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Research Article

MHD CONVECTIVE HEAT TRANSFER OF A NANO FLUID FLOW PAST AN INCLINED PERMEABLE PLATE WITH HEAT SOURCE AND RADIATION

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ABSTRACT
We investigated theoretically the MHD flow of a nano-fluid past an inclined, oscillatory and permeable semi-infinite flat plate. The constant heat source and the thermal radiation are also considered. The slip velocity is oscillatory on time. The governing equations of the boundary layer are solved numerically using the method of lines. The influence of various parameters on velocity and heat transfer is analyzed. The increase in inclination of the plate, thinning the momentum boundary and enhancing the thermal boundary layers.

Key Words*: *Nano - Fluid, Mhd, Inclined Plate, Radiation, Method of Lines.

List of symbols

B_0 Constant applied magnetic field (Wb m^{-2})
C_p Specific heat at constant pressure (J kg^{-1} K^{-1})
g Gravity acceleration (m s^{-2})
J Current density
M Dimensionless magnetic field parameter
n Dimensionless frequency
Nu Local Nusselt number
Pr Prandtl number
Q Dimensional heat source (kJ s^{-1})
Q_H Dimensionless heat source parameter (kJ s^{-1})
S Dimensionless suction parameter
t Dimensionless time (s)
T Local temperature of the nano-fluid (K)
T_w Wall temperature (K)
T_∞ Temperature of the ambient nano-fluid (K)
u, w Dimensionless velocity components (m s^{-1})
U_0 Characteristic velocity (m s^{-1})
w_0 Mass flux velocity
k Thermal conductivity
R_a Radiation parameter

Greek symbols

α Thermal diffusivity (m^2 s^{-1})
β Thermal expansion coefficient (K^{-1})
ε Dimensionless small quantity $(<< 1)$
ϕ Solid volume fraction of the nano-particles
μ Dynamic viscosity (Pa s)
ψ Kinematic viscosity (m^2 s^{-1})

θ Dimensionless temperature
σ Electrical conductivity ($m^2 s^{-1}$)
σ_1 Stefan-Boltzmann constant
δ Mean absorption Coefficient
γ Inclination angle of the plate
ρ Density

Superscript

– Dimensional quantities

Subscripts

f Fluid
s Solid
nf Nano-fluid

INTRODUCTION

Research in the field of Heat transfer challenging the cooling of many systems used in day to day life of mankind. The heat transfer enhances enormously when nano-particles are suspended in liquids like water, ethylene glycol etc. This has substantiated by Das *et al.*, (2006) in their review paper. In this scenario cooling systems demand the very low heat transfer rate through nano – fluids and heat energy systems like automobiles demanding the high heat transfer rate through nano – fluids.

Kuznetsov and Nield (2010) studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid (Newtonian fluid) past a vertical flat plate to the case of nano-fluids. In these papers the authors have used the nano-fluid model proposed by Buongiorno (2006). Although this author discovered that seven slip mechanisms take place in the convective transport in nano-fluids, it is only the Brownian diffusion and the thermophoresis that are the most important when the turbulent flow effects are absent. More recently, Khan and Aziz (2011) studied Natural convection flow of a nano-fluid over a vertical plate with uniform surface heat flux. Hamad and Pop (2011) presented in their recent paper that the solid volume and heat source enhances the heat transfer rate. This brief survey clearly indicates that a definitive conclusion regarding the role of nano-particles in enhancing natural convective transport is yet to be reached.

In this paper we aim to investigate the MHD Cu – water nano-fluid flow and the heat transfer past a vertical infinite permeable inclined oscillating flat plate under heat source, suction and radiation.

