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Research Article

ELASTOSTATIC PROBLEM OF AN INFINITE ROW OF PARALLEL CRACKS IN AN ORTHOTROPIC MEDIUM UNDER GENERAL LOADING

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ABSTRACT
Elastostatic problem of an orthotropic elastic plane containing an infinite row of parallel cracks has been considered. After expressing the stress and displacement fields in terms of two holomorphic functions defined in appropriate complex domains the problem has been solved by Riemann-Hilbert technique. Expressions for local stress fields near the crack tip are determined and graphically displayed for various orthotropic materials.

Key Words: *Orthotropic Elastic Medium, Holomorphic Functions, Riemann-Hilbert Technique, Local Stress Field and Stress Intensity Factor (Sif)*

INTRODUCTION
Problems with Griffith cracks were considered by Dhaliwal (1973); Satpathy and Parhi (1978); Piva and Viola (1988); Cinar and Erdogan (1983); Lowengrub and Srivastav (1968); De and Patra (1990); Kassir and Tse (1983) and others. De and Patra (1993) have solved the problem of propagation of two collinear Griffith cracks in an orthotropic strip. Atkinson (1965) studied the steady-state propagation of a semi-infinite crack in an aeolotropic material by means of the Cauchy integral formula. Piva and Viola (1988) applied complex variable approach to solve the elastodynamic crack problems in an orthotropic medium. Georgiadis (1986) solved the problem of a cracked orthotropic strip.

Several authors including Stallybrass (1970) and Rooke and Sneddon (1969) have considered thde elastostatic problems involving a cruciform crack and star-shaped crack in an isotropic elastic material. They have used Muskhelishvilli-Kolosev potential function, integral transform method and Wiener-Hoff technique. Das and Debnath (2000) studied a static cruciform crack problem in an infinite orthotropic elastic medium by reducing the problem to the solution of two simultaneous singular integral equations.

Recently Das and Debnath (2003) studied interaction between Griffith cracks in a sandwiched orthotropic layer. The Fourier transform technique is used to reduce the elastostatic problem to a set of integral equations which have been solved by using the finite Hilbert transform.

The elastostatic problem of an orthotropic body having a central inclined crack and subjected to a uniform biaxial load at infinity has been studied by Kirilyuk (2005; 2007). Various static problems on crack problems are found to be present in the works of Das (2002); Garg *et al.*, (2003); Mukherjee and Das (2007); Selim and Ahmed (2006) and Nobile and Carloni (2002; 2004) and many other researchers.

The present investigation is intended to study the elastostatic problem of an infinite row of parallel cracks in an orthotropic medium under general loading. By application of the complex variable theory dealing with sectionally holomorphic functions, the problem is reduced to Riemann-Hilbert problem. The expressions for quantities of physical interest e.g. stress intensity factor (S.I.F.), the local stress field near crack tip have been derived. Numerical results for different orthotropic materials have been displayed in the form of graphs.

THE BASIC EQUATIONS
Consider an infinite orthotropic elastic medium parallel to xy-plane. The displacement component along the z axis and all its derivatives with respect to z are assumed to be zero. The stress displacements relations are given by

$$\sigma_{xx} = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} \quad \ldots (1)$$

$$\sigma_{yy} = C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y} \quad \ldots (2)$$

$$\sigma_{xy} = C_{66}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \quad \ldots (3)$$

The equilibrium equations in terms of displacement components for elastostatic problem are

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 v}{\partial x \partial y} = 0$$

$$C_{66}\frac{\partial^2 v}{\partial x^2} + C_{22}\frac{\partial^2 v}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 u}{\partial x \partial y} = 0 \quad \ldots (4)$$

where C_{ij} (i,j=1,2,3) are elastic constants, $u = u(x, y)$, $v = v(x, y)$ are displacements in x, y directions.
The system of equations (4) may be rewritten as

$$I\frac{\partial \Phi}{\partial x} + A\frac{\partial \Phi}{\partial y} = 0 \quad \ldots (5)$$

in which I is the 4×4 identity matrix and

$$A = \begin{pmatrix} 0 & \alpha & 2\beta & 0 \\ -1 & 0 & 0 & 0 \\ 2\beta_1 & 0 & 0 & \alpha_1 \\ 0 & 0 & -1 & 0 \end{pmatrix},\ \Phi(x, y) = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} = \begin{pmatrix} u_{,x} \\ u_{,y} \\ v_{,x} \\ v_{,y} \end{pmatrix} \quad \ldots (6)$$

where

$$2\beta = \frac{C_{12} + C_{66}}{C_{11}}, 2\beta_1 = \frac{C_{12} + C_{66}}{C_{66}}$$

$$\alpha = \frac{C_{66}}{C_{11}}, \alpha_1 = \frac{C_{22}}{C_{66}} \quad \ldots (7)$$

Eigenvalues of the matrix A can be obtained from the equation

$$m^4 + 2a_1 m^2 + a_2 = 0 \quad \ldots (8)$$

where $2a_1 = \alpha + \alpha_1 - 4\beta\beta_1$, and $a_2 = \alpha\alpha_1$. … (9)

Two types of orthotropic materials can be defined: (i) Type I: when $a_1 > \sqrt{a_2}$ and (ii) Type II: when $|a_1| < \sqrt{a_2}$; based on the existence of the real part of the solution.

