

ANISOTROPIC COSMOLOGICAL MODEL GENERAL RELATIVITY

Sumeet Goyal¹ and *Harpreet²

¹Chandigarh Engineering College, Landran, Mohali, Punjab, India

²Sant Baba Bhag Singh Institute of Engineering & Technology, Department of Applied Sciences, Khiala, Padhiana, Jalandhar- 144030, Punjab, India

*Author for Correspondence

ABSTRACT

Spatially homogeneous and anisotropic Cosmological models play significant role in the description of the early stages of evolution of the universe. The problem of the cosmological constant is still unsettled. The authors recently considered time dependent G and Λ with Bianchi type-I Cosmological model. In this paper we have studied Bianchi type-I space time with variable gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$. The field equations have been solved by assuming $\Lambda \propto R^{-6}$. The exact Bianchi type-I model have been obtained. Expressions for some important cosmological parameter have been obtained for the model and the physical behavior of the model is discussed in detail. The model represent shearing, non-rotating and expanding model of the universe with a big-bang start. It is interesting that the proposed variation law provides an alternative approach to obtain exact solutions of Einstein's field equations. Also gravitational constant $G(t)$ is zero at initial singularity and it is increasing with time increase. This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

Key Words: Bianchi Type-I Universe, Varying G and Λ , Cosmology.

INTRODUCTION

In the Einstein field equations Gravitational constant G and cosmological constant Λ are really the constants forever and every where. However the smallness of the observed cosmological constant Λ in comparison to theoretical expectations can be justified by assuming dynamically decaying Λ due to its coupling with matter fields of the universe (Abdel Rahman, 1990; Beesham, 1986; Berman, 1991; Gasperini, (1987, 1988); Kolb *et al.*, 1990; Ozer and Taha, (1986, 1987); Weinberg, 1989). Linde 1974 proposed that Λ is a function of temperature and is related with the process of broken symmetry. Besides the cosmological constant Λ , Newtonian constant of gravity G playing the role of coupling constant between geometry and matter in Einstein field equations was proposed as a function of time in evolving universe (Abdussattar and Vishwakarma, 1996).

Solutions of the field equations may also be generated by law of variation of scale factor which was proposed by Pawan (1991). The behavior of the cosmological scale factor $R(t)$ in solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies. In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Hoyle *et al.*, 1997; Olson and Jordan 1987; Pavon 1991; Maia and Silva 1994; Silveira and Waga (1994, 1997); Bloomfield Torres and Waga 1996. Chen and Wu 1990 considered Λ varying R^{-2} (R is the scale factor), Carvalho *et al.*, 1992 generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-

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Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background.

The idea of gravitational constant G in the frame work of general relativity was first proposed by Dirac 1937. Lau 1985 working in the frame work of general relativity proposed modification linking variation of G with that of Λ . This modification allows us to use Einstein field equations formally unchanged since variation of Λ is accompanied by a variation in G . Arbab (1997,1998) has considered cosmological models with viscous fluid containing G and Λ . A lot of work has been done by Saha (2005, 2006a, 2006b); Vishwakarma and Abdussattar (1996a,1996b, 1999), Vishwakarma (2000, 2001) studying the FRW Models and Bianchi type-I cosmological model in general relativity with varying G and Λ . We have studied FRW model and Bianchi type-I cosmological models with time dependent G and Λ (Tiwari (2007, 2008, 2009, 2010)).

In this paper we study homogeneous Bianchi type-I space time with variables G and Λ containing matter in the form of perfect fluid. We obtain solutions of the field equations assuming that cosmological term is proportional to R^{-6} .

Model and Field Equations

We consider the space-time admitting Bianchi type-I group of motion in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (1)$$

The non-vanishing Christoffel symbols of the second kind are:

$$\begin{aligned} \Gamma_{11}^4 &= A\dot{A}, \quad \Gamma_{14}^1 = \frac{\dot{A}}{A}, \quad \Gamma_{41}^1 = \frac{\dot{A}}{A}, \quad \Gamma_{22}^4 = B\dot{B}, \quad \Gamma_{42}^2 = \frac{\dot{B}}{B} \\ \Gamma_{33}^4 &= C\dot{C}, \quad \Gamma_{34}^3 = \frac{\dot{C}}{C} \end{aligned}$$

The non-zero components of the Ricci tensor R_{ij}

$$\begin{aligned} R_{11} &= -\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)A\dot{A} - A\ddot{A}, \quad R_{22} = -\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)B\dot{B} - B\ddot{B} \\ R_{33} &= -\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)C\dot{C} - C\ddot{C}, \quad R_{44} = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} \\ R_{14} &= -2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \end{aligned}$$

The Ricci scalar

$$R = -2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right)$$

We assume the universe to be filled with the distribution of matter represented by the energy momentum tensor of perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \quad (2)$$

where ρ is the energy density of the cosmic matter, p its equilibrium pressure and v_i the unit flow vector of the fluid taken orthogonally to the hyper surfaces of homogeneity.

