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Research Article

EFFECT OF TWO-TEMPERATURE ON THERMOVISCOELASTIC PROBLEM WITH RHEOLOGICAL PROPERTIES

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ABSTRACT
A thermo-visco-elastic investigation of an isotropic material in two dimensional systems is presented in this work. The problem deals with rheological properties of volume as well as density in the material having temperature-dependent mechanical properties with the effect of two-temperature. The generalized heat conduction equation due to Lord-Shulman (LS), Green-Lindsay (GL) and classical coupled theory (CD) are employed. The expression for stresses, displacement, conductive temperature and thermo-dynamic temperature are obtained by means of normal mode analysis and results are presented graphically.

Key Words: *Two-Temperature, Thermo-Visco-Elasticity, Rheological Property, Relaxation Function, Normal Mode.*

INTRODUCTION
In the theory of thermoelastic diffusion the coupled thermoelastic model is used which implies infinite speeds of propagation of thermoelastic waves. Lord and Shulman (1967) obtained a wave-type heat equation by constructing a new law of heat conduction to replace the classical Fourier's law which ensure finite speeds of propagation for heat and elastic waves which is known as the first generalization of the coupled thermo-elasticity theory. The second generalization to the coupled theory of elasticity is known as the theory of thermo-elasticity with two relaxation times. Green and Lindsay (1972) obtained an explicit version of the constitutive equations by incorporating temperature rate term into the constitutive equations. Both the first and second generalized theory of thermo-elasticity overcomes the drawback of propagating thermal signal having infinite speed.
Thermo-visco-elastic theory has been applied with great success in the fields of geophysics, plasma physics, nuclear device and related topics. Various materials used in engineering applications exhibit visco-elastic behavior. Drozdov (1996) derived a constitutive model in thermo-visco-elasticity which accounts for changes in elastic moduli and relaxation times. The thermo-visco-elastic problem with composite cylinder under a remote uniform heat flow was discussed by Chao *et al.,* (2007). Using generalized theory proposed by Lord-Shulman and Green-Lindsay, the problem on visco-elastic materials has been discussed by Mukhopadhyay (1999), Rakshit and Mukhopadhyay (2005) and many other authors. Generalized thermo-visco-elastic problem with magnetic effect under the consideration of relaxation time has been studied by Othman and Song (2008). Acharya and Roy (2009) observed radial vibration of an infinite magnetoviscoelastic medium containing a cylindrical cavity. Kar and Kanoria (2009) developed the idea about thermo-visco-elastic stresses in an isotropic visco-elastic homogeneous spherical shell. Ezzat *et al.,* (2010) expend their valuable efforts to recognize the effects of modified Ohm's and Fourier's laws on generalized magneto-thermo-visco-elasticity with relaxation volume properties. With the effects of viscous properties in elastic materials Song et al., (2006) use up their precious hard work to develop the area.
At high temperature the mechanical properties of the material are temperature-dependent. Most investigation in thermo-visco-elasticity was done by ignoring the temperature-dependent mechanical properties. Thermo-visco-elasticity including the temperature-dependent mechanical properties increases the field area of elastic theory to the research workers. The temperature-dependent properties were proposed by Ferry (1953) in his valuable paper. Problem on such consideration was analyzed by Aouadi and El-Karamany (2004) in their paper. The study of thermo-visco-elasticity with two-temperature is of interest in some branches of material science,

metallurgy, applied mathematics etc. Now a day the effect of two-temperature has become an important area of research. According to Gurtin and William (1967) the second law of thermodynamics for continuous bodies may involve with twin temperatures. In the theory of thermodynamics the temperature caused by the thermal process is known as conductive temperature φ and the temperature due to mechanical process in the material is known as thermodynamic temperature θ. The theory of heat conduction depending on the above two temperatures was originated by Chen and Gurtin (1968). Propagation of wave and the propagation of harmonic plane wave in the theory of two-temperature thermo-elasticity ware investigated by Warren and Chen (1973), Puri and Jordan (2006) respectively. Quintanilla (2004) analyzed the existence, structural stability, convergence and spatial behavior for the theory two-temperature thermo-elasticity. By means of two-temperature generalized thermo-elasticity Youssef and Al-Harby (2007) explained the state-space approach on an infinite body with spherical cavity. Ailawilia *et al.*, (2009) investigated the deformation of a rotating generalized thermoelastic medium with two temperatures under the influence of gravity subjected to different type of sources. Banik and Kanoria (2011, 2012) investigated the effects of two-temperature on generalized thermo-elasticity for infinite medium with spherical cavity. Mondal and Mukhopadhyay (2013) discussed the effects of rheological volume and density properties on their problem having temperature dependent mechanical properties.
During the last few decades it has been seen that the different kinds of problems related with thermo-elasticity involve with two-temperature. In the recent history of thermodynamics a relevant interest on two-temperature thermo-elasticity has been found in various papers by research workers. In the present work we concentrate upon the analysis of effects of two-temperature in the material having temperature dependent mechanical properties on generalized thermo-visco-elastic problem. In the context of LS, GL and CD theories we have investigated the stress, conductive temperature, thermodynamic temperature and displacement in an infinite isotropic elastic material using the two-temperature generalized thermo-visco-elasticity theory. To guess the effects of the temperature discrepancy among the different theories several comparisons has been exposed in figures.