MATHEMATICAL FORMULATION

Consider the unsteady three dimensional free convection flow of a nano-fluid past a vertical permeable semi-infinite plate in the presence of an applied magnetic field with constant heat source and radiation. We consider a Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$. The flow is assumed to be in the $\bar{x}$ direction, which is taken along the plate, and $\bar{z}$ - axis is normal to the plate. We assume that the plate has an oscillatory movement on time $\bar{t}$ and frequency $\bar{n}$ with the velocity u(0, t),which is given u(0, t) = $U_0(1 + \varepsilon \cos(nt))$, where ε is a small constant parameter ($\varepsilon << 1$) and U_0 is the characteristic velocity. We consider that initially (t<0) the fluid as well as the plate are at rest. A uniform external magnetic field B_0 is taken to be acting along the $\bar{z}$-axis. We consider the case of a short circuit problem in which the applied electric field E = 0, and also assume that the induced magnetic field is small compared to the external magnetic field B_0. The surface temperature is assumed to have the constant value T_w while the ambient temperature has the constant value T_∞, where $T_w > T_\infty$. The conservation equation of current density $\nabla . J = 0$ gives J_z = constant. Since the plate is electrically non-conducting, this constant is zero. It is assumed that the plate is infinite in extent and hence all physical quantities do not depend on $\bar{x}$ and $\bar{y}$ but depend only on $\bar{z}$ and $\bar{t}$,

Research Article

$$\text{i.e} \qquad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$$

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Bossinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial w}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial z^2} + (\rho\beta)_{nf}\, g\,(T - T_\infty)\cos\gamma - \sigma B_0^2 u\right] \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \alpha_{nf}\frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}}(T - T_\infty) - \frac{\alpha_{nf}}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial z} \quad \ldots\ldots\ldots\ldots (3)$$

The appropriate initial and boundary conditions for the problem are given by

$$u(z,t) = 0, T = T_\infty \quad for\ t < 0\ \forall\ z$$

$$\left.\begin{array}{l} u(0,t) = \left[1 + \frac{\varepsilon}{2}\left(e^{int} + e^{-int}\right)\right], T(0,t) = T_w \\ u(\infty,t) \to 0, T(\infty,t) \to T_\infty,\ \varepsilon << 1 \end{array}\right\} \quad t \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_f + \phi(\rho c_p)_s,$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$k_{nf} = k_f\left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The thermo-physical properties of the materials used are as follows.

Table 1: Thermo-physical properties

Physical Properties	Water	Copper(Cu)
C_p $(J/kg\ K)$	4,179	385
ρ (kg/m^3)	997.1	8,933
κ $(W/m\ K)$	0.613	400
$\beta\times10^{-5}$ $(1/K)$	21	1.67

We consider the solution of Esq. (1) as $w = -w_0$ ……………… (6)

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where the constant w_0 represents the normal velocity at the plate which is positive for suction ($w_0>0$) and negative for blowing or injection ($w_0<0$). Thus, we introduce the following dimensionless variables:

$$z = (\frac{\psi_f}{U_0})Z,\; t = (\frac{\psi_f}{U_0^2})t^*,\; n = (\frac{U_0^2}{\psi_f})\eta,\; u = UU_0,\; \theta = \frac{T-T_\infty}{T_w-T_\infty},$$

$$q_r = -\frac{4\sigma_1}{3\delta}\frac{\partial T^4}{\partial y} \quad \text{………. (7)}$$

We assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty^3 - 3T_\infty^4$$

By using above, we obtain

$$\frac{\partial q_r}{\partial z} = -\frac{16\sigma_1}{3\delta}\frac{\partial^2 T^4}{\partial z^2} \quad \text{………………. (8)}$$

