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Research Article

DETERMINATION OF OPTIMAL ORDER QUANTITY OF INTEGRATED AN INVENTORY MODEL USING YAGER RANKING METHOD

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ABSTRACT
The integrated vendor – buyer production inventory models are gaining much significance in recent times. Several methods such as classical differential calculus, cost difference rate comparison approach are employed to determine the optimal lot size with the assumption that the demand and the costs involved are deterministic in nature, which is not possible in this world of uncertainity. This leads to the transformation of deterministic parameters to fuzzy parameters. In this paper the demand and the associated costs are taken as fuzzy variables. To determine the optimal inventory policies the Yager's ranking methods for fuzzy numbers is utilized. A set of numerical data is employed to analyse the characteristics of the proposed model.

Key Words: *Inventory, Integrated Vendor Buyer, Fuzzy Number, Yager Ranking Method*

INTRODUCTION

In the past decades the inventory models were devised for vendor and the buyer separately, but during the last few years the concept of integrated vendor buyer inventory model has earned favour of the top management of the business concern as it minimize the total costs. As co-operation, the main backbone of the successful functioning of the production sector, the phenomena of integrated vendor-buyer inventory model has great welcome among the enterprise manager.The researchers who have wide range of interest in integrated models to determine the optimal order quantity. A brief literature review of their contributions is given by Chun-Jen Chung. To mention a few Yang *et al.,* (2007); Minner (2007) and Teng (2009). To these integrated models the concept of backordering is merged to have better cost control of the inventory system. One common point that has to be noted in all the earlier models is that the nature of demand and the cost are deterministic which is quite impossible at all times. The reasons for the cause of such situations are inflation, sudden rise and fall in the economy of the mighty nations and so on. The effects are, the fluctuations of demand and costs which pave way for fuzzy parameters.

In accordance to it integrated inventory problems are addressed under fuzzy environment to determine the optimal order quantity. For instance Park (1987) discusses the EOQ model with fuzzy cost coefficients. Ishii and Kunno (1998), Petrovic *et al.,* (1996) and Kao and Hsu (2002) investigate the newsboy inventory model with fuzzy cost coefficients and demands respectively. Roy and Maiti (1997), Chang (2009) construct a fuzzy EOQ model with fuzzy defective rate and fuzzy demand. Most of the paper directly supposes the model parameters as a triangular fuzzy number and then finds their optimal solutions. Yao and Chiang (2003); Lee and Yao (1998) and Chang and Yao (1998) develop the EOQ model with fuzzy ordering quantities. In this paper a fuzzy integrated vendor buyer production model is discussed.

The structure of this paper is organized as follows. In section 2, the preliminaries are given. In Section 3, the total inventory cost of the integrated problem under deterministic nature is discussed. In Section 4, fuzzy optimal ordering quantity is determined using Yager's ranking (1981) method. Finally the characteristics of proposed models will be illustrated and some conclusions will be made.

PRELIMINARIES

Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$. In the pair $\{(x, \mu_{\tilde{A}}(x))\}$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

α - Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree □ is called the α level set or α - cut. $A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha$

Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,

(iii) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,

(iv) $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,

(v) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(vi) $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

where a_1, a_2, a_3 and a_4 are real numbers.

Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \leq a_2 \leq a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by (Q, r) Inventory Model with Fuzzy Lead Time

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_1 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{Otherwise} \end{cases}$$

Yagers' Ranking Method

If the α cut of any fuzzy number $\tilde{A}$ is $[A_L(\alpha), A_g(\alpha)]$then its ranking index $I(\tilde{A})$ is $\frac{1}{2}\int_0^1 \left[A_L(\alpha) + A_g(\alpha)\right] d\alpha$.

THE DETERMINISTIC INTEGRATED VENDOR BUYER INVENTORY MODEL

The assumptions and the notations of the model are as follows.

Assumptions

The algebraic model for the integrated two-stage vendor–buyer inventory model is developed on the basis of the following assumptions.