Formulation of the Problem

We shall assume here a homogeneous isotropic thermally conducting material in a thermo-visco-elastic infinite medium. In the context of two-temperature generalized thermo-visco-elasticity the governing equations and the constitutive relations for the material in the absence of external forces and heat sources with rheological properties of volume as well as density can be taken as:

The equation of motion

$$\sum_{j=1}^{3} \sigma_{ij,j} = \rho \ddot{u}_i \ . \tag{1}$$

The generalized equation of heat conduction

$$k\,\nabla^2\varphi = C_E \int_0^t R_3(t-\tau)\frac{\partial}{\partial\tau}\left(\frac{\partial\theta}{\partial\tau} + \tau_2\frac{\partial^2\theta}{\partial\tau^2}\right)d\tau + 3\varphi_0\alpha_T\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left(\frac{\partial e}{\partial\tau} + \tau_3\frac{\partial^2 e}{\partial\tau^2}\right)d\tau. \tag{2}$$

The constitutive relations

$$S_{ij} = \int_0^t R_1(t-\tau)\frac{\partial e_{ij}}{\partial\tau}d\tau\,, \tag{3}$$

$$\sigma = \int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left\{e - 3\alpha_T\left(\theta - \varphi_0 + \tau_1\frac{\partial\theta}{\partial\tau}\right)\right\}d\tau \tag{4}$$

where

$$S_{ij} = \sigma_{ij} - \sigma\delta_{ij}\ ;\ \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right);\ e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij}\ ; e = \sum_{k=1}^{3} e_{kk}\ ;\ \sigma = \frac{1}{3}\sum_{k=1}^{3}\sigma_{kk}\ .$$

The conductive temperature φ is related with the thermo-dynamic temperature θ as

$$\varphi - \theta = a\nabla^2\varphi \tag{5}$$

where $a \geq 0$ is the two-temperature parameter (Youssfe, 2006).

Here u_i are the components of displacement vector; θ, φ are the thermo-dynamic temperature and the conductive temperature respectively both measured from a constant reference temperature φ_0; σ_{ij}, ε_{ij} are the components of the stress tensor and strain tensor respectively; $e,\ \rho,\ k,\ C_E,\ \alpha_T$ are dilatation, density, thermal conductivity, specific heat at constant strain, co-efficient of linear thermal expansion; τ_1, τ_2, τ_3 are thermal relaxation times; $R_1(t), R_2(t), R_3(t)$ are non-negative relaxation function, relaxation function characterized by rheological properties of volume and density respectively; δ_{ij} is the Kronecker delta. The comma notations are used to present the partial derivatives with respect to the space variables and the over-headed dots denote partial derivative with respect to time variable t. Here $\tau_1 = \tau_2 = \tau_3 = 0$ corresponds to classical coupled theory (CD), whereas $\tau_1 = 0, \tau_2 = \tau_3 \neq 0$ corresponds to Lord and Shulman (LS) theory and $\tau_3 = 0, \tau_2 \neq 0, \tau_1 \neq 0$ corresponds to Green and Lindsay (GL) theory.

The relaxation functions are taken in the form (El-Karamany, 1983)

$$\left.\begin{aligned} R_1(t) &= 2\mu\left(1 - M_1\int_0^t g(t)dt\right) \\ R_2(t) &= K\left(1 - M_2\int_0^t g(t)dt\right) \\ R_3(t) &= \rho\left(1 - M_3\int_0^t g(t)dt\right) \end{aligned}\right\}. \tag{6}$$

Here μ is the lame' constant; K is the bulk modulus. The function $g(t)$ generally taken in the form $g(t) = e^{-\beta t}\ t^{\alpha-1}$ where

$0 < \alpha < 1, \beta > 0, 0 \leq M_2 \leq M_3 \leq M_1 < \Gamma(\alpha), 0 \leq t < \infty.$

Using equations (3) and (4) we have

$$\sigma_{ij} = \int_0^t R_1(t-\tau)\frac{\partial e_{ij}}{\partial \tau}\ d\tau + \delta_{ij}\int_0^t R_2(t-\tau)\frac{\partial}{\partial \tau}\left\{e - 3\alpha_T\left(\theta - \varphi_0 + \tau_1\frac{\partial \theta}{\partial \tau}\right)\right\}d\tau. \tag{7}$$

With the help of equation (7) and eliminating θ between equations (1) and (5) we have

$$\rho\ddot{u}_i = \int_0^t R_1(t-\tau)\frac{\partial}{\partial \tau}\left(\frac{\nabla^2 u_i}{2} + \frac{e_{,i}}{6}\right)d\tau + \int_0^t R_2(t-\tau)\frac{\partial}{\partial \tau}[e_{,i} - 3\alpha_T\{(\varphi - a\,\nabla^2\varphi)_{,i} + \tau_1(\dot{\varphi} - a\,\nabla^2\dot{\varphi})_{,i}\}]d\tau. \tag{8}$$

In the above we consider the material having temperature dependent mechanical properties which are in the form (Lomakin 1976) $\mu = \mu_0\psi_0(\theta)$, $K = K_0\psi_0(\theta)$, $\rho = \rho_0\psi_1(\theta)$, $\alpha_T = \alpha_T^0\psi_2(\theta)$ where $\psi_i(\theta) = 1 - \alpha_i(\theta - T_r)$, $i = 0,1,2$ and $\alpha_i > 0, i = 0,1$ and $\alpha_2 < 0$ where μ_0, K_0, ρ_0 and α_T^0 are the Lame' constant, bulk modulus, density and Co-efficient of linear thermal expansion at room temperature T_r.