Using equations 5, 6, 7 and 8 the Eqs. 2-3 can be written in the following dimensionless form:

$$\left[1-\phi+\phi(\frac{\rho_s}{\rho_f})\right]\left(\frac{\partial U}{\partial \tau}-S\frac{\partial U}{\partial Z}\right) = \frac{1}{(1-\phi)^{2.5}}\frac{\partial^2 U}{\partial Z^2} + \left[1-\phi+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\theta\cos\gamma - MU \quad \text{………………(9)}$$

$$\left[1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]\left(\frac{\partial \theta}{\partial \tau}-S\frac{\partial \theta}{\partial Z}\right) = \frac{1}{p_r}\left[\frac{k_{nf}}{k_f}\frac{\partial^2\theta}{\partial Z^2}\right] - \frac{1}{p_r}Q_H\theta + \frac{1}{p_r}\frac{4}{3}R_a\frac{\partial^2\theta}{\partial Z^2} \quad \text{……………(10)}$$

where the corresponding boundary conditions (4) can be written in the dimensionless form as:

$$\begin{aligned} & U(z,t) = 0,\; \theta(z,t) = 0 \quad for\; t<0\;\forall\; z \\ & \left.\begin{aligned} & U(0,t) = \left[1+\frac{\varepsilon}{2}(e^{int}+e^{-int})\right],\; \theta(0,t)=1 \\ & U(\infty,t)\to 0,\; \theta(\infty,t)\to 0 \end{aligned}\right\}\forall\; t\ge 0 \quad \text{………………… (11)} \end{aligned}$$

Here p_r is the Prandtl number, S is the suction (S>0) or injection (S<0) parameter, M is the magnetic parameter, R_a is the Radiation parameter and Q_H is the heat source parameter, which are defined as:

$$p_r = \frac{\psi_f}{\alpha_f},\; S = \frac{w_o}{U_0},\; M = \frac{\sigma B_O^2\psi_f}{\rho_f U_0^2},\; R_a = \frac{4\alpha\sigma_1 T_\infty^3}{\delta k_{nf}},\; Q_H = \frac{Q\psi_f^2}{k_f U_0^2}$$

where the velocity characteristic U_0 is defined as

$$U_0 = \left[g\beta_f(T_w-T_\infty)\psi_f\right]^{1/3} \quad \text{………………. (12)}$$

The local Nusselt number Nu in dimension less form:

$$Nu = -\frac{k_{nf}}{k_f}\theta'(0) \quad \text{………………… (13)}$$

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RESULTS AND DISCUSSIONS

The governing equations are solved by using Method of lines with the help of Mathematica *package*. The variations of velocity U and temperature θ are graphically exhibited and the Heat Transfer rate (Nu) is exhibited in Table – 2 for various values of ϕ, S, M, α ,Q_H and R_a by keeping Pr = 6.2, nt = π/2 and ε = 0.02. The effect of various parameters is as follows.

The increase of solid volume fraction reduces the velocity Fig.1 and enhances the temperature Fig.7. The thickness of momentum and the thermal boundary layers decreases with increase in ϕ. The variations of velocity and the temperature with R_a are depicted in Fig. 2 and 8. The increase in R_a enhances the momentum and thermal boundary layers.