The Case of Type I Orthotropic material

The roots of the equation (8) will be purely imaginary, if $a_2 > 0, a_1 > \sqrt{a_2}$ … (10)

The two eigenvalues can be taken as $m_1 = ip$ and $m_2 = iq$ … (11)

where $p = \left[a_1 - (a_1^2 - a_2)^{1/2}\right]^{1/2}$, and $q = \left[a_1 + (a_1^2 - a_2)^{1/2}\right]^{1/2}$ … (12)

We now consider the transformation $\Phi(x, y) = U\Psi(x, y)$, where $U = \begin{pmatrix} 0 & \frac{2\beta p^2}{\alpha - p^2} & 0 & \frac{2\beta q^2}{\alpha - q^2} \\ \frac{2\beta p^2}{\alpha - p^2} & 0 & \frac{2\beta q^2}{\alpha - q^2} & 0 \\ -p & 0 & -q & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ and $\Psi(x, y) = [\Psi_1 \quad \Psi_2 \quad \Psi_3 \quad \Psi_4]^T$

It can be shown that $\Omega_1(z_1) = \Psi_1(x_1, y_1) + i\Psi_2(x_1, y_1)\, and\, \Omega_2(z_2) = \Psi_3(x_2, y_2) + i\Psi_4(x_2, y_2)$ are two holomorphic functions of $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, respectively,

where $x_1 = x, x_2 = x, y_1 = \frac{y}{p}, y_2 = \frac{y}{q}$.

The stress components can be expressed in terms of two analytic functions $\Omega_j(z_j), (j = 1, 2)$ as follows:

$$\sigma_{xx} = \frac{C_{66}}{\alpha} \operatorname{Im}[l_1\Omega_1(z_1) + l_2\Omega_2(z_2)] \quad \ldots (13)$$

$$\sigma_{yy} = C_{66} \operatorname{Im}[p^2 l_3 \Omega_1(z_1) + q^2 l_4 \Omega_2(z_2)] \quad \ldots (14)$$

$$\sigma_{xy} = C_{66} \operatorname{Re}[p l_5 \Omega_1(z_1) + q l_6 \Omega_2(z_2)] \quad \ldots (15)$$

where

$$l_1 = \frac{2\beta p^2}{(\alpha - p^2)} + (2\beta - \alpha),\; l_2 = \frac{2\beta q^2}{(\alpha - q^2)} + (2\beta - \alpha)$$

$$l_3 = 1 - \frac{2\beta}{\alpha - p^2},\; l_4 = 1 - \frac{2\beta}{\alpha - q^2},\; l_5 = -l_3,\; l_6 = -l_4 \quad \ldots (16)$$

The corresponding displacement components are

$$u(x, y) = \operatorname{Im}[\frac{2\beta p^2}{\alpha - p^2}\omega_1(z_1) + \frac{2\beta q^2}{\alpha - q^2}\omega_2(z_2)] \quad \ldots (17)$$

$$v(x, y) = -\operatorname{Re}[p\omega_1(z_1) + q\omega_2(z_2)] \quad \ldots (18)$$

where

$$\omega_1(z_1) = \int \Omega_1(z_1)dx = \frac{i}{p}\int \Omega_1(z_1)dy$$

$$\omega_2(z_2) = \int \Omega_2(z_2)dx = \frac{i}{q}\int \Omega_2(z_2)dy \quad \ldots (19)$$

The Case of Type II Orthotropic material

When the elastic properties of the orthotropic material are such that

$$|a_1| < \sqrt{a_2},\; a_2 > 0 \quad \ldots (20)$$

The roots of the equation (8) are complex.

The eigen values can be taken as,

$$m_1 = \gamma_1 + i\gamma_2, \quad m_2 = -\overline{m}_1,\; \gamma_2 > 0 \quad \ldots (21)$$

where a bar denotes complex conjugate and

$$\gamma_1 = \left[\frac{1}{2}(\sqrt{a_2} + a_1)\right]^{1/2}, \quad \gamma_2 = \left[\frac{1}{2}(\sqrt{a_2} - a_1)\right]^{1/2} \quad \ldots (22)$$

Following Piva and Voila (1988), the basic complex variables can be written as,

$$z_1 = \left(x - \frac{\gamma_1}{\gamma_1^{\ 2} + \gamma_2^{\ 2}} y \right) + i \left(\frac{\gamma_2}{\gamma_1^{\ 2} + \gamma_2^{\ 2}} y \right) \qquad \ldots (23)$$

$$z_2 = \left(x + \frac{\gamma_1}{\gamma_1^{\ 2} + \gamma_2^{\ 2}} y \right) + i \left(\frac{\gamma_2}{\gamma_1^{\ 2} + \gamma_2^{\ 2}} y \right) \qquad \ldots (24)$$

The stress components can be written as,

$$\sigma_{xx} = C_{66}\{k_1 \operatorname{Im}[\Omega_1(z_1) + \Omega_2(z_2)] - k_2 \operatorname{Re}[\Omega_1(z_1) - \Omega_2(z_2)]\} \qquad \ldots (25)$$

$$\sigma_{yy} = C_{66}\{k_3 \operatorname{Im}[\Omega_1(z_1) + \Omega_2(z_2)] - k_4 \operatorname{Re}[\Omega_1(z_1) - \Omega_2(z_2)]\} \qquad \ldots (26)$$

$$\sigma_{xy} = C_{66}\{k_5 \operatorname{Re}[\Omega_1(z_1) + \Omega_2(z_2)] + k_6 \operatorname{Im}[\Omega_1(z_1) - \Omega_2(z_2)]\} \qquad \ldots (27)$$

where

$$k_1 = \frac{C_{12}}{C_{66}} - 2\beta p_3 \frac{C_{11}}{C_{66}}, \ k_2 = 2\beta p_4 \frac{C_{11}}{C_{66}}$$

$$k_3 = \frac{C_{22}}{C_{66}} - 2\beta p_3 \frac{C_{12}}{C_{66}}, \ k_4 = 2\beta p_4 \frac{C_{12}}{C_{66}}$$

$$k_5 = 2\beta p_2 - \gamma_2, \ \ k_6 = 2\beta p_1 - \gamma_1,$$

$$\frac{m_1}{\alpha + m_1^2} = p_1 + ip_2 \text{ , and } \frac{m_1^2}{\alpha + m_1^2} = p_3 + ip_4 \qquad \ldots (28)$$