For simplicity of the problem we introduce the following dimensionless terms and notations:

$$x_i^{'} = c_0\eta_0 x_i\ ,\ u_i^{'} = c_0\eta_0 u_i\ ,\ t^{'} = c_o^2\eta_0 t\ ,\ \tau_i^{'} = c_o^2\eta_0\tau_i\ ,\ \theta^{'} = \frac{\gamma(\theta-\varphi_0)}{\rho_0 c_0^2},\ \varphi^{'} = \frac{\gamma(\varphi-\varphi_0)}{\rho_0 c_0^2},$$
$$\sigma_{ij}^{'} = \frac{\sigma_{ij}}{K_0}\ ,\ R_1^{'}(t) = \frac{2R_1(t)}{3K_0},\ R_2^{'}(t) = \frac{R_2(t)}{K_0},\ R_3^{'}(t) = \frac{R_3(t)}{\rho_0},\ c_0^2 = \frac{\lambda_0+2\mu_0}{\rho_0},\ \eta_0 = \frac{\rho_0 C_E}{k}.$$

Using the non-dimensional variables the equations (2) and (4)-(7) after dropping the primes take the form

$$\nabla^2\varphi = \int_0^t R_3(t-\tau)\frac{\partial}{\partial\tau}\left[\left(\frac{\partial}{\partial\tau}+\tau_2\frac{\partial^2}{\partial\tau^2}\right)(\varphi-a_1\nabla^2\varphi)\right]d\tau$$
$$+\varepsilon\,\psi_2\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left(\frac{\partial e}{\partial\tau}+\tau_3\frac{\partial^2 e}{\partial\tau^2}\right)d\tau, \qquad (9)$$

$$\sigma = \int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left[e-a_2\psi_2\left\{\left(\varphi+\tau_1\frac{\partial\varphi}{\partial\tau}\right)-a_1\left(1+\tau_1\frac{\partial}{\partial\tau}\right)\nabla^2\varphi\right\}\right]d\tau\ , \qquad (10)$$

$$\varphi-\theta = a_1\,\nabla^2\varphi\ , \qquad (11)$$

$$\left.\begin{aligned} R_1(t) &= \beta_1\psi_0\left(1-M_1\int_0^t g(t)dt\right)\\ R_2(t) &= \psi_0\left(1-M_2\int_0^t g(t)dt\right)\\ R_3(t) &= \psi_1\left(1-M_3\int_0^t g(t)dt\right)\end{aligned}\right\}, \qquad (12)$$

$$\sigma_{ij} = \frac{3}{2}\int_0^t R_1(t-\tau)\frac{\partial}{\partial\tau}\left(\varepsilon_{ij}-\frac{e}{3}\delta_{ij}\right)d\tau$$
$$+\delta_{ij}\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left[e-a_2\psi_2\left\{\left(\varphi+\tau_1\frac{\partial\varphi}{\partial\tau}\right)-a_1\left(1+\tau_1\frac{\partial}{\partial\tau}\right)\nabla^2\varphi\right\}\right]d\tau \qquad (13)$$

Where

$$a_1 = ac_0^2\,\eta_0^2\ ,a_2 = \frac{\rho_0 C_0^2}{K_0}\ ,\varepsilon = \frac{3\,\gamma\varphi_0\,\alpha_T^0 K_0}{k\,\rho_0\,C_o^2\,\eta_0},\ \ \beta_1 = \frac{4\mu_0}{3K_0}.$$

We consider the two dimensional problem subjected to the plain strain parallel to xy-plane and the displacement components $(u, v, 0)$ are the function of the space variables x, y and the time variable t .

Using the non-dimensional terms and ignoring the primes from equation (8) we have

$$a_2\psi_1\frac{\partial^2 u}{\partial t^2} = \int_0^t R_1(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{\partial^2 u}{\partial x^2}+\frac{3}{4}\frac{\partial^2 u}{\partial y^2}+\frac{1}{4}\frac{\partial^2 v}{\partial x\partial y}\right]d\tau+\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right.$$
$$\left.-a_2\psi_2\left\{\left(\frac{\partial\varphi}{\partial x}-a_1\frac{\partial^3\varphi}{\partial x^3}-a_1\frac{\partial^3\varphi}{\partial x\partial y^2}\right)+\tau_1\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial\tau}-a_1\frac{\partial^3\varphi}{\partial x^2\partial\tau}-a_1\frac{\partial^3\varphi}{\partial y^2\partial\tau}\right)\right\}\right]d\tau, \qquad (14)$$

$$a_2\psi_1\frac{\partial^2 v}{\partial t^2}=\int_0^t R_1(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{\partial^2 v}{\partial y^2}+\frac{3}{4}\frac{\partial^2 v}{\partial x^2}+\frac{1}{4}\frac{\partial^2 u}{\partial x\partial y}\right]d\tau+\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 u}{\partial x\partial y}\right.$$
$$\left.-a_2\psi_2\left\{\left(\frac{\partial\varphi}{\partial y}-a_1\frac{\partial^3\varphi}{\partial y^3}-a_1\frac{\partial^3\varphi}{\partial x^2\partial y}\right)+\tau_1\frac{\partial}{\partial y}\left(\frac{\partial\varphi}{\partial\tau}-a_1\frac{\partial^3\varphi}{\partial y^2\partial\tau}-a_1\frac{\partial^3\varphi}{\partial x^2\partial\tau}\right)\right\}\right]d\tau\,. \quad (15)$$