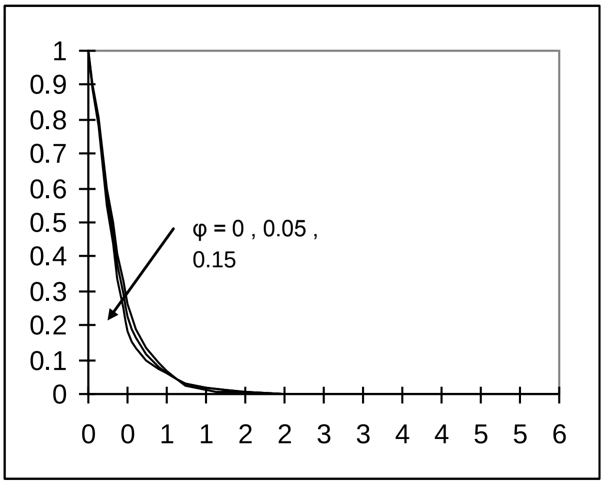

Figure 1: Variation of U with ϕ

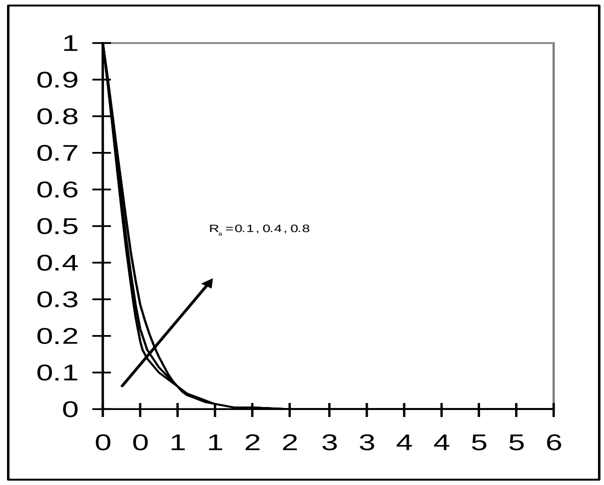

Figure 2: Variation of U with R_a

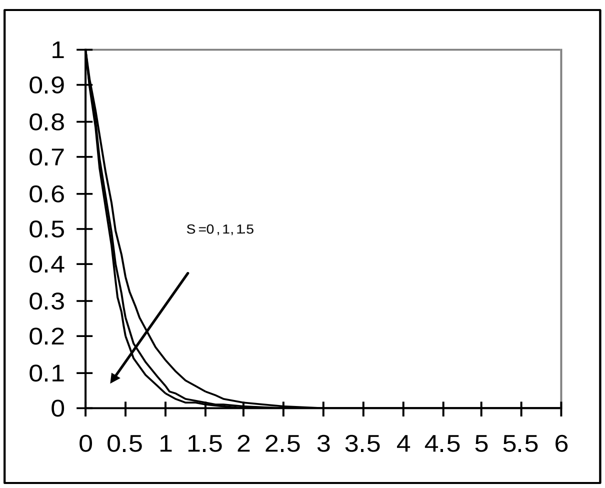

Figure 3: Variation of U with S

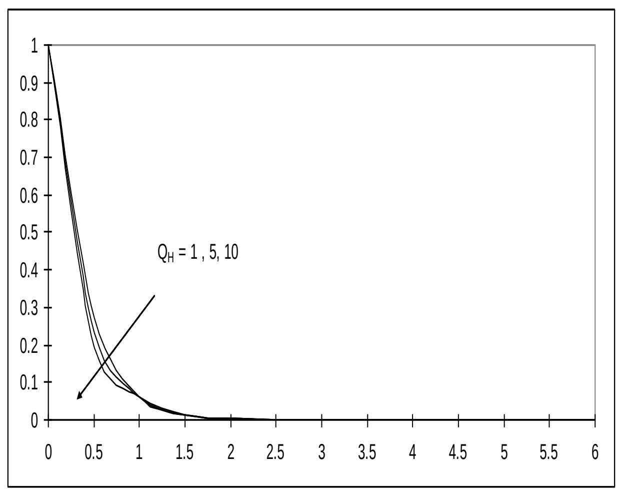

Figure 4: Variation of U with Q_H

The increase in suction reduces the momentum and thermal boundary layer in Fig. 3 and 9. The variations of velocity and temperature with heat source Q_H are exhibited in Fig. 4 and 10. The increase in Q_H decreases the velocity and the temperature. The variations of velocity and temperature with magnetic parameter M are depicted in Fig. 5 and 11. The effects of inclination angle α on velocity and temperature are exhibited in Fig. 6 and 12. The increase in inclination reduces the velocity and enhances the temperature.

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The rate of heat transfer (Nu) for various values of R_a, ϕ and Q_H are given in the Table-2. The Nu is increasing with increase in ϕ and Q_H. But Nu decreases with increase in R_a.

