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Research Article

AN INNOVATIVE ANALYSIS TO DEVELOP NEW THEOREMS ON IRREGULAR POLYGON

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ABSTRACT
The irregular Polygon is a four sided polygon of two dimensional geometrical figures. The triangle, square, rectangle, tetragon, pentagon, hexagon, heptagon, octagon, nonagon, dodecagon, parallelogram, rhombus, rhomboid, trapezium or trapezoidal, kite and dart are the members of the irregular polygon family. A polygon is a two dimensional example of the more general prototype in any number of dimensions. However the properties are varied from one to another. The author has attempted to develop two new theorems for the property of irregular polygon for a point anywhere inside of the polygon with necessary illustrations, appropriate examples and derivation of equations for better understanding.

Key Words: *Irregular Polygon, Triangle, Right-angled triangle, Perpendicular and Vertex*

INTRODUCTION
Polygon (Weisstein, 2003) is a closed two dimensional figure formed by connecting three or more straight line segments, where each line segment end connects to only one end of two other line segments. Polygon is one of the most all-encompassing shapes in two- dimensional geometry. The sum of the interior angles is equal to 180 degree multiplied by number of sides minus two. The sum of the exterior angles is equal to 360 degree. From the simple triangle up through square, rectangle, tetragon, pentagon, hexagon, heptagon, octagon, nonagon, dodecagon (Weisstein, 2003,) and beyond is called n-gon. Depending on its interior vertex (Weisstein, 2003) angle, the polygon is divided into various categories viz.,
Convex polygon (Weisstein, 2003) a polygon with all diagonals in the interior of the polygon. All vertices point outward concave, at least one vertex point inward towards the center of the polygon. A convex polygon is defined as a polygon with all its interior angles less than 180°.
Concave polygon (Weisstein, 2003) a non-convex. The concave polygon is a simple polygon having at least one interior angle greater than 180°.
Simple polygon (Weisstein, 2003) it does not cross itself.
Complex polygon (Weisstein, 2003) its path may self-intersect. It looks as combined polygons.
Equiangular polygon: all its corner angles are equal.
Cyclic polygon: all corners lie on a single circle.
Isogonal or vertex-transitive polygon: all corners lie within the same symmetry orbit. The polygon is also cyclic and equiangular.
Equilateral polygon or Regular Polygon (Weisstein, 2003) all edges are of the same length. (A polygon with five or more sides can be equilateral without being convex).
Isotoxal or edge-transitive polygon: all sides lie within the same symmetry orbit. The polygon is also equilateral.
Tangential polygon: all sides are tangent to an inscribed circle.
The particular cases of the polygons are: *Square* (Weisstein, 2003) and *Rhombus* (Weisstein, 2003).

NEW THEOREM ON IRREGULAR POLYGON
Theorem 1:

Suppose P is a point anywhere inside of the irregular polygon, points $A_1,\ A_2,\ A_3,\ \dots\ A_n$ are the vertices of the polygon and points $M_1,\ M_2,\ M_3,\ \dots\ M_n$ are the perpendicular projection of the point P on the sides

$\overline{A_1A_2}$, $\overline{A_2A_3}$, $\overline{A_3A_4}$ … $\overline{A_nA_1}$ of the polygon. If $\overline{M_1A_1}$, $\overline{M_2A_2}$, $\overline{M_3A_3}$, …, $\overline{M_nA_n}$ are left part line segment and $\overline{M_1A_2}$, $\overline{M_2A_3}$, $\overline{M_3A_4}$, …, $\overline{M_nA_1}$ are right part of the line segment of the sides respectively then the sum of the squares of all the left side line segments is equal to the sum of the squares of all the right side line segments (ref. fig.1). The mathematical form of the theorem is

$$\left(\overline{M_1A_1}\right)^2 + \left(\overline{M_2A_2}\right)^2 + \left(\overline{M_3A_3}\right)^2 + \cdots + \left(\overline{M_nA_n}\right)^2 = \left(\overline{M_1A_2}\right)^2 + \left(\overline{M_2A_3}\right)^2 + \left(\overline{M_3A_4}\right)^2 + \cdots + \left(\overline{M_nA_1}\right)^2$$