And also we have

$$e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\,. \quad (16)$$

From equations (9) and (13) we have

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=\int_0^t R_3(t-\tau)\frac{\partial}{\partial\tau}\left[\left(\frac{\partial}{\partial\tau}+\tau_2\frac{\partial^2}{\partial\tau^2}\right)\left(\varphi-a_1\frac{\partial^2\varphi}{\partial x^2}-a_1\frac{\partial^2\varphi}{\partial y^2}\right)\right]d\tau$$
$$+\varepsilon\,\psi_2\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left(\frac{\partial e}{\partial\tau}+\tau_3\frac{\partial^2 e}{\partial\tau^2}\right)d\tau, \quad (17)$$

$$\left[\sigma_{xx}\,,\sigma_{yy}\,,\sigma_{zz}\,\right]=\frac{3}{2}\int_0^t R_1(t-\tau)\frac{\partial}{\partial\tau}\left\{\left[\frac{\partial u}{\partial x},\frac{\partial v}{\partial y},0\right]-\frac{e}{3}\right\}d\tau$$
$$+\int_0^t R_2(t-\tau)\frac{\partial}{\partial\tau}\left[e-a_2\psi_2\left\{\left(\varphi+\tau_1\frac{\partial\varphi}{\partial\tau}\right)-a_1\left(1+\tau_1\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)\right\}\right]d\tau\,, \quad (18)$$

$$\sigma_{xy}=\frac{3}{4}\int_0^t R_1(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]d\tau. \quad (19)$$

Solution of the Problem

A solution of the considered physical variable can be expressed at the form of plane wave in terms of normal modes by expressing it in the following exponential form

$$\left[e,\theta,\varphi,u,v,\sigma_{ij}\right](x,y,t)=\left[e^*(y),\theta^*(y),\varphi^*(y),u^*(y),v^*(y),\sigma_{ij}^*(y)\right]\exp(\omega t+\mathbb{i}bx) \quad (20)$$

Where ω is the frequency and b is the wave number in x-direction and $\mathbb{i}=\sqrt{-1}$.

Considering any function $f(x,y,t)$ satisfying the condition that the first order or higher order partial derivatives with respect to t are zero for $-\infty<t\le 0$, we have

$$\int_0^t R(t-\tau)\frac{\partial}{\partial\tau}f(x,y,\tau)d\tau=\omega f^*(y)\bar{R}(\omega)\exp(\omega t+\mathbb{i}bx) \quad (21)$$

Where

$$\bar{R}(\omega)=\int_0^\infty R(t)\exp(-\omega t)dt \quad (22)$$

Using equations (20) and (21) we have from equations (14)-(19)

$$a_2\psi_1\omega u^*=\bar{R}_1\left[\frac{1}{4}\mathbb{i}be^*+\frac{3}{4}(D^2-b^2)u^*\right]$$
$$+\bar{R}_2\mathbb{i}b\left[e^*-a_2\psi_2(1+\tau_1\omega)\left(1-a_1(D^2-b^2)\right)\varphi^*\right], \quad (23)$$
$$a_2\psi_1\omega v^*=\bar{R}_1\left[\frac{1}{4}De^*+\frac{3}{4}(D^2-b^2)v^*\right]$$

$$+\bar{R}_2\left[De^* - a_2\psi_2(1+\tau_1\omega)\left(1-a_1(D^2-b^2)\right)D\varphi^*\right], \tag{24}$$

$$e^* = \mathbb{i}bu^* + Dv^*, \tag{25}$$

$$b_1\psi_2 e^* = (D^2-b^2)\varphi^* - b_2\varphi^*, \tag{26}$$

$$\left[\sigma^*_{xx}, \sigma^*_{yy}, \sigma^*_{zz}\right] = \frac{3}{2}\omega\bar{R}_1\left\{[\mathbb{i}bu^*, Dv^*, 0] - \frac{1}{3}e^*\right\} + \sigma^*, \tag{27}$$

$$\sigma^*_{xy} = \frac{3}{4}\omega\bar{R}_1(Du^* + \mathbb{i}bv^*) \tag{28}$$

where

$$\sigma^* = \omega\bar{R}_2\left[e^* - a_2\psi_2(1+\tau_1\omega)\left(1-a_1(D^2-b^2)\right)\varphi^*\right],\ b_1 = \frac{\varepsilon\omega^2\bar{R}_2(1+\tau_3\omega)}{1+a_1\omega^2\bar{R}_3(1+\tau_2\omega)},$$

$$b_2 = \frac{\omega^2\bar{R}_3(1+\tau_2\omega)}{1+a_1\omega^2\bar{R}_3(1+\tau_2\omega)}, D \equiv \frac{d}{dy}.$$

With the help of (25) and (26) the combination form of (23) and (24) may be written as

$$[D^2 - \theta_1 + b_2]e^* = \frac{a_2b_2\psi_2\bar{R}_2}{b_3}(1+\tau_1\omega)(1-a_1b_2)\varphi^* \tag{29}$$

where

$$\theta_1 = b^2 + b_2 + \frac{a_2\omega\psi_1}{b_3} + \frac{a_2b_1\psi_2^2\bar{R}_2}{b_3}(1+\tau_1\omega)(1-a_1b_2),$$

$$b_3 = \bar{R}_1 + \bar{R}_2 + a_1a_2b_1\psi_2\bar{R}_2(1+\tau_1\omega).$$

Eliminating φ^* between (26) and (29) we obtain the following forth order ordinary differential equation:

$$(D^2 - m_1^2)(D^2 - m_2^2)e^*(y) = 0 \tag{30}$$

where

$$(m_1^2, m_2^2) = \frac{1}{2}(r_1 \pm \omega_1), \omega_1 = (r_1^2 - 4r_2)^{\frac{1}{2}}, r_1 = b^2 + \theta_1, r_2 = b^2\theta_1 + \frac{a_2b_2\omega\psi_1}{b_3}.$$

The solution of the equation (30) may be taken as

$$e^*(y) = \sum_{j=1}^{2} P_j(y) \tag{31}$$

Where

$P_j(y) = \left\{A_j \mathrm{Cosh}(m_j y) + A_{j+2}\mathrm{Sinh}(m_j y)\right\};\ j = 1,2$ and A_k, $k = 1,2,3,4$ are parameters depending with b and ω.