The displacement components are given by

$$u(x, y) = -2\beta \operatorname{Im}[(p_3 + ip_4)\omega_1(z_1) + (p_3 - ip_4)\omega_2(z_2)] \qquad \ldots (29)$$

$$v(x, y) = -\operatorname{Im}[(\gamma_1 + i\gamma_2)\omega_1(z_1) - (\gamma_1 - i\gamma_2)\omega_2(z_2)] \qquad \ldots (30)$$

where

$$\omega_1(z_1) = \int \Omega_1(z_1)dx = \frac{-(\gamma_1 - i\gamma_2)}{\gamma_1^2 + \gamma_2^2} \int \Omega_1(z_1)dy \qquad \ldots (31)$$

$$\omega_2(z_2) = \int \Omega_2(z_2)dx = \frac{(\gamma_1 + i\gamma_2)}{\gamma_1^2 + \gamma_2^2} \int \Omega_2(z_2)dy \qquad \ldots (32)$$

BOUNDARY CONDITIONS

Lest us consider an infinite row of parallel cracks, at a distance of period 2π, having a constant length $2a$, along the y axis, with uniform function $p_0 f(x)$ applied to its edges, in an elastic medium so that the boundary conditions of the problem are

$$\sigma_{yy}(x,0) = -p_0 f(x), \qquad |x| < a \qquad \ldots (33)$$

$$\sigma_{xy}(x,0) = 0, \qquad |x| < \infty \qquad \ldots (34)$$

$$v(x,0) = 0, \qquad |x| > a \qquad \ldots (35)$$

$$\sigma_{xx}, \sigma_{yy}, \sigma_{xy} = 0, \qquad |z_j| \to \infty \qquad \ldots (36)$$

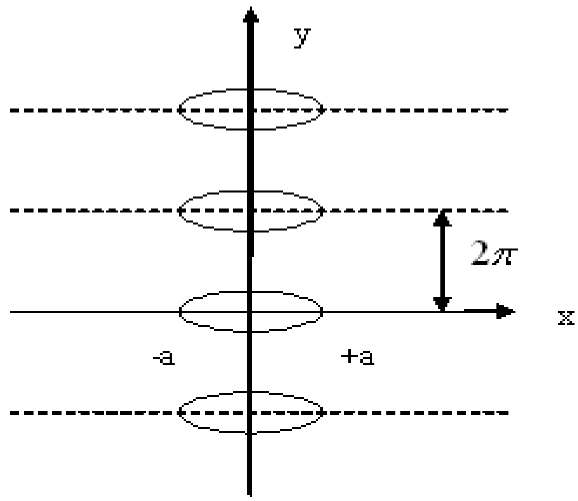

Figure 1: An infinite row of parallel cracks, at a distance of period 2π , having a constant length $2a$, along the y axis

SOLUTION OF THE PROBLEM

Case I: Type I material

Setting

$$\Lambda_1(z_1) = \frac{ip\Delta}{l_6}\Omega_1(z_1) \qquad \dots(37)$$

$$\Lambda_2(z_2) = \frac{-iq\Delta}{l_5}\Omega_2(z_2) \qquad \dots(38)$$

The stress components can be written as,

$$\sigma_{xx} = \frac{C_{66}}{\alpha\Delta}\mathrm{Re}[\frac{l_2 l_5}{q}\Lambda_2(z_2) - \frac{l_1 l_6}{p}\Lambda_1(z_1)] \qquad \dots(39)$$

$$\sigma_{yy} = \frac{C_{66}}{\Delta}\mathrm{Re}[q l_4 l_5 \Lambda_2(z_2) - p l_3 l_6 \Lambda_1(z_1)] \qquad \dots(40)$$

$$\sigma_{xy} = \frac{C_{66} l_5 l_6}{\Delta}\mathrm{Im}[\Lambda_1(z_1) - \Lambda_2(z_2)] \qquad \dots(41)$$

where

$$\Delta = p l_3 l_6 - q l_4 l_5 \qquad \dots(42)$$

Equation (45) and boundary condition (34) lead to

$$\Lambda_1(x) = \Lambda_2(x), |x| < \infty \qquad \dots(43)$$

Using the symmetry property $\Lambda_j(z_j) = \overline{\Lambda}_j(z_j)$ boundary conditions (33) and (35) by virtue of (43) , lead to

$$\mathrm{Re}\Lambda_j(x) = \frac{p_0 f(x)}{C_{66}}, \ |x| < a \ \text{ on } \ y = 0 \qquad \dots (44)$$

$$\mathrm{Im}\Lambda_j(x) = 0 , |x| > a \ \text{ on } \ y = 0 \qquad \dots(45)$$

where $\Lambda_j(z_j)$ is a periodic function of period 2π .