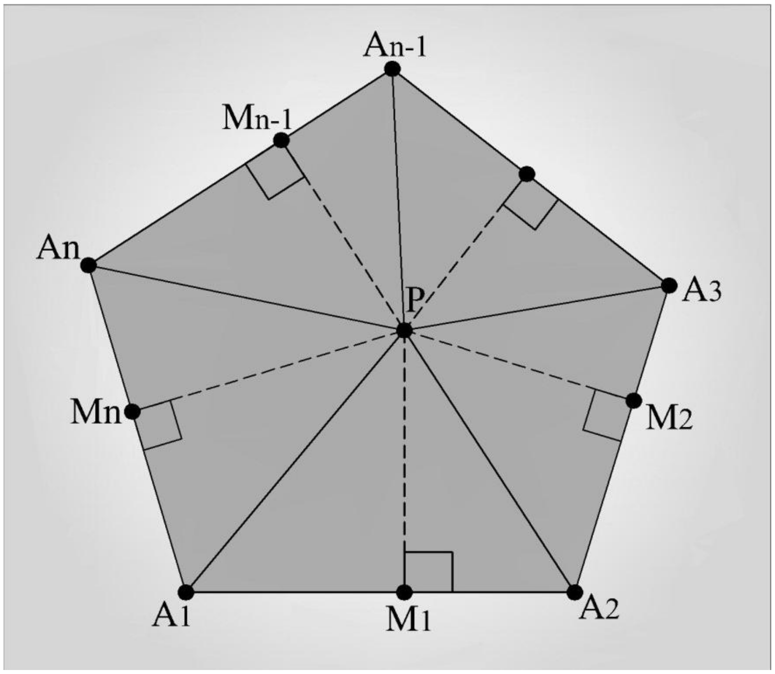

Figure 1: A diagram of irregular cyclic Polygon $A_1A_2 \dots A_{n-1}A_n$

Derivation of equations for theorem-1

Point P is anywhere in the ellipse, points A_1, A_2, A_3, ... A_n are the vertices of the polygon and points M_1, M_2, M_3, ... M_n are the projection of the point P on the sides of the polygon $\overline{A_1A_2}$, $\overline{A_2A_3}$, $\overline{A_3A_4}$ … $\overline{A_nA_1}$ respectively. Therefore, $\overline{PM_1} \perp \overline{A_1A_2}$, $\overline{PM_2} \perp \overline{A_2A_3}$ ... $\overline{PM_n} \perp \overline{A_nA_1}$. Point P is joined with vertices of the polygon. $\overline{PA_1}, \overline{PA_2}, \overline{PA_3}, \dots \overline{PA_n}$ are the distance of the vertices from point P.

Let, $\overline{PA_1} = a_1$, $\overline{PA_2} = a_2$, $\overline{PA_3} = a_3$, ... $\overline{PA_n} = a_n$

Let, $\overline{PM_1} = h_1$, $\overline{PM_2} = h_2$, $\overline{PM_3} = h_3$, ... $\overline{PM_n} = h_n$

Let, $\overline{A_1A_2} = b_1$, $\overline{A_2A_3} = b_2$, $\overline{A_3A_4} = b_3$, ... $\overline{A_nA_1} = b_n$

Let, $\overline{M_1A_1} = c_1, \overline{M_1A_2} = d_1$, $\overline{M_2A_2} = c_2, \overline{M_2A_3} = d_2$, ... $\overline{M_nA_n} = c_n, \overline{M_nA_1} = d_n$

Referring fig. 2,

In right triangle PM_1A_1, $\overline{PA_1}^2 = \overline{PM_1}^2 + \overline{M_1A_1}^2$

Substituting, $PA_1 = a_1$, $\overline{PM_1} = h_1$, $\overline{M_1A_1} = c_1$, we get

$a_1^2 = h_1^2 + c_1^2$ ———————————————————————————— [1]

In right triangle PM_1A_2, $\overline{PA_2}^2 = \overline{PM_1}^2 + \overline{M_1A_2}^2$

Substituting, $\overline{PA_2} = a_2$, $\overline{PM_1} = h_1$, $\overline{M_1A_2} = d_1$, we get

$a_2^2 = h_1^2 + d_1^2$ ———————————————————————————— [2]

Subtracting [2] from [1],

$a_1{}^2 - a_2{}^2 = c_1{}^2 - d_1{}^2$ ———————————————————————————— [3]