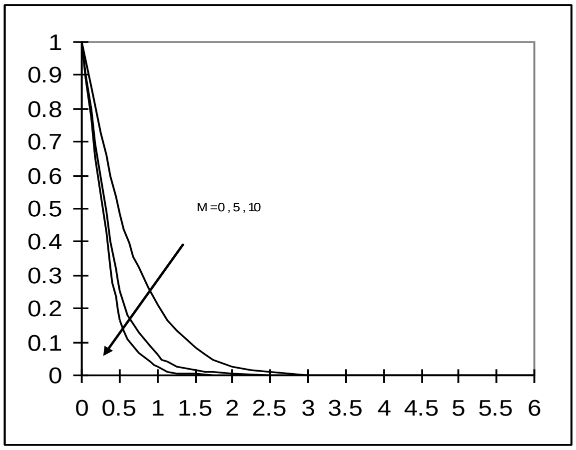

Figure 5: Variation of U with M

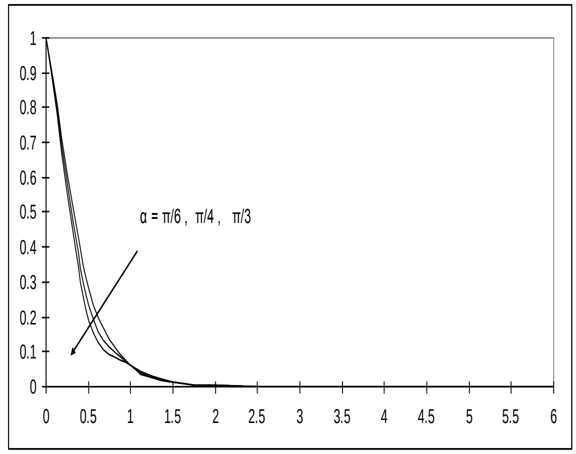

Figure 6: Variation of U with α

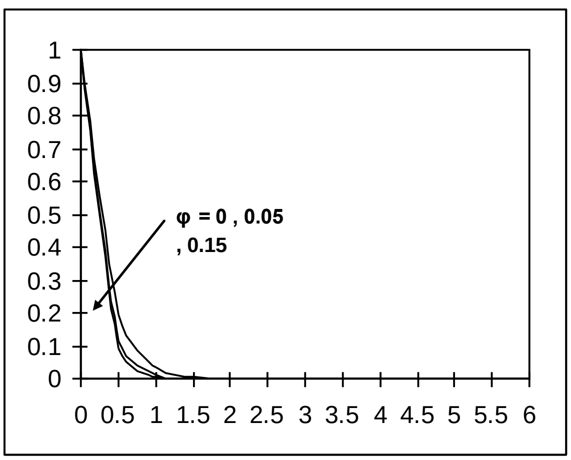

Figure 7: Variation of θ with ϕ

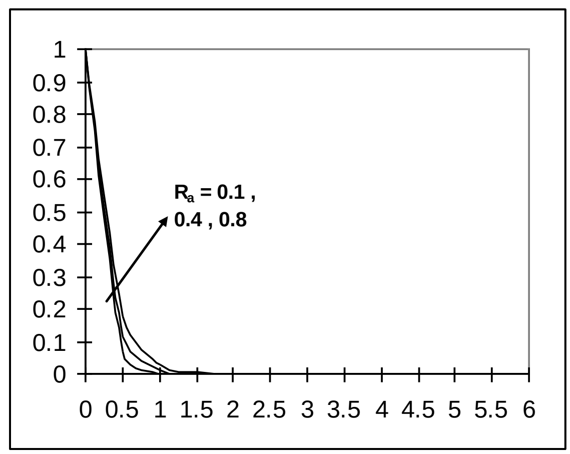

Figure 8: Variation of θ with R_a

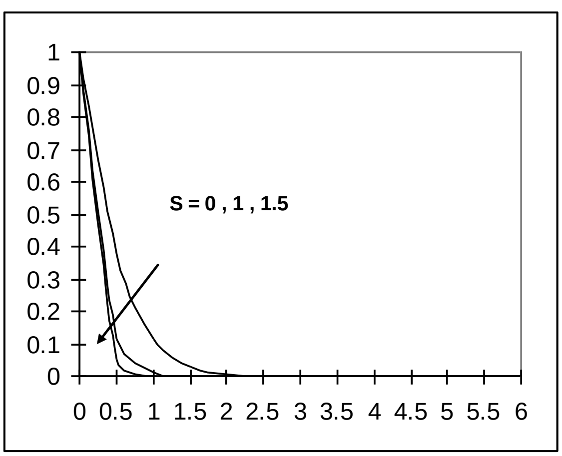

Figure 9: Variation of θ with S

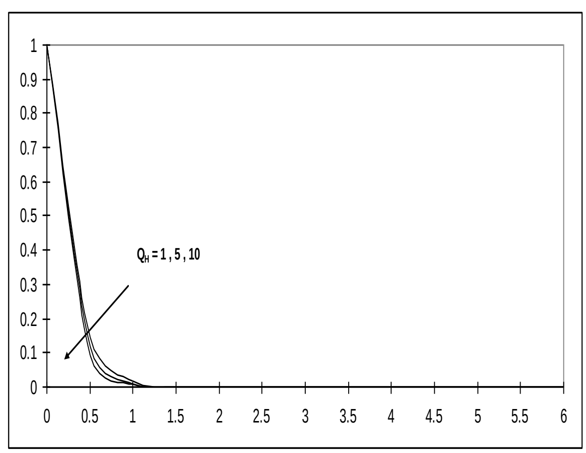

Figure 10: Variation of θ with Q_H