Since the stress distribution is identical in the strips $2n\pi < \pm y < 2(n+1)\pi, n = 0,1,2,...$ the stress field should be periodic function of period $2\pi i$.

Equations (44) and (45) can be written as

$$\Lambda_j^+(z_j) + \Lambda_j^-(z_j) = \frac{2p_0 f(x)}{C_{66}}, \qquad \text{on} \quad \text{L:} \quad |x| > a \qquad \ldots(46)$$

$$\text{and } \Lambda_j^+(z_j) - \Lambda_j^-(z_j) = 0 \text{, otherwise} \qquad \ldots(47)$$

where $\Lambda_j(x + i.0) = \Lambda_j^+(z_j)$ and $\Lambda_j(x - i.0) = \Lambda_j^-(z_j)$.

Following the approach of Gakov (1966) to solve this Riemann boundary value problem we set

$$\Lambda_j(z_j) = \int_0^a \frac{F(t)\sinh t}{\cosh t - \cosh z_j} dt, \qquad \text{which is singular on L , where y = 0} \qquad \ldots(48)$$

Applying the Sokhotski's formula we obtain,

$$\Lambda_j^+(z_j) = \pi i F(z_j) + \int_0^a \frac{F(t)\sinh t}{\cosh t - \cosh z_j} dt \qquad \text{and} \qquad \Lambda_j^-(z_j) = -\pi i F(z_j) + \int_0^a \frac{F(t)\sinh t}{\cosh t - \cosh z_j} dt$$

From (46) and above Sokhotski's formula, we have

$$\int_0^a \frac{F(t)\sinh t}{\cosh t - \cosh z_j} dt = \frac{p_0 f(x)}{C_{66}}$$

Using Finite Hilbert transform and the Airfoil equation, the unknown $F(t)$ is calculated.

where

$$F(t) = \frac{(\cosh a - 1)}{2A} + \frac{p_0}{\pi^2 \mu A} \int_0^a \frac{\sqrt{(\cosh x - 1)(\cosh a - \cosh x)}}{\cosh t - \cosh x} \sinh x f(x) dx$$

$$\text{and } A = \sqrt{(\cosh t - 1)(\cosh a - \cosh t)}$$

Particular case

When f(x) =1, the solution of the Riemann boundary value problem defined by (44) and (45) is given by

$$\Lambda_j(z_j) = \frac{p_0}{C_{66}} F_j(z_j), (j = 1,2) \qquad \ldots(49)$$

where

$$F_j(z_j) = 1 - \sqrt{\frac{\cosh z_j - 1}{\cosh z_j - \cosh a}} \qquad , \quad (j = 1,2) \qquad \ldots (50)$$

The stress components may be rewritten as

$$\sigma_{xx} = \frac{p_0}{\alpha\Delta} \text{Re}[\frac{l_2 l_5}{q} F_2(z_2) - \frac{l_1 l_6}{p} F_1(z_1)] \qquad \ldots (51)$$

$$\sigma_{yy} = \frac{p_0}{\Delta} \text{Re}[q l_4 l_5 F_2(z_2) - p l_3 l_6 F_1(z_1)] \qquad \ldots (52)$$

$$\sigma_{xy} = \frac{p_0 l_5 l_6}{\Delta} \mathrm{Im}[F_1(z_1) - F_2(z_2)] \qquad \ldots (53)$$

It can also be shown from (17) and (18) that

$$u(x,y) = \frac{-2\beta p_0}{\Delta C_{66}} \mathrm{Re}[\frac{p l_6}{\alpha - p^2} G_1(z_1) - \frac{q l_5}{\alpha - q^2} G_2(z_2)] \qquad \ldots (54)$$

$$v(x,y) = \frac{p_0}{\Delta C_{66}} \mathrm{Im}[l_5 G_2(z_2) - l_6 G_1(z_1)] \qquad \ldots (55)$$

with

$$G_j(z_j) = \int F_i(z_i) dz_i = z_j - \cosh^{-1}\left\{\frac{2\cosh z_j - \cosh a + 1}{\cosh a + 1}\right\}, \text{(j=1,2)} \qquad \ldots (56)$$

The local stress and displacement fields: It is well known that the crack features are controlled by the local stress field. Introducing polar coordinates (r_j, ϑ_j), the local stress fields are given by

$$\sigma_{xx} = \sigma_{xx}^{(1)} + \sigma_{xx}^{(0)}$$

$$\approx \frac{-p_0}{\alpha\Delta}\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left\{\frac{l_2 l_5}{q\sqrt{C_2(\vartheta)}}\cos\frac{\vartheta_2}{2} - \frac{l_1 l_6}{p\sqrt{C_1(\vartheta)}}\cos\frac{\vartheta_1}{2}\right\} + \frac{p_0}{\alpha\Delta}\left\{\frac{l_2 l_5}{q} - \frac{l_1 l_6}{p}\right\} \qquad \ldots (57)$$

$$\sigma_{yy} = \sigma_{yy}^{(1)} + \sigma_{yy}^{(0)}$$

$$\approx \frac{-p_0}{\Delta}\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left\{q l_4 l_5 \frac{\cos\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}} - p l_3 l_6 \frac{\cos\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}}\right\} - p_0 \qquad \ldots (58)$$

$$\sigma_{xy} = \sigma_{xy}^{(1)} + \sigma_{xy}^{(0)}$$

$$\sigma_{xy} \approx \frac{-p_0 l_5 l_6}{\Delta}\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left\{\frac{\sin\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}} - \frac{\sin\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}\right\} \qquad \ldots (59)$$

where $\sigma_{xx}^{(1)}$ denotes the singular part of stress and $\sigma_{xx}^{(0)}$ denotes the non-singular part of stress and similarly for other stress fields.