Putting (31) in equation (29) we have

$$\varphi^*(y) = \sum_{j=1}^{2} F_j . P_j(y) \tag{32}$$

where

$$F_j = \frac{b_3}{a_2b_2\psi_2\bar{R}_2}\frac{(m_j^2 - \theta_1 + b_2)}{(1+\tau_1\omega)(1-a_1b_2)};\ j = 1,2.$$

Using normal mode from equation (11) we have

$$\theta^*(y) = \sum_{j=1}^{2}\left[F_j(1-a_1b_2) - a_1b_1\psi_2\right].P_j(y) \tag{33}$$

Two-temperature parameter $a = 0$ implies $a_1 = 0$ that is for the null temperature discrepancy we observe from equations (32) and (33) that the conductive temperature and thermodynamic temperature are identical and the problem converted to unique temperature problem.

With the help of equation (26) and substituting $e^*$and φ^* we obtain from equation (23)

$$u^*(y) = Q(y) + ib\sum_{j=1}^{2} H_j\ .P_j(y) \tag{34}$$

where

$$Q(y) = B_1\text{Cosh}(m_3y) + B_2\text{Sinh}(m_3y)\,, m_3^2 = b^2 + \frac{4a_2\psi_1\omega}{3\bar{R}_1},\ H_j = \frac{\left(\bar{T}_1F_j - \bar{T}_2\right)}{\left(m_j^2 - m_3^2\right)}\ ; j = 1,2\,,$$

$$\bar{T}_1 = \frac{4a_2\psi_2\bar{R}_2}{3\bar{R}_1}(1+\tau_1\omega)(1-a_1b_2), \bar{T}_2 = \frac{4}{3\bar{R}_1}\left(\bar{R}_2 + \frac{1}{4}\bar{R}_1 + a_1a_2b_1\psi_2^2\bar{R}_2(1+\tau_1\omega)\right)$$

and B_1, B_2 are parameter depending with b and ω.

Again from (25), (27) and (28) we have

$$v^*(y) = \frac{-ib}{m_3^2}DQ(y) + \sum_{j=1}^{2}\frac{\left(1+b^2H_j\right)}{m_j^2}DP_j(y), \tag{35}$$

$$\left[\sigma_{xx}^*\ , \sigma_{yy}^*\ , \sigma_{zz}^*\right](y) = \frac{3ib\omega}{2}\bar{R}_1[1,-1,0].\,Q(y) + \sum_{j=1}^{2}\left[K_j, L_j, K_j + \frac{3}{2}b^2\omega\bar{R}_1H_j\right].P_j(y), \tag{36}$$

$$\sigma_{xy}^*(y) = \frac{3\omega\bar{R}_1}{4}\frac{(m_3^2+b^2)}{m_3^2}DQ(y) + \frac{3\omega ib\bar{R}_1}{4}\sum_{j=1}^{2}\frac{S_j}{m_j}\ DP_j(y) \tag{37}$$

where

$$K_j = -\frac{\omega\bar{R}_1}{2}\left(1+3b^2H_j\right) + \omega\bar{R}_2\{1 + a_1a_2b_1\psi_2^2(1+\tau_1\omega) - a_2\psi_2(1+\tau_1\omega)(1-a_1b_2)F_j\}\,,$$

$$L_j = \omega\bar{R}_1\left(1+\frac{3}{2}b^2H_j\right) + \omega\bar{R}_2\{1 + a_1a_2b_1\psi_2^2(1+\tau_1\omega) - a_2\psi_2(1+\tau_1\omega)(1-a_1b_2)F_j\}\,,$$

$$S_j = H_jm_j + \frac{\left(1+b^2\ H_j\right)}{m_j}\ \ ; j = 1,2.$$

Boundary Conditions

We shall consider a homogeneous isotropic thermo-visco-elastic infinite thick flat plate of a finite thickness 2L occupying the region R=$\{(x,y,z);\ |x| < \infty, |z| < \infty, |y| \le L\}$ where the plane $y = 0$ coincide with the middle surface of the plate. The boundary conditions on the surface $y = \pm L$ are taken to be

$$\left.\begin{aligned} \varphi &= \varphi_1(x,t) \\ \sigma_{xy} &= 0 \\ \sigma_{yy} &= \varphi_2(x,t) \end{aligned}\right\} \tag{38}$$

Where φ_1, φ_2 are known functions of x and t. On consideration of symmetry with respect to y-axis we can assume $A_3 = A_4 = B_2 = 0$.