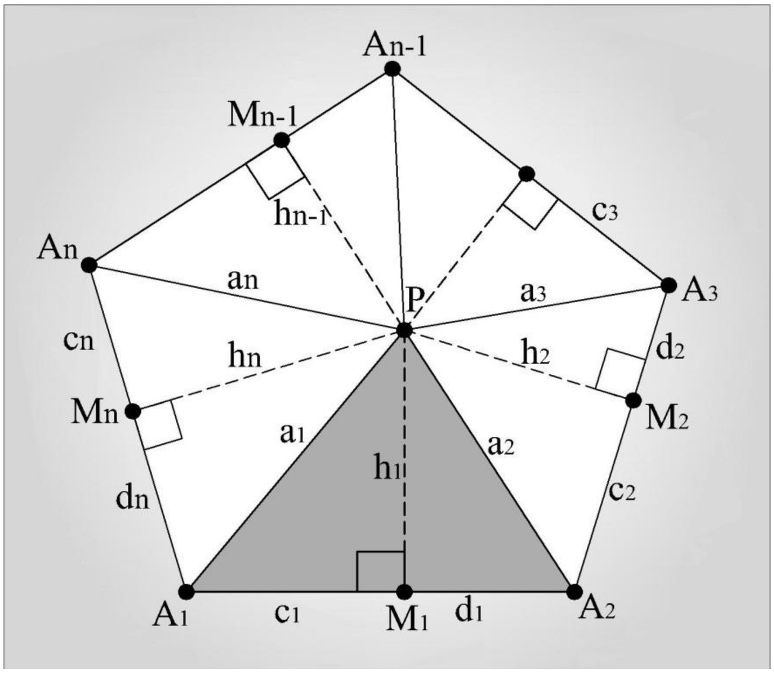

Figure 2: **A diagram of irregular polygon with ΔPA_1A_2**

Rreferring fig. 3,

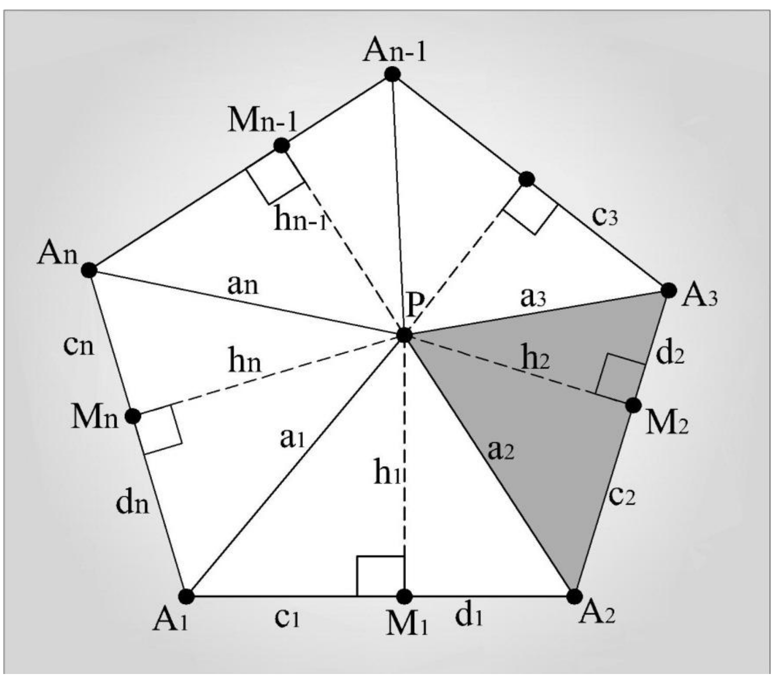

Figure 3: **A diagram of irregular polygon with ΔPA_2A_3**

In right triangle PM_2A_2, $\overline{PA_2}^2 = \overline{PM_2}^2 + \overline{M_2A_2}^2$
Substituting, $\overline{PA_2} = a_2$, $\overline{PM_2} = h_2$, $\overline{M_2A_2} = c_2$, we get
$a_2{}^2 = h_2{}^2 + c_2{}^2$ ———————————————————————————— [4]

In right triangle PM_2A_3, $\overline{PA_3}^2 = \overline{PM_2}^2 + \overline{M_2A_3}^2$
Substituting, $\overline{PA_3} = a_3$, $\overline{PM_2} = h_2$, $\overline{M_2A_3} = d_2$, we get
$a_3{}^2 = h_2{}^2 + d_2{}^2$ -- [5]
Subtracting [5] from [4],
$a_2{}^2 - a_3{}^2 = c_2{}^2 - d_2{}^2$ -- [6]
…
…
…
Similarly referring fig. 4,
In right triangle PM_nA_1, $\overline{PA_n}^2 = \overline{PM_n}^2 + \overline{M_nA_n}^2$
Substituting, $\overline{PA_n} = a_n$, $\overline{PM_n} = h_n$ and $\overline{M_nA_n} = c_n$ we get
$a_n{}^2 = h_n{}^2 + c_n{}^2$ -- [7]
In right triangle PM_nA_1, $\overline{PA_1}^2 = \overline{PM_n}^2 + \overline{M_nA_1}^2$
Substituting, $\overline{PA_1} = a_1$, $\overline{PM_n} = h_n$ and $\overline{M_nA_1} = d_n$ we get
$a_1{}^2 = h_n{}^2 + d_n{}^2$ -- [8]