$$\text{Again, } z_j = a + re^{i\vartheta_j} = a(1 + \delta_j e^{i\vartheta_j}), \delta_j = \frac{r}{a} C_j(\vartheta), (j = 1,2) \qquad \ldots (60)$$

$$\text{and } C_j(\vartheta) = (\cos^2\vartheta + \frac{\sin^2\vartheta}{p_j^2})^{1/2}, \vartheta_j tg^{-1}\left(\frac{tg\,\vartheta}{p_j}\right), p_j = \begin{cases} p & j = 1 \\ q & j = 2 \end{cases} \qquad \ldots (61)$$

The circumferential stress distribution is given by

$$\sigma_{\vartheta\vartheta} = \sigma_{\vartheta\vartheta}^{(1)} + \sigma_{\vartheta\vartheta}^{(0)} \qquad \ldots (62)$$

where

$$\sigma_{\vartheta\vartheta}^{(1)} = \sigma_{xx}^{(1)} \sin^2\vartheta + \sigma_{yy}^{(1)} \cos^2\vartheta - \sigma_{xy} \sin 2\vartheta, \qquad \ldots (63)$$

and $$\sigma_{\vartheta\vartheta}^{(0)} = p_0\left[\frac{1}{\alpha\Delta}\left\{\frac{l_2 l_5}{q} - \frac{l_1 l_6}{p}\right\}\sin^2\vartheta - \cos^2\vartheta\right] \qquad \ldots(64)$$

The stress intensity factor near the crack tip (x = a) is given by the relation

$$K_a = \lim_{x\to a^+}\left[2(x-a)\right]^{1/2}\sigma_{yy}(x,0)$$

$$= \sqrt{2}\,p_0\sqrt{\tanh\frac{a}{2}} \qquad \ldots (65)$$

Case II: Type II material

Setting

$$\Lambda_1(z_1) = \frac{-2\Delta_1}{k_5 + ik_6}\Omega_1(z_1)$$

$$\Lambda_2(z_2) = \frac{2\Delta_1}{k_5 - ik_6}\Omega_2(z_2)$$

where $\Delta_1 = k_3k_6 - k_4k_5$ and following the same approach as that of Type I material, for f(x) =1, the stress components can be written as,

$$\sigma_{xx} = \frac{-p_0}{2\Delta_1}[(k_1k_5 + k_2k_6)\operatorname{Im}\{F_1(z_1) - F_2(z_2)\} + (k_1k_6 - k_2k_5)\operatorname{Re}\{F_1(z_1) + F_2(z_2)\}]$$

$$\sigma_{yy} = \frac{-p_0}{2\Delta_1}[\Delta_1\operatorname{Re}\{F_1(z_1) + F_2(z_2)\} + (k_3k_5 + k_4k_6)\operatorname{Im}\{F_1(z_1) - F_2(z_2)\}]$$

$$\sigma_{xy} = \frac{-p_0}{2\Delta_1}(k_5^2 + k_6^2)\operatorname{Re}[F_1(z_1) - F_2(z_2)]$$

where

$$\Lambda_j(z_j) = \frac{p_0}{C_{66}}F_j(z_j),\quad (j = 1,2)$$

The displacements fields are

$$u(x,y) = \frac{\beta p_0}{\Delta_1 C_{66}}[(p_3k_5 - p_4k_6)\operatorname{Im}\{G_1(z_1) - G_2(z_2)\} + (p_3k_6 + p_4k_5)\operatorname{Re}\{G_1(z_1) + G_2(z_2)\}]$$

$$v(x,y) = \frac{p_0}{2\Delta_1 C_{66}}[(\gamma_1k_5 - \gamma_2k_6)\operatorname{Im}\{G_1(z_1) + G_2(z_2)\} + (\gamma_1k_6 + \gamma_2k_5)\operatorname{Re}\{G_1(z_1) - G_2(z_2)\}]$$

with $G_j(z_j) = \int F_i(z_i)dz_i, (j = 1,2)$

The local stress and displacement fields: The local stress fields are given by

$$\sigma_{xx} = \sigma_{xx}^{(1)} + \sigma_{xx}^{(0)}$$

$$\approx \frac{-p_0}{2\Delta_1}\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left[(k_1k_5 + k_2k_6)\left\{\frac{\sin\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}} - \frac{\sin\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}\right\} - (k_1k_6 - k_2k_5)\left\{\frac{\cos\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}} + \frac{\cos\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}\right\}\right] - \frac{p_0}{\Delta_1}(k_1k_6 - k_2k_5)$$

$$\sigma_{yy} = \sigma_{yy}^{(1)} + \sigma_{yy}^{(0)}$$

$$\approx \frac{p_0}{2\Delta_1}\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left[(k_3k_6-k_4k_5)\left\{\frac{\cos\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}}+\frac{\cos\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}\right\}+(k_3k_5+k_4k_6)\left\{\frac{\sin\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}}-\frac{\sin\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}\right\}\right]-p_0$$

$$\sigma_{xy}=\sigma_{xy}^{(1)}+\sigma_{xy}^{(0)}$$

$$\approx \frac{-p_0}{2\Delta_1}(k_5^2+k_6^2)\sqrt{\frac{\tanh\frac{a}{2}}{r}}\left\{\frac{\cos\frac{\vartheta_2}{2}}{\sqrt{C_2(\vartheta)}}-\frac{\cos\frac{\vartheta_1}{2}}{\sqrt{C_1(\vartheta)}}\right\}$$

where $C_j(\vartheta)=(\cos^2\vartheta+l^2\sin^2\vartheta+\varepsilon_j l^2\sin 2\vartheta)^{1/2}, l^2=(\gamma_1^2+\gamma_2^2)^{-1}$

$$\vartheta_j=tg^{-1}\left(\frac{\gamma_2 l^2\sin\vartheta}{\cos\vartheta+\varepsilon_j\gamma_1 l^2\sin\vartheta}\right), \varepsilon_j=\begin{cases}-1 & j=1\\ 1 & j=2\end{cases}$$