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(a) Both the production and demand rates are constant and the production rate is greater than the demand rate.
(b) The integrated system of single-vendor and single-buyer is considered.
(c) The vendor and the buyer have complete knowledge of each other's information.
(d) The buyer's shortage is allowed.

Notations

Q = Buyer's lot size per delivery.
n = Number of deliveries from the vendor to the buyer per vendor's replenishment interval.
nQ = Vendor's lot size per delivery.
S = Vendor's setup cost per setup.
A = Buyer's ordering cost per order.
Cv = Vendor's unit production cost.
Cb = Unit purchase cost paid by the buyer.
πb = Unit backordering cost of the buyer.
r = Annual inventory carrying cost per dollar invested in stocks.
P = Production rate per year, where P > D.
D = Demand rate per year.
K = The backordering ratio.
(1 − K) = The non-backordering ratio.
TC(Q) = The average integrated total cost of the vendor–buyer inventory model considering backordering.
The total integrated cost is
TC (Q) = {the vendor's setup cost + carrying cost} + {the buyer's ordering cost + carrying cost}+{the buyer's backordering cost}

The total integrated cost is
TC(Q) = {the vendor's setup cost + carrying cost} + {the buyer's ordering cost + carrying cost} + {the buyer's backordering cost}

$$= \left\{ \frac{DS}{nQ} + \frac{rQC_v}{2}\left[(n-1)\left(1 - \frac{D}{P}\right) + \frac{P}{P} \right] \right\} + \left\{ \frac{DA}{Q} + \frac{r(1-F)^2 QC_B}{2} \right\} + \left\{ \frac{F^2 Q\pi_b}{2} \right\}$$

$$\frac{\partial\, TC(Q)}{\partial\, Q} = \frac{-DS}{nQ^2} + \frac{rC_v}{2}\left[(n-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right] - \frac{DA}{Q^2} + \frac{r(1-F)^2 QC_B}{2} + \frac{F^2 Q\pi_b}{2}$$

$$\frac{\partial\, TC(Q)}{\partial\, Q} = 0$$

$$Q = \sqrt{\frac{2D(S+nA)}{n\left[rCr\left\{ (n-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right\} + r(1-F)^2 C_B + F^2 \pi_b \right]}}$$

(or) TC(Q) can be written as

$$TC(Q) = \frac{DS}{nQ} + \frac{(n-1)rQC_v}{2} + \frac{D}{P}\frac{(2-n)rQC_v}{2}$$

$$+\ \frac{DA}{Q} + \frac{r(1 - F)^2QC_b}{2} + \frac{F^2Q\pi_b}{2}$$

$$\frac{\partial\ TC(Q)}{\partial\ Q} = \frac{1}{Q^2}\left[\frac{DS}{nQ} + DA\right] + \frac{1}{2P}\left[D(2 - n)rC_v + P[(n-1)rC_v + r(1 - F)^2C_b + F^2\pi_b\right]$$

$$Q = \sqrt{\frac{2PD(S + nA)}{\left[D(2 - n)rC_v + P((n - 1)rC_v + r(1 - F)^2C_b + F^2\pi_b\right]}}$$

FUZZY INTEGRATED VENDOR-BUYER INVENTORY MODEL

Let $\tilde{D},\ \tilde{S},\ \tilde{C}_v,\ \tilde{C}_b,\ \tilde{\pi}_v,\ \tilde{A}$ be fuzzy trapezoidal numbers and they are defined as follows. [ie. they are described by the α-cuts]

$D(\alpha_D) = \left[L_D^{-1}(\alpha_D),\ R_D^{-1}(\alpha_D)\right]$

$S(\alpha_S) = \left[L_S^{-1}(\alpha_S),\ R_S^{-1}(\alpha_S)\right]$

$C_v(\alpha C_v) = \left[LC_v^{-1}(\alpha C_v),\ R_{C_v}^{-1}(\alpha C_v)\right]$