The Einstein field equations with time dependent G and Λ is given by

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$$R_i^j - \frac{1}{2} R_l^l g_i^j = -8\pi G(t) \left[T_i^j - \frac{\Lambda(t)}{8\pi G(t)} g_i^j \right] \quad (3)$$

In view of vanishing divergence of Einstein tensor we have

$$\dot{\Lambda} = -8\pi\rho\dot{G} - 8\pi G \left[\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] \quad (4)$$

From eq (4), we observe that Λ is a constant in the absence of matter (*i.e.* $T_{ij} = 0$), implying that the presence of matter is essential for a time varying Λ . Incorporation of the time varying cosmological constant $\Lambda(t)$ in the Einstein field equation (3), amounts to assuming that along with its usual energy momentum tensor of the matter content T_{ij} , there is additional term $-\frac{\Lambda}{8\pi G} g_{ij}$ representing the energy momentum tensor of vacuum with its energy density ρ_v and homogeneous isotropic pressure p_v satisfying the equations of state

$$p_v = -\rho_v = \frac{-\Lambda}{8\pi G} \quad (5)$$

The conservation law for energy-momentum tensor $T_i^j{}_{;j} = 0$ leads to

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (6)$$

Equation (4) and (6) lead to

$$\dot{\Lambda} = -8\pi\rho\dot{G} \quad (7)$$

For the metric (1) and energy-momentum tensor (2) in co-moving system of co-ordinates, the field equation (3) yield

$$8\pi G \left(p - \frac{\Lambda}{8\pi G} \right) = \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} \quad (8)$$

$$8\pi G \left(p - \frac{\Lambda}{8\pi G} \right) = \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} \quad (9)$$

$$8\pi G \left(p - \frac{\Lambda}{8\pi G} \right) = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} \quad (10)$$

$$8\pi G \left(\rho + \frac{\Lambda}{8\pi G} \right) = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \quad (11)$$

we take the equation of state

$$p = \omega\rho, \quad 0 \leq \omega \leq 1 \quad (12)$$

using eq. (12) in eq. (6) and then integrating, we get

$$\rho = \frac{k}{(ABC)} \omega + 1 \quad (13)$$

Where $k > 0$, is constant of integration.

Taking $ABC = f(t)$, from equations, (8), (9) and (10) we obtain.

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$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \quad (14)$$

and
$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \quad (15)$$

where k_1 and k_2 are constant of integration.

The Hubble parameter H , volume expansion θ , shear σ and deceleration parameter q are given by

$$H = \frac{\theta}{3} = \frac{\dot{R}}{R^3} \quad (16)$$

$$\sigma = \frac{k}{\sqrt{3}/R^3} \quad (17)$$

$$q = -1 - \frac{\dot{H}}{H^2} \quad (18)$$

Equations (8) – (11) and (6) can be written in terms of H , σ and q as

$$H^2(2q-1) - \sigma^2 = 8\pi G \left(p - \frac{\Lambda}{8\pi G} \right) \quad (19)$$

$$3H^2 - \sigma^2 = 8\pi G \left(\rho + \frac{\Lambda}{8\pi G} \right) \quad (20)$$

$$\dot{\rho} + (\rho + p) \frac{3\dot{R}}{R} = 0 \quad (21)$$

Overduin and Cooperstock (1998), define

$$\rho_c = \frac{3H^2}{8\pi G} \quad (22)$$

and
$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2} \quad (23)$$

are respectively critical density and density parameter. From (19), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G \rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore $0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}$ and $0 \leq \frac{8\pi G \rho}{\theta^2} \leq \frac{1}{3}$ for $\Lambda \geq 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.

From (19), and (20), we have

$$\frac{d\theta}{dt} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ , and for positive Λ , universe will slows down the rate of decrease.

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Also $\dot{\sigma} = \frac{-\sigma 3\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of f .

Solution of the Field Equations

The system of equations (7) - (11) and (12) supply only six equations in seven unknowns (A, B, C, ρ, p, G and Λ). One extra equation is needed to solve the system completely. The phenomenological Λ decay scenarios, have been considered by number of authors, Chen and Wu 1990 considered $\Lambda \propto a^{-2}$ (a is the scale factor of the Robertson-Walker metric).