With the help of equation (38), from the equations (32), (37) and second of (36) we have

$$\sum_{j=1}^{2} F_j A_j \operatorname{Cosh}(m_j L) = \varphi_1^*, \tag{39}$$

$$\mathbb{i}b \sum_{j=1}^{2} S_j A_j \operatorname{Sinh}(m_j L) + \frac{(m_3^2 + b^2)}{m_3} B_1 \operatorname{Sinh}(m_3 L) = 0, \tag{40}$$

$$\sum_{j=1}^{2} L_j A_j \operatorname{Cosh}(m_j L) - \frac{3\mathbb{i}b\omega}{2} \bar{R}_1 B_1 \operatorname{Cosh}(m_3 L) = \varphi_2^* . \tag{41}$$

Solving the system of linear equations (39)-(41) for A_1, A_2 and B_1 we have

$$A_1 = \frac{d_1}{d}, A_2 = \frac{d_2}{d}, B_1 = \frac{d_3}{d} \quad ; \text{ where}$$

$$d = \frac{(m_3^2 + b^2)}{m_3} \operatorname{Cosh}(m_1 L) \operatorname{Cosh}(m_2 L) \operatorname{Sinh}(m_3 L)(F_2 L_1 - F_1 L_2)$$
$$- \frac{3}{2} b^2 \omega \bar{R}_1 \operatorname{Cosh}(m_3 L)[F_2 S_1 \operatorname{Cosh}(m_2 L) \operatorname{Sinh}(m_1 L) - F_1 S_2 \operatorname{Cosh}(m_1 L) \operatorname{Sinh}(m_2 L)] \ ,$$

$$d_1 = \frac{3}{2} b^2 \omega \bar{R}_1 \varphi_1^* S_2 \operatorname{Cosh}(m_3 L) \operatorname{Sinh}(m_2 L) + \frac{(m_3^2 + b^2)}{m_3} \operatorname{Cosh}(m_2 L) \operatorname{Sinh}(m_3 L)(F_2 \varphi_2^* - L_2 \varphi_1^*),$$

$$d_2 = -\left[\frac{3}{2} b^2 \omega \bar{R}_1 \varphi_1^* S_1 \operatorname{Cosh}(m_3 L) \operatorname{Sinh}(m_1 L) + \frac{(m_3^2 + b^2)}{m_3} \operatorname{Cosh}(m_1 L) \operatorname{Sinh}(m_3 L)(F_1 \varphi_2^* - L_1 \varphi_1^*)\right],$$

$$d_3 = -\mathbb{i}b[S_1 \operatorname{Cosh}(m_2 L) \operatorname{Sin}(m_1 L)(F_2 \varphi_2^* - L_2 \varphi_1^*) + S_2 \operatorname{Cosh}(m_1 L) \operatorname{Sinh}(m_2 L)(L_1 \varphi_1^* - F_1 \varphi_2^*)].$$

On the basis of equation (22) the relaxation functions (12) take the form

$$\bar{R}_1 = \frac{\beta_1 \psi_0}{\omega}\left[1 - M_1 \sqrt{\frac{\pi}{\omega + \beta}}\right]; \ \bar{R}_2 = \frac{\psi_0}{\omega}\left[1 - M_2 \sqrt{\frac{\pi}{\omega + \beta}}\right]; \ \bar{R}_3 = \frac{\psi_1}{\omega}\left[1 - M_3 \sqrt{\frac{\pi}{\omega + \beta}}\right].$$

Equations (31)-(37) together with the above derived values of A_1 , A_2 and B_1 for distinct cases provide the eventual solutions in normal form.

NUMERICAL RESULTS AND DISCUSSION

To study the behavior of the quantities in details and with the intention of demonstrating the outcomes obtain in the above we attempt to achieve the numerical values of the different characteristic parameters of the material. For execution of the graphical representation we take for granted the numerical values for a magnesium crystal-like material as (Ezzat *et al.*, 2010)

$$\rho_0 = 1.74 \times 10^3 kg/m^3 \,, C_E = 1020\, J/K\, kg, k = 156 W/K\, m, \lambda_0 = 3543 \times 10^7 N/m^2,$$
$$\mu_0 = 1518 \times 10^7 N/m^2 \,, \lambda_0 + 2\mu_0 = 6579 \times 10^7 N/m^2 \,, \alpha_T^0 = 25.2 \times 10^{-6}\, 1/K,$$
$$\varphi_0 = 298K, K_0 = 4555 \times 10^7 N/m^2 \,, \gamma = (3\lambda_0 + 2\mu_0)\alpha_T^0 = 3.444 \times 10^5 N/m^2\, K.$$

The above considered numerical values imply $a_2 = 1.4443, \varepsilon = 0.3027 \times 10^{-3}, \beta_1 = 0.444346$. For infinitesimal temperature deviations from reference temperature we can take $\psi_i(\varphi_0) = 1 - \alpha_i(\varphi_0 - T_r); i = 0,1,2$ such that $\alpha_0 > 0,\ \alpha_1 > 0\ ,\ \alpha_2 < 0$ and there after we consider $\psi_0 = 0.82\ , \psi_1 = 0.90\ , \psi_2 = 1.25$. For the graphical evaluation the other constants in this paper may be taken as $\beta = 0.05, \alpha = \frac{1}{2}, \omega = 4, b = 2, L = 2, M_1 = 0.106, M_2 = M_3 = 0.08$.

In the figures, comparisons are made between the theory of thermo-visco-elasticity with the presence and absence of the two-temperature parameter (temperature discrepancy). We can also compare the graphical results in between coupled theory and generalized theory. We moreover draw the graphs considering temperature independent mechanical properties (TIMP) and temperature dependent mechanical properties (TDMP).When $\psi_0 = \psi_1 = \psi_2 = 1$ our considered problem converted to the same with TIMP.