The circumferential stress distribution is given by

$$\sigma_{\vartheta\vartheta}=\sigma_{\vartheta\vartheta}^{(1)}+\sigma_{\vartheta\vartheta}^{(0)}$$

where $\sigma_{\vartheta\vartheta}^{(1)}=\sigma_{xx}^{(1)}\sin^2\vartheta+\sigma_{yy}^{(1)}\cos^2\vartheta-\sigma_{xy}\sin 2\vartheta,$

and $\sigma_{\vartheta\vartheta}^{(0)}=-p_0\left[\frac{1}{\Delta_1}(k_1k_6-k_2k_5)\sin^2\vartheta+\cos^2\vartheta\right]$

The stress intensity factor near the crack tip (x = a) is given by the relation

$$K_a=\lim_{x\to a^+}\left[2(x-a)\right]^{1/2}\sigma_{yy}(x,0)$$

$$=\sqrt{2}p_0\sqrt{\tanh\frac{a}{2}}$$

NUMERICAL RESULTS AND DISCUSSIONS

Numerical results for singular terms of stress field near the crack tip for different orthotropic materials in case of Type I orthotropic material have been considered. The values of the elastic constants have been taken from the paper of Kassir and Tse (1983) and Garg (1981). All units are in the order of 10^{12}dyns/cm^2.

Table 1: The values of the elastic constants for different orthotropic materials

Materials	C_{11}	C_{22}	C_{12}	C_{66}
Pinewood	0.0983	0.00413	0.000983	0.00736
Beryllium	3.148	3.649	0.888	1.124
Steel-Mylar	1.87	0.292	0.13	0.062
Beechwood	0.017	0.158	0.015	0.0103
Magnesium	0.575	0.601	0.195	0.167

In Fig. 2 - Fig. 4, variation of the singular terms of the stress components $\sigma_{xx}^{(1)}$, $\sigma_{yy}^{(1)}$, and $\sigma_{xy}^{(1)}$ with different angular positions near crack tip are displayed for the above mentioned materials.

In Fig.5, variation of singular term of the circumferential stress with different angular positions near the crack tip is presented.

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Fig. 6 shows the comparison of singular and non singular terms of the stress component σ_{xx} . Natures of curves for the above mentioned materials are the same with a linear shifting for the different stress components.

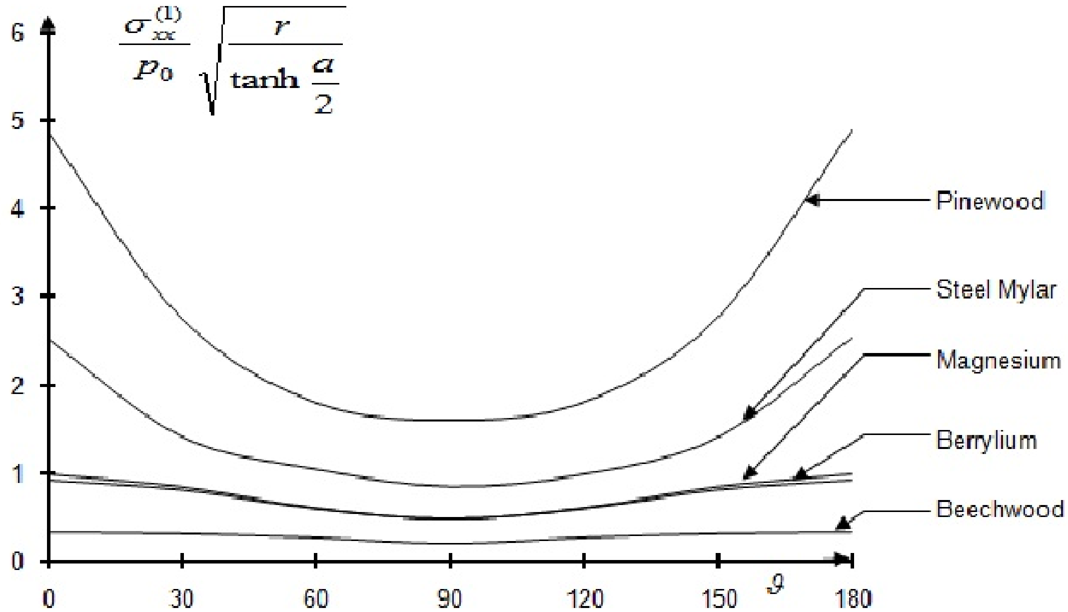

Figure 2: Plot of $\sigma_{xx}^{(1)}$ against angle (degree)