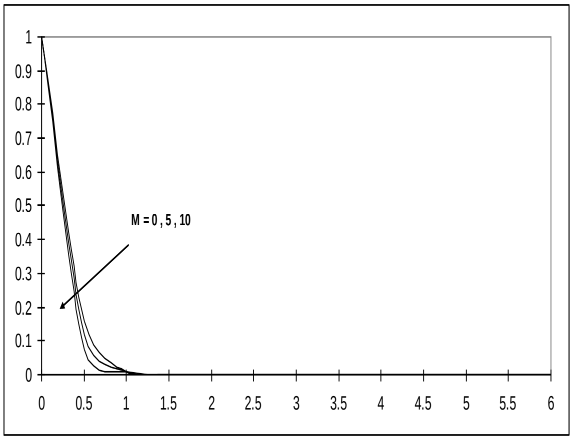

Figure 11: Variation of θ with M

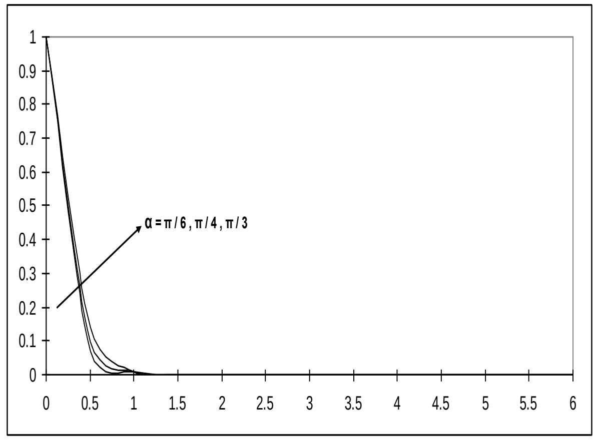

Figure 12: Variation of θ with α

Table 2: Nusselt Number

ϕ	Q_H = **5;** Ra = **0.4**	Q_H = **10;** Ra = **0.4**	Q_H = **5;** Ra = **0.8**
0	4. 5034	4. 8933	3. 6028
0.05	4. 9525	5. 4462	4. 0485
0.15	5. 8815	6. 6397	5. 0378
0.2	6. 3913	7. 3178	5. 6092

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