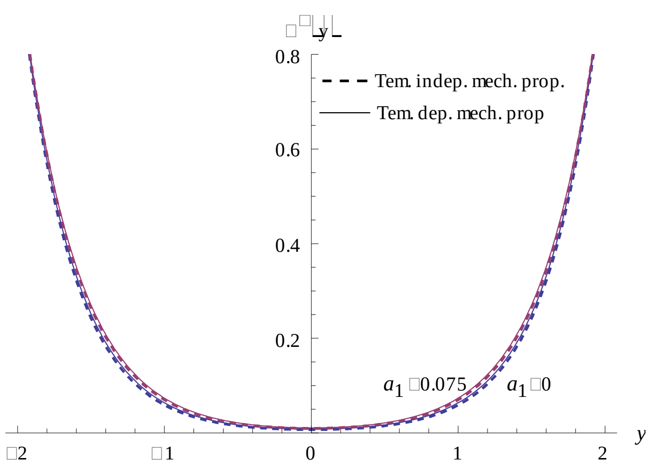

Figure1: Conductive temperature distribution

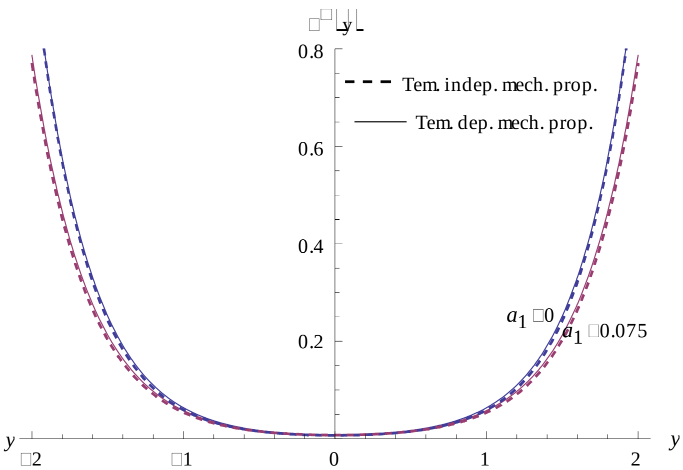

Figure 2: Thermodynamic temperature distribution

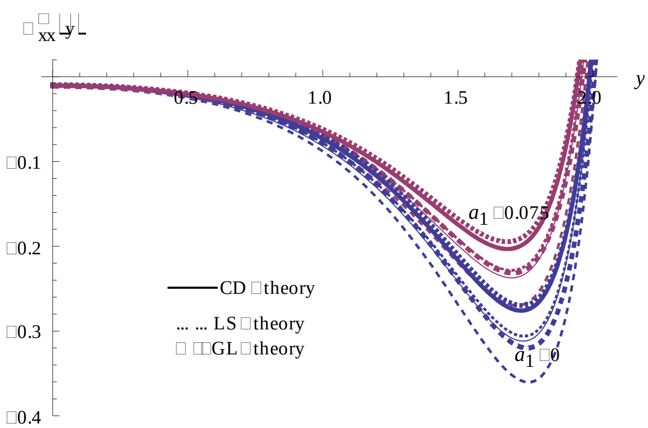

Figure 3: Stress distribution

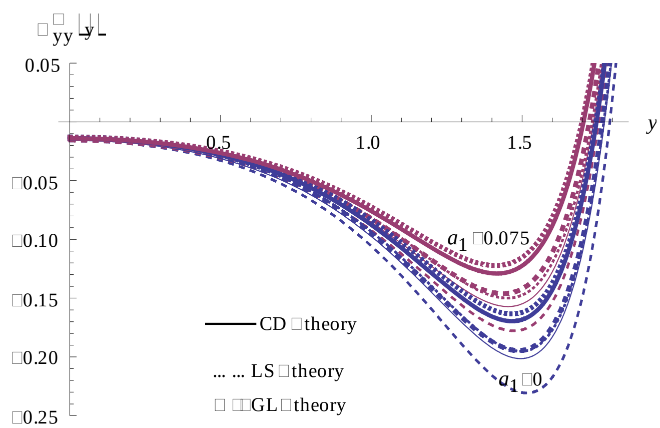

Figure 4: Stress distribution

With the purpose of study the effects of two-temperature parameter on displacement, temperature and stress we now present our outcomes of the numerical estimation in the structure of graphs. Two distinct values of a_1 are specified in all the figures corresponding to two-temperature theory ($a_1 = 0.075$) and unique temperature theory ($a_1 = 0$). Figure 1 and figure 2 demonstrate the variation of conductive temperature and thermodynamic temperature with respect to the space variable respectively. In both these graphs there is no variation between coupled theory and generalized theory. The graphs are shown in view of GL theory and comparison between the dashed curve and the solid curve are in favor of TIMP and TDMP respectively. For zero temperature discrepancy ($a_1 = 0$) the conductive temperature φ^* and thermodynamic temperature θ^* coincide to each other and the problem goes for unique temperature model. In the figures 3-7 solid curve, dotted curve and dashed curve stand for CD-theory, LS-theory and GL-theory respectively. The thick curves and the thin curves are stands for TIMP and TDMP respectively for the figures 3-7. Here the curves for the stress distributions σ_{xx}^* and σ_{yy}^* and the curves for displacement distribution u^* are symmetric about the vertical axis where as the