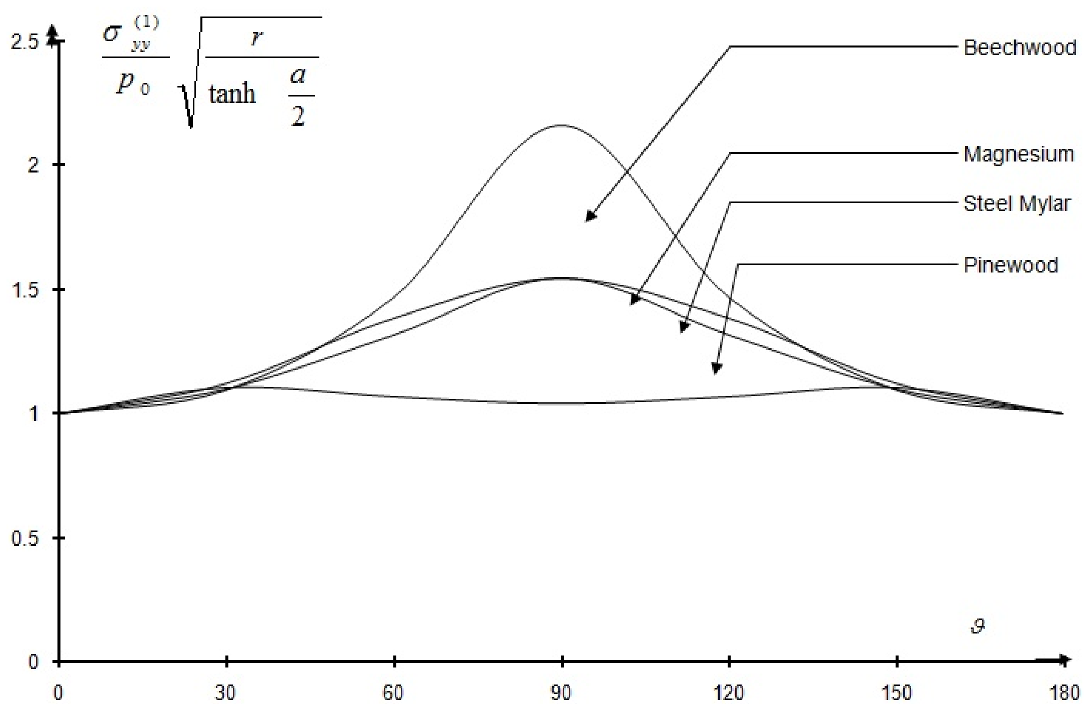

Figure 3: Plot of $\sigma_{yy}^{(1)}$ against angle (degree)

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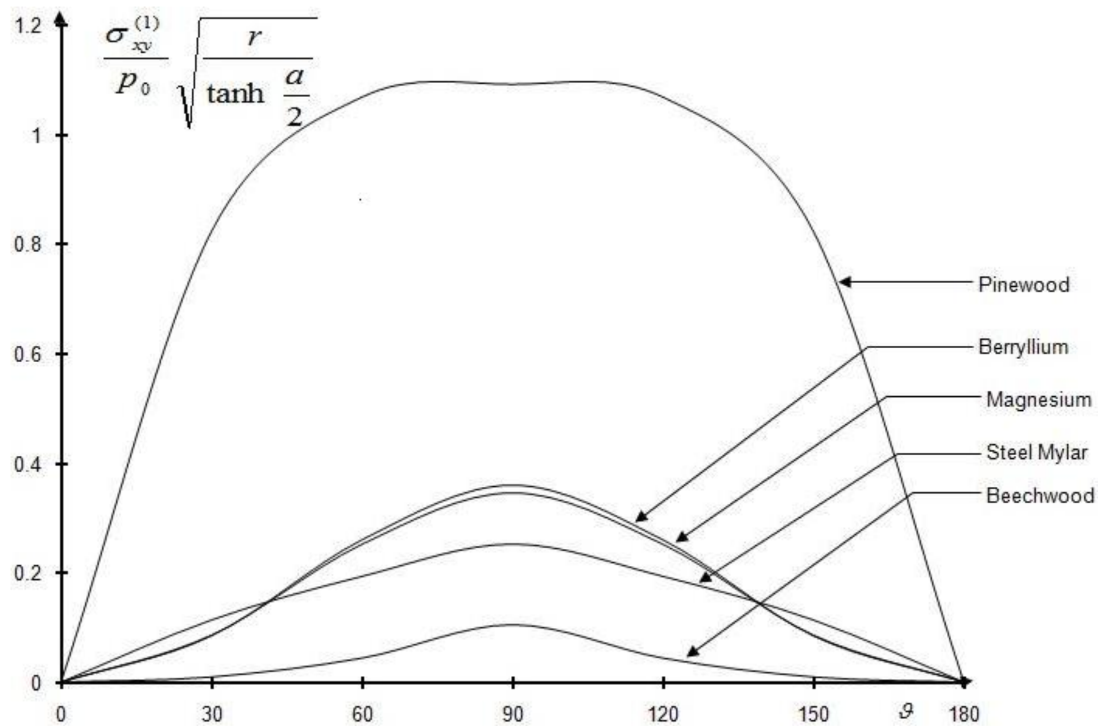

Figure 4: Plot of $\sigma_{xy}^{(1)}$ against angle (degree)