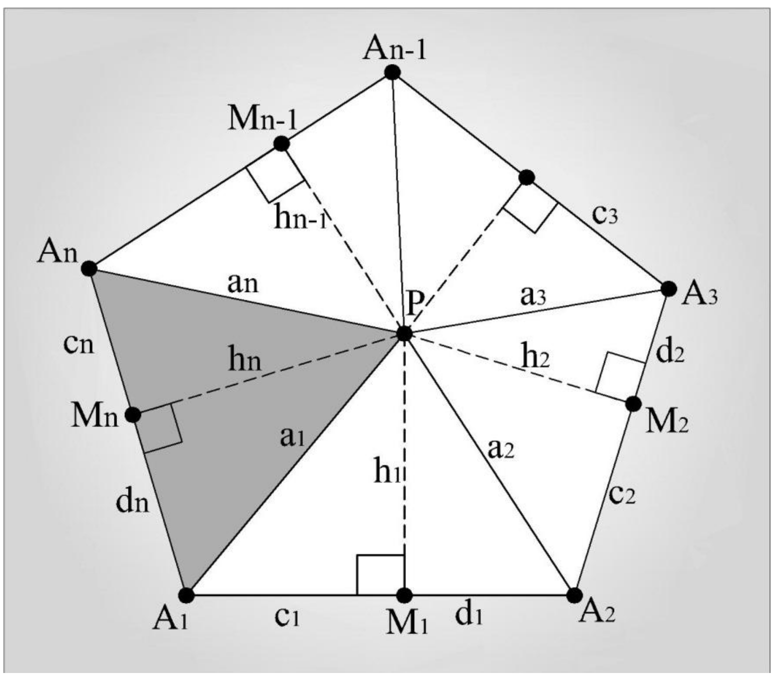

Figure 4: **A diagram of irregular polygon with ΔPA_nA_1**

Subtracting [8] from [7],
$a_n{}^2 - a_1{}^2 = c_n{}^2 - d_n{}^2$ -- [9]
Adding equations [3], [6] … [9], we get

$$(a_1{}^2 - a_2{}^2 + a_2{}^2 - a_3{}^2 \ldots + a_n{}^2 - a_1{}^2) = (c_1{}^2 - d_1{}^2 + c_2{}^2 - d_2{}^2 + \cdots + c_n{}^2 - d_n{}^2)$$

$$0 = c_1{}^2 - d_1{}^2 + c_2{}^2 - d_2{}^2 + \cdots + c_n{}^2 - d_n{}^2$$

Therefore, $c_1{}^2 - d_1{}^2 + c_2{}^2 - d_2{}^2 + c_3{}^2 - d_3{}^2 + \cdots + c_n{}^2 - d_n{}^2 = 0$
Therefore, $c_1{}^2 + c_2{}^2 + c_3{}^2 + \cdots + c_n{}^2 = d_1{}^2 + d_2{}^2 + d_3{}^2 + \cdots + d_n{}^2$
Rewritting, $c_1 = \overline{M_1A_1}$, $c_2, = \overline{M_2A_2}$, $c_3 = \overline{M_3A_3}$, … $c_n = \overline{M_nA_n}$,
$d_1 = \overline{M_1A_2}$, $d_2 = \overline{M_2A_3}$, $d_3 = \overline{M_3A_4}$, … $d_n = \overline{M_nA_1}$

$$\left(\overline{M_1A_1}\right)^2 + \left(\overline{M_2A_2}\right)^2 + \cdots + \left(\overline{M_nA_n}\right)^2 = \left(\overline{M_1A_2}\right)^2 + \left(\overline{M_2A_3}\right)^2 + \cdots + \left(\overline{M_nA_1}\right)^2 \quad ------[10]$$

The equation 10 is the mathematical form of the theorem-1.

Theorem 2:

Suppose P is a point anywhere inside of the irregular polygon, points $A_1,\ A_2,\ A_3,\ \ldots\ A_n$ are the vertices of the polygon and points $M_1,\ M_2,\ M_3,\ \ldots\ M_n$ are the perpendicular projection of the point P on the sides $\overline{A_1A_2}$, $\overline{A_2A_3}$, $\overline{A_3A_4}$ … $\overline{A_nA_1}$ of the polygon. If $\overline{M_1A_1}, \overline{M_2A_2}, \overline{M_3A_3}$, …, $\overline{M_nA_n}$ are left part line segment and $\overline{M_1A_2}, \overline{M_2A_3}, \overline{M_3A_4}$, …, $\overline{M_nA_1}$ are right part line segment of the sides respectively then the sum of the squares of all the sides of the polygon and twice the sum of the squares of all the perpendiculars is equal to twice the sum of the squares of the distance of point P from the vertices and twice the sum of the product of right side line segments and right side line segments (ref. fig.1). The mathematical form of the theorem is

$$\left(\overline{A_1A_2}^2+\overline{A_2A_3}^2+\overline{A_3A_4}^2+\cdots+\overline{A_nA_1}^2\right)+2\left(\overline{PM_1}^2+\overline{PM_2}^2+\overline{PM_3}^2+\cdots+\overline{PM_n}^2\right)$$
$$=2\left(\overline{PA_1}^2+\overline{PA_2}^2+\cdots+\overline{PA_n}^2\right)+2\left[\left(\overline{M_1A_1}\times\overline{M_1A_2}\right)+\left(\overline{M_2A_2}\times\overline{M_2A_3}\right)+\cdots+\left(\overline{M_nA_n}\times\overline{M_nA_1}\right)\right]$$

Derivation of equations for theorem-2

Adding [1] and [2],

$$\mathbf{a_1}^2+\mathbf{a_2}^2=\mathbf{h_1}^2+\mathbf{c_1}^2+\mathbf{h_1}^2+\mathbf{d_1}^2 \qquad ----------------------[\mathbf{11}]$$

Adding [4] and [5],

$$\mathbf{a_2}^2+\mathbf{a_3}^2=\mathbf{h_2}^2+\boldsymbol{c_2}^2+\mathbf{h_2}^2+\boldsymbol{d_2}^2 \qquad ----------------------[\mathbf{12}]$$