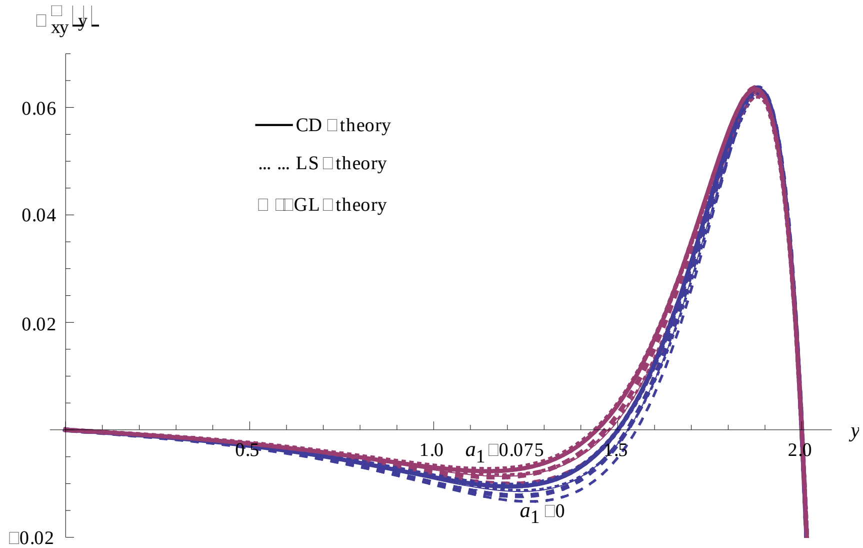

Figure 5: Stress distribution

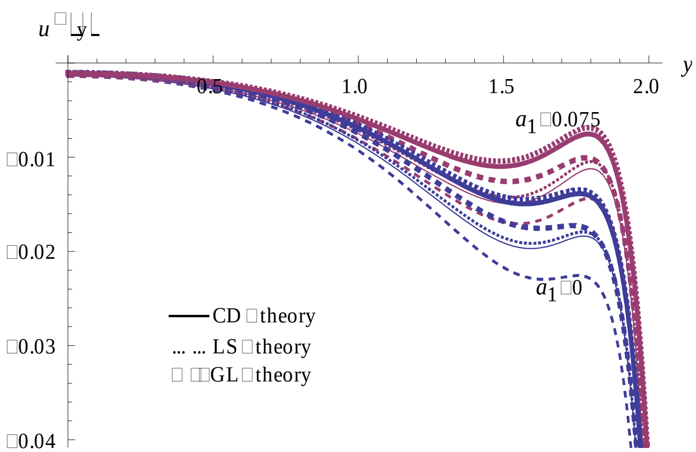

Figure 6: Displacement distribution

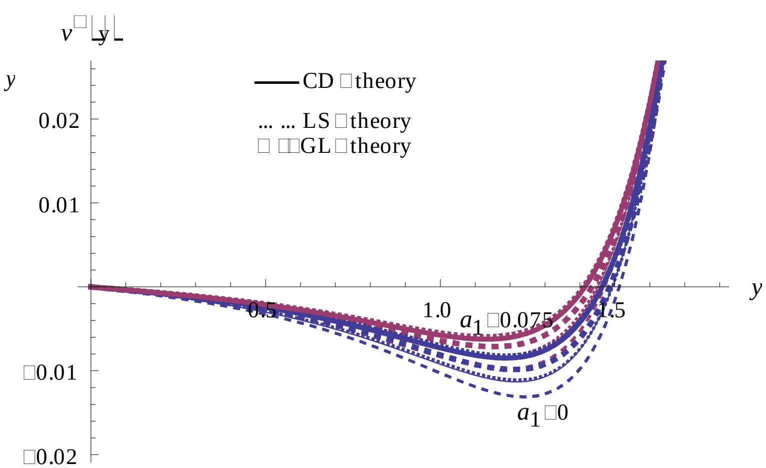

Figure 7: Displacement distribution

curves for the stress distributions σ_{xy}^{*} and the curves for displacement distribution v^{*} are symmetric about origin. So as we plotted only the right hand portion of the vertical axis of the graphs 3-7. Figures 3 and 4 represent stress distributions σ_{xx}^{*} and σ_{yy}^{*} with respect to space variable, respectively, for coupled theory and generalized theory considering $a_1 = 0.075$ and $a_1 = 0$. In both the figures 3 and 4 the curves at vicinity of the origin have almost the same values for all the above three theories and at a distance from the vertical axis the curves produce the notable variations. Figure 5 depict the stress distribution σ_{xy}^{*} for the different three theories with the comparison between TIMP and TDMP. Figure 6 exhibits the graph corresponding to the displacement field quantity u^{*} where the curves for three distinct theories are different in view of two-temperature ($a_1 = 0.075$) and unique temperature ($a_1 = 0$) and have remarkable deviation between TIMP and TDMP. It is observed from the figure that the magnitude of the displacement for two-temperature model is lesser than that of unique temperature model in favor of all the above three theories. Figure 7 shows the displacement distribution v^{*} and evaluations are made for two-temperature and unique temperature with TIMP and TDMP. It is interesting to note that the amplitudes of stress distribution σ_{xy}^{*} and displacement v^{*} vanish at the origin for all models.

Conclusion

In the context of classical theory and generalized theory the governing equations of thermo-visco-elasticity with two-temperature have been investigated by means of normal mode analysis with the effect of TDMP. The numerical computations of leading equations show the effect of two-temperature and TDMP on various theories. It is noticed that for the effect of non-null temperature discrepancy ($a_1 = 0.075$) the curves for stresses and displacements decrease compared with null temperature discrepancy ($a_1 = 0$). The analysis indicate that the stress functions are continuous and obey asymptotic nature in the neighborhood of the vertical axis $y = \pm 2$.

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