value of $\omega^{(de)}$ as desired by choosing the appropriate values of the parameter.

CONCLUSION

In the present paper, the Bianchi Type-IX cosmological models with PF and anisotropic DE have been studied. Here two models with exponential expansion and power law expansion have been studied. In both the models, the anisotropic DE isotropizes for large value of t. The anisotropy of the space isotropizes for exponential expansion. Also the anisotropy of the space isotropizes the power law expansion model provided $m > 1/3$. Also in case of exponential expansion model after certain time evolves towards λCDM cosmological model for different values of m. Most of the observations are similar to that of Akarsu and Kilinc (2010). The model for $m = 1$, i.e. $q = 2$, is not discussed as it is a decelerating model which is not consistent with the present-day observations.

In summary, two cosmological models which lead to a cosmological scenario in accordance with recent features of modern cosmology as an initial phase with decelerating expansion followed by an accelerating one at late time has been obtained. This is most relevant and significant to astrophysics. However, detailed studies are still needed to discuss concrete possible applications (if any) of the models presented in this paper which will make their phenomenological relevance clearer.

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