…
…
…

Adding [7] and [8],

$$\mathrm{a_n}^2+\mathrm{a_1}^2=\mathrm{h_n}^2+c_n^2+\mathrm{h_n}^2+d_n^2 \qquad ------------------------[13]$$

Adding [11] to [13], we get

$$\Rightarrow \mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2+\mathrm{a_1}^2$$
$$=\mathrm{h_1}^2+\mathrm{c_1}^2+\mathrm{h_1}^2+\mathrm{d_1}^2+\mathrm{h_2}^2+c_2^2+\mathrm{h_2}^2+d_2^2+\cdots+\mathrm{h_n}^2+c_n^2+\mathrm{h_n}^2+d_n^2$$

$$\Rightarrow 2(\mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2)$$
$$=2\left(\mathrm{h_1}^2+\mathrm{h_2}^2+\mathrm{h_3}^2+\cdots+\mathrm{h_n}^2\right)+\left(\mathrm{c_1}^2+\mathrm{d_1}^2+c_2^2+d_2^2+\cdots+c_n^2+d_n^2\right)$$

$$\Rightarrow 2(\mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2)$$
$$=2\left(\mathrm{h_1}^2+\mathrm{h_2}^2+\mathrm{h_3}^2+\cdots+\mathrm{h_n}^2\right)+(\mathrm{c_1}+\mathrm{d_1})^2-2\mathrm{c_1d_1}+(\mathrm{c_2}+\mathrm{d_2})^2-2\mathrm{c_2d_2}$$
$$+(\mathrm{c_3}+\mathrm{d_3})^2-2\mathrm{c_3d_3}+\cdots+(\mathrm{c_n}+\mathrm{d_n})^2-2\mathrm{c_nd_n}$$

Let, $\mathrm{c_1}+\mathrm{d_1}=\mathrm{A_1A_2},\ \mathrm{c_2}+\mathrm{d_2}=\mathrm{A_2A_3},\ \ldots\quad \mathrm{c_n}+\mathrm{d_n}=\mathrm{A_nA_1}$ and substituting in above eqn.

$$\Rightarrow 2(\mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2)$$
$$=2\left(\mathrm{h_1}^2+\mathrm{h_2}^2+\mathrm{h_3}^2+\cdots+\mathrm{h_n}^2\right)+\overline{\mathrm{A_1A_2}}^2-2\mathrm{c_1d_1}+\overline{\mathrm{A_2A_3}}^2-2\mathrm{c_2d_2}$$
$$+\overline{\mathrm{A_3A_4}}^2-2\mathrm{c_3d_3}+\cdots+\overline{\mathrm{A_nA_1}}^2-2\mathrm{c_nd_n}$$

$$\Rightarrow 2(\mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2)$$
$$=2\left(\mathrm{h_1}^2+\mathrm{h_2}^2+\mathrm{h_3}^2+\cdots+\mathrm{h_n}^2\right)+\left(\overline{\mathrm{A_1A_2}}^2+\overline{\mathrm{A_2A_3}}^2+\overline{\mathrm{A_3A_4}}^2+\cdots+\overline{\mathrm{A_nA_1}}^2\right)$$
$$-2(\mathrm{c_1d_1}+\mathrm{c_2d_2}+\mathrm{c_3d_3}+\cdots+\mathrm{c_nd_n})$$

$$\Rightarrow \overline{\mathrm{A_1A_2}}^2+\overline{\mathrm{A_2A_3}}^2+\overline{\mathrm{A_3A_4}}^2+\cdots+\overline{\mathrm{A_nA_1}}^2$$
$$=2(\mathrm{a_1}^2+\mathrm{a_2}^2+\mathrm{a_3}^2+\cdots+\mathrm{a_n}^2)+2(\mathrm{c_1d_1}+\mathrm{c_2d_2}+\mathrm{c_3d_3}+\cdots+\mathrm{c_nd_n})$$
$$-2\left(\mathrm{h_1}^2+\mathrm{h_2}^2+\mathrm{h_3}^2+\cdots+\mathrm{h_n}^2\right)$$

$$\Rightarrow \overline{\mathrm{A_1A_2}}^2+\overline{\mathrm{A_2A_3}}^2+\overline{\mathrm{A_3A_4}}^2+\cdots+\overline{\mathrm{A_nA_1}}^2$$
$$=2\left(\overline{\mathrm{PA_1}}^2+\overline{\mathrm{PA_2}}^2+\cdots+\overline{\mathrm{PA_n}}^2\right)+2\left(\left(\overline{M_1A_1}\times\overline{M_1A_2}\right)+\left(\overline{M_2A_2}\times\overline{M_2A_3}\right)+\cdots+\left(\overline{M_nA_n}\times\overline{M_nA_1}\right)\right)$$