Hoyle et al. 1997 considered $\Lambda \propto a^{-3}$, whereas $\Lambda \propto a^{-m}$ (a is scale factor and m is constant) considered by Olson and Jordan 1987; Pavon 1991; Bloomfield Torres and Waga 1996. Thus we take the decaying vacuum energy density

$$\Lambda(t) = \frac{a}{R^6} \quad (24)$$

Where ' a ' is a constant.

Using equation (13) and (24) in equation (7), we get

$$G = \frac{a f^{(\omega-1)}}{4\pi k(\omega-1)} \quad (25)$$

from eq. (19), (20), and (24) we get.

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 = 0 \quad (26)$$

which on integration gives

$$R^3 = k_3 t + k_4 \quad (27)$$

where k_3 and k_4 are constant of integration. By using eq. (27) in (14) and (15), the metric (1) assumes the form.

$$ds^2 = -dT^2 + m_1^2 (T)^{\frac{2k_3+2k_2+4k_1}{3k_3}} dx^2 + m_2^2 (T)^{\frac{2k_3+2k_2-2k_1}{3k_3}} dy^2 + m_3^2 (T)^{\frac{2k_3-4k_2-2k_1}{3k_3}} dz^2 \quad (28)$$

where m_1, m_2 and m_3 are constants such that $m_1 m_2 m_3 = 1$ and $T = k_3 t + k_4$.

Conclusion

For the model (28) pressure p and density ρ are given by

$$p = \omega k (T)^{-(\omega+1)}$$

$$\rho = k (T)^{-(\omega+1)}$$

Which are positive provided $k > 0$. The energy conditions $-\rho < p < \rho$ is always satisfied. As T tends to zero pressure and density become infinite. The expansion scalar θ and shear σ for the model are $\theta = k_3 (T)^{-1}$, $\sigma = \frac{k}{\sqrt{3}} (T)^{-1}$. As t increases, θ and σ decrease and tend to zero

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as $t \rightarrow \infty$, thus the model starts expanding with a big-bang from its singular state $t=0$ and continues to expand till $T = \infty$. The anisotropy σ/θ for the model is constant. Gravitational constant G and cosmological constant Λ are

$$G = \frac{a(k_3 t + k_4)^{\omega-1}}{4\pi k(\omega-1)}$$

$$\Lambda = \frac{a}{(k_3 t + k_4)^2}$$

Gravitational constant $G(t)$ is zero at $T=0$ and as t increases, $G(t)$ also increases. A partial list of cosmological models in which the gravitational constant G is increasing with time are contained in Abdel-Rahman 1990, Chow 1981, Levitt 1980 and Milne 1935.

Further, we observe that $\Lambda \propto \frac{1}{T^2}$ which follows from the model of Kalligas et al. 1992,

Berman 1990, Berman and Som 1990, Berman et al. 1989] and Bertolami (1986a,1986b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

Critical density, vacuum density and density parameter for the model (28) are

$$\rho_c = \frac{k k_3^2 (\omega-1)}{6(k_3 t + k_4)^{\omega+1}}$$

$$\rho_v = \frac{k(\omega-1)}{2(k_3 t + k_4)^{\omega+1}}$$

and

$$\Omega = \frac{\rho}{\rho_c} = \frac{6}{k_3^2 (\omega-1)}$$

ρ, p, ρ_c, ρ_v and Λ decrease as t increases and tends to zero as $t \rightarrow \infty$. Therefore the model would essentially give an empty universe for large time t .

In this paper we have studied Bianchi type-I space time with variable gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$. The field equations have been solved by assuming $\Lambda \propto R^{-6}$. The exact Bianchi type-I model have been obtained. Expressions for some important cosmological parameter have been obtained for the model and the physical behavior of the model is discussed in detail. The model represent shearing, non-rotating and expanding model of the universe with a big-bang start. It is interesting that the proposed variation law provides an alternative approach to obtain exact solutions of Einstein's field equations. In present a unified description of the evolution of the universe which starts with a decelerating expansion and expands with acceleration at late times. Recent observational data (Knop et al. 2003; Riess et al. (1998, 2004), Spergel et al. 2007, Tegmark et al. 2004, Perlmutter et al. 1998) strongly suggest this acceleration. Also gravitational constant $G(t)$ is zero at initial singularity and it is increasing with time increase. The cosmological constant $\Lambda \propto 1/t^2$, which follows from the model of Kalligas et al. 1992, Berman and Som 1990; Berman et al. 1989 and Bertolami (1986a, 1986b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. Finally the solutions presented in the paper are new and useful for

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better understanding of the evolution of the universe in Bianchi type-I space time with variable G and Λ .

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