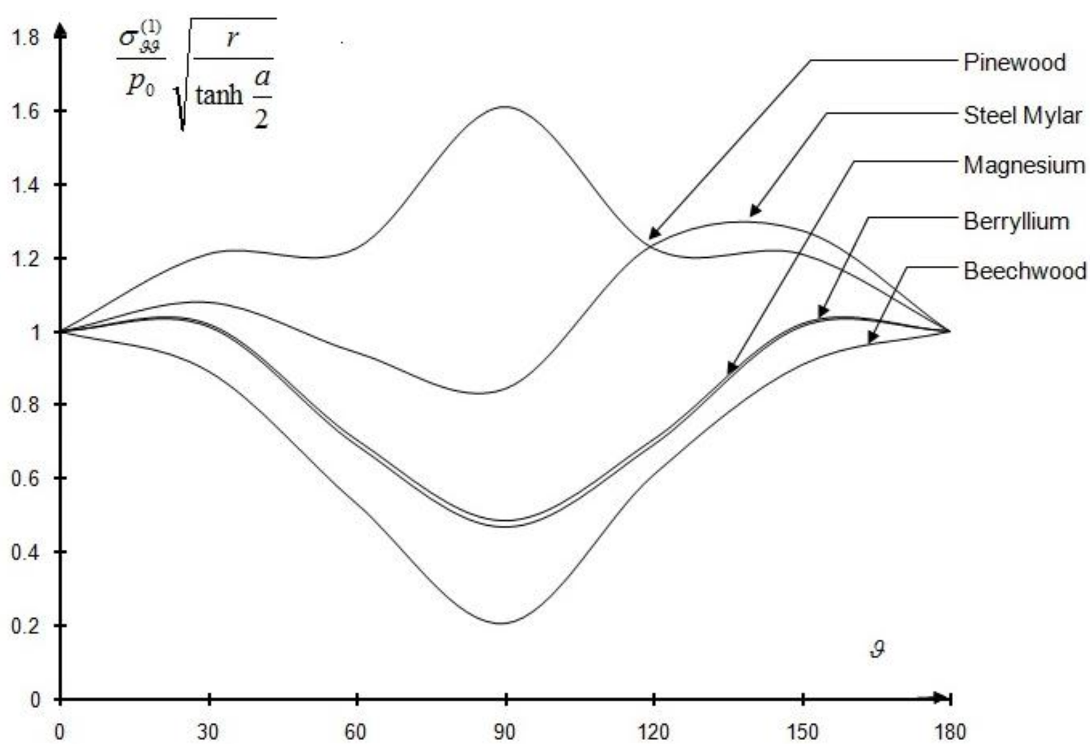

Figure 5: Plot of circumferential stress $\sigma_{\theta\theta}^{(1)}$ against angle (degree)

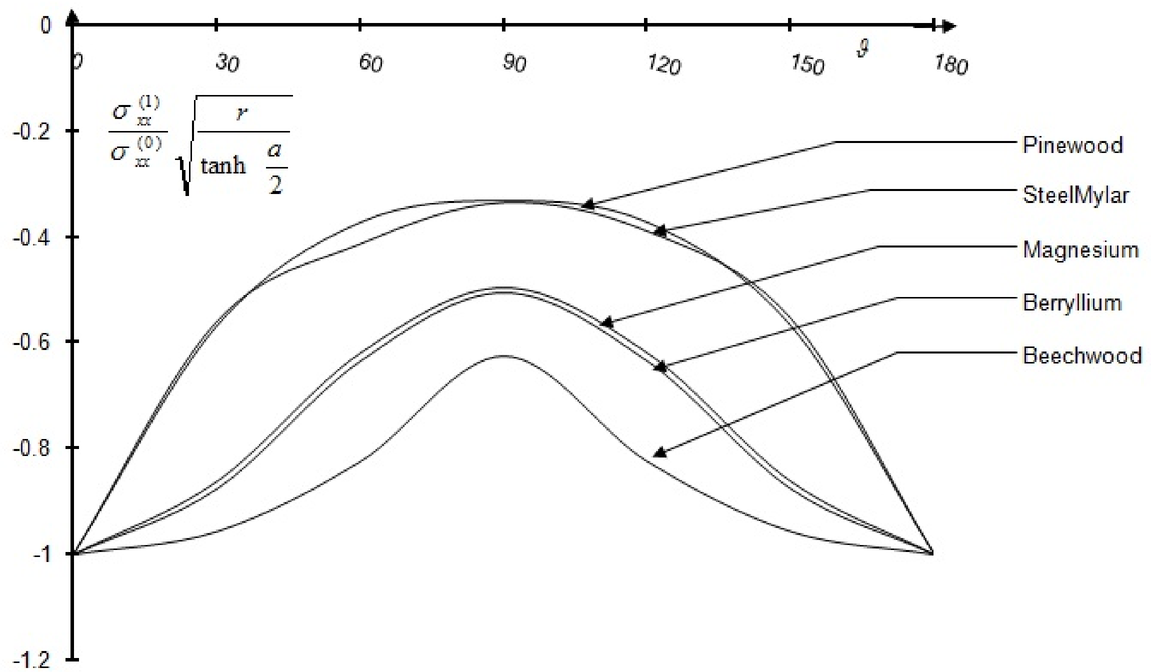

Figure 6: Comparison between non singular and singular terms of the stress component σ_{xx}

Conclusions

The outcomes of the present investigations may be summarized as investigation of elastostatic problem of an infinite row of parallel cracks in an orthotropic medium under general loading. The problem is reduced to Riemann-Hilbert problem with the help of the complex variable theory dealing with sectionally holomorphic functions. The expressions for quantities of physical interest e.g. stress intensity factor (S.I.F.), the local stress field near crack tip, have been completely and thoroughly investigated.Results for singular terms of stress field near the crack tip for different orthotropic materials in case of Type I orthotropic material have been displayed in the form of graphs. It becomes plausible that similar problems in general anisotropic material may be caried out with similar techniques.

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