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$$-2\left(\overline{PM_1}^2+\overline{PM_2}^2+\overline{PM_3}^2+\cdots+\overline{PM_n}^2\right)$$

$$\Rightarrow \overline{A_1A_2}^2+\overline{A_2A_3}^2+\overline{A_3A_4}^2+\cdots+\overline{A_nA_1}^2$$
$$=2\left[\overline{PA_1}^2+\overline{PA_2}^2+\cdots+\overline{PA_n}^2+\left(\overline{M_1A_1}\times\overline{M_1A_2}\right)+\left(\overline{M_2A_2}\times\overline{M_2A_3}\right)+\cdots+\left(\overline{M_nA_n}\times\overline{M_nA_1}\right)\right.$$
$$\left.-\left(\overline{PM_1}^2+\overline{PM_2}^2+\overline{PM_3}^2+\cdots+\overline{PM_n}^2\right)\right] \quad -----------------[14]$$

Eqn.[14] is the mathematical form of theorem-2

RESULTS AND DISCUSSION

Example-1

An irregular hexagon is considered for an example (ref. fig. 5).

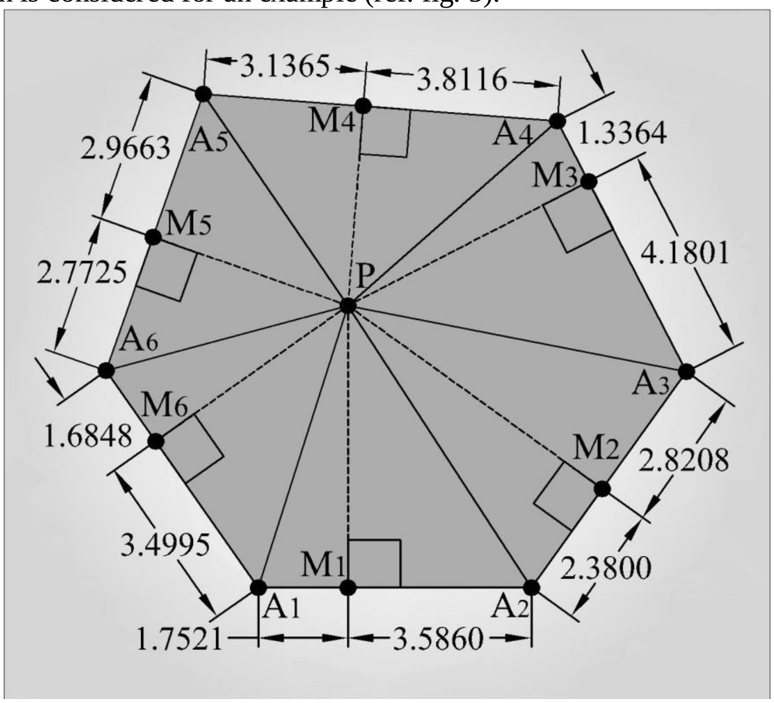

Figure 5: **An AutoCAD drawing of irregular hexagon with dimensions**

Table 1: **Dimesions of an irregular hexagon**

Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side
$\overline{M_1A_1}$	1.7521	$\overline{M_1A_2}$	3.5860	$\overline{A_1A_2}$	5.3381	$\overline{PA_1}$	5.7839	$\overline{PM_1}$	5.5121
$\overline{M_2A_2}$	2.3800	$\overline{M_2A_3}$	2.8208	$\overline{A_2A_3}$	5.2008	$\overline{PA_2}$	6.5759	$\overline{PM_2}$	6.1301
$\overline{M_3A_3}$	4.1801	$\overline{M_3A_4}$	1.3364	$\overline{A_3A_4}$	5.5165	$\overline{PA_3}$	6.7480	$\overline{PM_3}$	5.2974
$\overline{M_4A_4}$	3.8116	$\overline{M_4A_5}$	3.1365	$\overline{M_4A_4}$	6.9481	$\overline{PA_4}$	5.4634	$\overline{PM_4}$	3.9141
$\overline{M_5A_5}$	2.9663	$\overline{M_5A_6}$	2.7725	$\overline{M_5A_5}$	5.7388	$\overline{PA_5}$	5.0158	$\overline{PM_5}$	4.0446
$\overline{M_6A_6}$	1.6848	$\overline{M_6A_1}$	3.4995	$\overline{M_6A_6}$	5.1843	$\overline{PA_6}$	4.9037	$\overline{PM_6}$	4.6033

Theorem-1 (eqn.10) is:

$$\left(\overline{M_1A_1}\right)^2+\left(\overline{M_2A_2}\right)^2+\left(\overline{M_3A_3}\right)^2+\cdots+\left(\overline{M_nA_n}\right)^2=\left(\overline{M_1A_2}\right)^2+\left(\overline{M_2A_3}\right)^2+\left(\overline{M_3A_4}\right)^2+\cdots+\left(\overline{M_nA_1}\right)^2$$

$$\text{LHS}=\left(\overline{M_1A_1}\right)^2+\left(\overline{M_2A_2}\right)^2+\left(\overline{M_3A_3}\right)^2+\cdots+\left(\overline{M_nA_n}\right)^2$$