$C_b(\alpha C_b) = \left[L\pi_b^{-1}(\alpha\pi_b),\ R_{\pi_b}^{-1}(\alpha\pi_b)\right]$

$A(\alpha_A) = \left[L_A^{-1}(\alpha_A),\ R_A^{-1}(\alpha_A)\right]$

$r(\alpha_r) = \left[L_r^{-1}(\alpha_r),\ R_r^{-1}(\alpha_r)\right]$

TC(Q) can be rewritten as

$$TC(Q) = \frac{DS}{nQ} + (n - 1)\left[\frac{rQC_v}{2}\right] + \frac{D}{P}(2 - n)\frac{rQC_v}{2}$$

$$+\ \frac{DA}{Q} + \frac{r(1 - F)^2QC_b}{2} + \frac{F^2Q\pi_b}{2}$$

$$K_1(\alpha_D,\ \alpha_D) = \frac{1}{4}\left\{\int_0^1 L_D^{-1}(\alpha_D)dL_D\ .\ \int_0^1 L_S^{-1}(\alpha_S)d\alpha_S + \int_0^1 L_S^{-1}(\alpha_S)d\alpha_S\ .\ \int_0^1 R_S^{-1}(\alpha_S)d\alpha_S\right\}$$

$$K_2(\alpha_v,\ \alpha_v) = \frac{1}{4}\int_0^1 LC_v^{-1}(\alpha C_v) + RC_v^{-1}(\alpha C_v)d\alpha_r\ .\ \int_0^1 L_r^{-1}(\alpha_r)d\alpha_r\ .\ \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r$$

$$K_3(\alpha_\infty,\ \alpha C_v) = \frac{1}{4}\left\{\int_0^1 L_D^{-1}(\alpha_D)dL_D . \int_0^1 LC_r^{-1}(\alpha C_r)d\alpha_r + \int_0^1 R_D^{-1}(\alpha_D)d\alpha_D . \int_0^1 RC_v^{-1}(\alpha_v)d\alpha C_v\right\}$$

$$K_4(\alpha_\infty,\ \alpha_A) = \frac{1}{4}\left\{\int_0^1 L_D^{-1}(\alpha_D)dL_D . \int_0^1 L_A^{-1}(\alpha_A)d\alpha_A + \int_0^1 R_D^{-1}(\alpha_D)d\alpha_D . \int_0^1 R_A^{-1}(\alpha_A)d\alpha C_A\right\}$$

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$$K_5(\alpha_r, \alpha C_b) = \frac{1}{4}\int_0^1 \left[LC_b^{-1}(\alpha C_b) + RC_b^{-1}(\alpha C_b) \right] d\alpha C_b \;.\; \int_0^1 L_r^{-1}(\alpha_r) d\alpha_r \;.\; \int_0^1 R_r^{-1}(\alpha_r) d\alpha_r$$

$$K_6(\alpha\pi_b) = \frac{1}{2}\int_0^1 \left[L\pi_b^{-1}(\alpha\pi_b) + R\pi_b^{-1}(\alpha\pi_b) \right] d\alpha_\pi$$

$$TC(Q) = \frac{K_1(\alpha_D, \alpha_S)}{nQ} + \frac{(n-1)rC_rK_2(\alpha C_r)}{2} + \frac{rQ(2-n)K_3(\alpha_D, \alpha C_v)}{2P} + \frac{K_4(\alpha_D, \alpha_A)}{Cr} + \frac{r(1-F)^2QK_5(\alpha C_b)}{2} + \frac{F^2K_6(\alpha\pi b)C_r}{2}$$