Substituting the values given in table-1 in the above equation,

$$\therefore \text{LHS}=1.7521^2+2.3800^2+4.1801^2+3.8116^2+2.9663^2+1.6848^2$$

$$\therefore \text{LHS}=52.3733\ \text{sq. units} \quad ------------------------------[15]$$

$$\text{RHS}=\left(\overline{M_1A_1}\right)^2+\left(\overline{M_2A_2}\right)^2+\left(\overline{M_3A_3}\right)^2+\cdots+\left(\overline{M_nA_n}\right)^2$$

Substituting the values given in table-1 in the above equation,

$$\therefore \text{RHS}=3.586^2+2.8208^2+1.3364^2+3.1365^2+2.7725^2+3.4995^2$$

$$\therefore \text{RHS}=52.3732\ \text{sq. units} \quad ------------------------------[16]$$

Comparing eqns. [15] and [16], LHS = RHS

Hence, the theorem-1 is proved.

Theorem-2 (eqn.14) is:

$$\overline{A_1A_2}^2+\overline{A_2A_3}^2+\overline{A_3A_4}^2+\cdots+\overline{A_nA_1}^2+2\left(\overline{PM_1}^2+\overline{PM_2}^2+\overline{PM_3}^2+\cdots+\overline{PM_n}^2\right)$$
$$=2\left(\overline{PA_1}^2+\overline{PA_2}^2+\cdots+\overline{PA_n}^2\right)+2\left[\left(\overline{M_1A_1}\times\overline{M_1A_2}\right)+\left(\overline{M_2A_2}\times\overline{M_2A_3}\right)+\cdots+\left(\overline{M_nA_n}\times\overline{M_nA_1}\right)\right]$$

$$\text{LHS}=\overline{A_1A_2}^2+\overline{A_2A_3}^2+\overline{A_3A_4}^2+\cdots+\overline{A_6A_1}^2+2\left(\overline{PM_1}^2+\overline{PM_2}^2+\overline{PM_3}^2+\cdots+\overline{PM_6}^2\right)$$

Substituting the values given in table-1 in the above equation,

$$\therefore \text{LHS}=5.3381^2+5.2008^2+5.5165^2+6.9481^2+5.7388^2+5.1843^2$$
$$+2(5.5121^2+6.1301^2+5.2974^2+3.9141^2+4.0446^2+4.6033^2)$$

$$\therefore \text{LHS}=194.0623+2(148.8932)=491.8487\ \text{sq. units} \quad ---------------[17]$$

$$\text{RHS}=2\left(\overline{PA_1}^2+\overline{PA_2}^2+\cdots+\overline{PA_6}^2\right)+2\left[\left(\overline{M_1A_1}\times\overline{M_1A_2}\right)+\left(\overline{M_2A_2}\times\overline{M_2A_3}\right)+\cdots+\left(\overline{M_6A_6}\times\overline{M_6A_1}\right)\right]$$

Substituting the values given in table-1 in the above equation,

$$\text{RHS}=2[5.7839^2+6.5759^2+6.748^2+5.4634^2+5.0158^2+4.9037^2]$$
$$+2[6.283+6.7135+5.5863+12.826+8.2241+5.896]$$

$$\therefore \text{RHS}=2(201.2847)+2(45.5504)=493.6702\ \text{sq. units} \quad --------------[18]$$

Comparing eqns. [17] and [18], LHS = RHS

Hence, the theorem-2 is also proved.

Example-2

A scalene triangle is considered for an example (ref. fig. 6).

Table 2: Dimesions of scalene triangle

Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side	Notation of the Side	Length of the side
$\overline{M_1A_1}$	2.0134	$\overline{M_1A_2}$	7.8001	$\overline{A_1A_2}$	9.8135	$\overline{PA_1}$	3.7453	$\overline{PM_1}$	3.1580
$\overline{M_2A_2}$	8.3789	$\overline{M_2A_3}$	4.9378	$\overline{A_2A_3}$	13.3167	$\overline{PA_2}$	8.4152	$\overline{PM_2}$	0.7802
$\overline{M_3A_3}$	4.9370	$\overline{M_3A_1}$	3.6620	$\overline{A_3A_1}$	8.5990	$\overline{PA_3}$	4.9991	$\overline{PM_3}$	0.7853

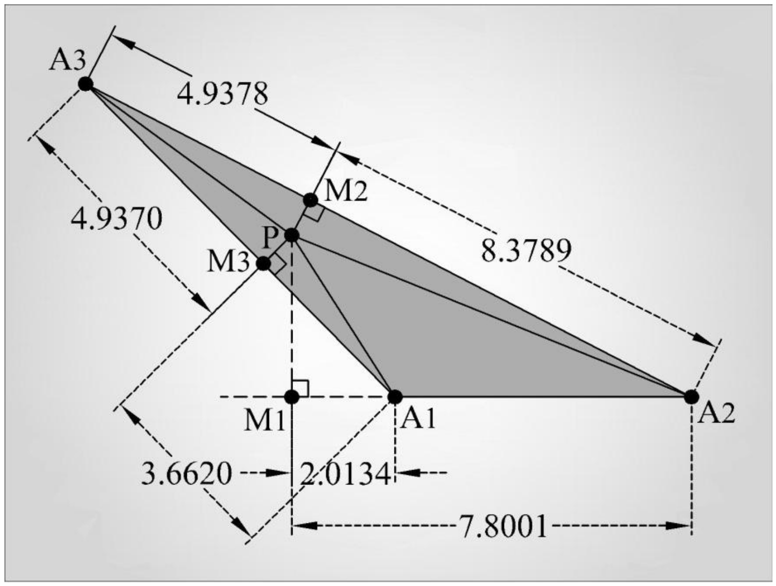

Figure 6: **An AutoCAD drawing of scalene triangle with dimensions**

(i) Theorem-1 (eqn.10) is:

$$\left(\overline{M_1A_1}\right)^2 + \left(\overline{M_2A_2}\right)^2 + \left(\overline{M_3A_3}\right)^2 + \cdots + \left(\overline{M_nA_n}\right)^2 = \left(\overline{M_1A_2}\right)^2 + \left(\overline{M_2A_3}\right)^2 + \left(\overline{M_3A_4}\right)^2 + \cdots + \left(\overline{M_nA_1}\right)^2$$

$$\text{LHS} = \left(\overline{M_1A_1}\right)^2 + \left(\overline{M_2A_2}\right)^2 + \left(\overline{M_3A_3}\right)^2 + \cdots + \left(\overline{M_nA_n}\right)^2$$

Substituting the values given in table-2 in the above equation,

$\therefore$ LHS $= 2.0134^2 + 8.3789^2 + 4.937^2$

$\therefore$ LHS $= 98.6337$ sq. units ——————————————————————————[19]

Substituting the values given in the table-1 are substituted in the above equation,

$$\text{RHS} = \left(\overline{M_1A_2}\right)^2 + \left(\overline{M_2A_3}\right)^2 + \left(\overline{M_3A_4}\right)^2 + \cdots + \left(\overline{M_nA_1}\right)^2$$

Substituting the values given in table-2 in the above equation,

$\therefore$ RHS $= 7.8001^2 + 4.9378^2 + 3.662^2$

$\therefore$ RHS $= 98.6337$ sq. units ——————————————————————————[20]

Comparing eqns. [19] and [20], LHS = RHS

Hence, the theorem-1 is proved.

(ii) Theorem-2 (eqn.14) is:

$$\overline{A_1A_2}^2 + \overline{A_2A_3}^2 + \overline{A_3A_4}^2 + \cdots + \overline{A_nA_1}^2 + 2\left(\overline{PM_1}^2 + \overline{PM_2}^2 + \overline{PM_3}^2 + \cdots + \overline{PM_n}^2\right)$$
$$= 2\left(\overline{PA_1}^2 + \overline{PA_2}^2 + \cdots + \overline{PA_n}^2\right) + 2\left[\left(\overline{M_1A_1} \times \overline{M_1A_2}\right) + \left(\overline{M_2A_2} \times \overline{M_2A_3}\right) + \cdots + \left(\overline{M_nA_n} \times \overline{M_nA_1}\right)\right]$$

$$\text{LHS} = \overline{A_1A_2}^2 + \overline{A_2A_3}^2 + \overline{A_3A_1}^2 + 2\left(\overline{PM_1}^2 + \overline{PM_2}^2 + \overline{PM_3}^2 + \cdots + \overline{PM_n}^2\right)$$

Substituting the values given in table-2 in the above equation,

$\therefore$ LHS $= 9.8135^2 + 13.3167^2 + 8.599^2 + 2(3.158^2 + 0.7802^2 + 0.7853^2)$

$\therefore$ LHS $= 347.5821 + 2(11.1984)$

$\therefore$ LHS $= 369.9789$ sq. units ————————————————————————— [21]

$$\text{RHS} = 2\left(\overline{PA_1}^2 + \overline{PA_2}^2 + \overline{PA_3}^2\right) + 2\left[\left(\overline{M_1A_1} \times \overline{M_1A_2}\right) + \left(\overline{M_2A_2} \times \overline{M_2A_3}\right) + \left(\overline{M_3A_3} \times \overline{M_3A_1}\right)\right]$$

Substituting the values given in table-2 in the above equation,

RHS $= 2[3.7453^2 + 8.4152^2 + 4.9991^2] + 2[15.7047 + 41.3733 + 18.0793]$

$\therefore$ RHS $= 2(109.8339) + 2(75.1573)$

$\therefore$ RHS $= 369.9824$ sq. units ————————————————————————[22]

Comparing eqns. [21] and [22], LHS = RHS
Hence, the theorem-2 is also proved.

Conclusion

In this article, a new theorem on irregular cyclic polygon with appropriate illustration where it is necessary, including necessary equations derived for new properties including step by step derivations of equations and illustrations where ever necessary. The property discussed in this article has been proved with two appropriate examples. The theorems, which have been defined in this article, may be useful for whose work is related to geometry, research or further study in the irregular Polygon.

REFERENCES

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