To determine the optimal order quantity

$$\frac{\partial\, TC(Q)}{\partial\, Q} = \frac{-\left[K_1(\alpha_D, \alpha_S) + K_4(\alpha_D, \alpha_A)\right]}{Q^2} + \frac{(n-1)rK_2(\alpha C_v)}{2} + \frac{r(2-n)K_3(\alpha_D, \alpha C_v)}{2P} + \frac{r(1-F)^2K_5(\alpha C_b)}{2} + \frac{K_6(\alpha\pi b)F^2}{2}$$

$$\frac{\partial\, TC(Q)}{\partial\, Q} = 0$$

$$\Rightarrow Q = \sqrt{\frac{2P\left[K_1(\alpha_D, \alpha_S) + K_4(\alpha_D, \alpha_A)\right]}{P((n-1)K_2(\alpha(r) + r(2-n)K_3(\alpha_D, \alpha(v)))) + P\left[r(1-F)^2K_5(\alpha C_h) + K_5(\alpha\pi_b)F^2\right]}}$$

NUMERICAL EXAMPLE

To validate the proposed model consider the data

D	=	(560, 580, 620, 640)	r	=	(8, 9, 11, 12)
S	=	(80, 90, 110, 120)	n	=	.1
C_v	=	(3, 4, 6, 7)	F	=	10%
C_b	=	(2, 3, 5, 6)			
π_b	=	(2, 3, 5, 6)			
A	=	(90, 100, 120, 130)			
P	=	700			

$D(\alpha_D)$	=	$(560 + 20\alpha, 640 - 20\alpha)$	
$S(\alpha_S)$	=	$(80 + 10\alpha, 120 - 10\alpha)$	
$C_v(\alpha_{C_v})$	=	$(3 + \alpha, 7 - \alpha)$	
$C_b(\alpha_{C_b})$	=	$(2 + \alpha, 6 - \alpha)$	
$\pi_b(\alpha_{\pi_b})$	=	$(2 + \alpha, 6 - \alpha)$	
$A(\alpha_A)$	=	$(90 + 10\alpha, 130 - 10\alpha)$	
$P(\alpha_r)$	=	$(8 + \alpha, 12 - \alpha)$	P = 700

$K_1(\alpha_\infty, \alpha_s)$

$$= \frac{1}{4}\left\{\int_0^1 560+20\alpha\, d\alpha \;.\; \int_0^1 80+10\alpha\, d\alpha + \int_0^1 640\text{-}20\alpha\, d\alpha \;.\; \int_0^1 120\text{-}10\alpha\, d\alpha\right\}$$

$$= \frac{1}{4}\left\{\left[560\alpha+\frac{20\alpha^2}{2}\right]_0^1 . \left[80\alpha+\frac{10\alpha^2}{2}\right]_0^1 + \left[640\alpha-\frac{20\alpha^2}{2}\right]_0^1 . \left[120\alpha-\frac{10\alpha^2}{2}\right]_0^1\right\}$$

$$= \frac{1}{4}\{(560+10)\cdot(80+5)+(640-10)\cdot(120-5)\}$$

$$= \frac{1}{4}\{(570.85)+(630)(115)\}$$

$$= \frac{1}{4}[48450+72450]$$

$K_1(\alpha_\infty, \alpha_s) \quad = 30225$

$K_2(\alpha_r, \alpha_{C_v})$

$$= \frac{1}{4}\left\{\int_0^1 (8+\alpha)\, d\alpha \;.\; \int_0^1 (3+\alpha)\, d\alpha + \int_0^1 (12\text{-}\alpha)\, d\alpha \;.\; \int_0^1 (7\text{-}\alpha)\, d\alpha\right\}$$

$$= \frac{1}{4}\left\{\left(8+\frac{1}{2}\right).\left(3+\frac{1}{2}\right)+\left(8-\frac{1}{2}\right)\left(7-\frac{1}{2}\right)\right\}$$

$$= \frac{1}{4}\{(8.5)\cdot(3.5)+(11.5)\cdot(6.5)\}$$

$$= \frac{1}{4}[29.75+74.75]$$

$= 26.125$

$K_3(\alpha_\infty, \alpha_r, \alpha_{C_v})$

$$= \frac{1}{8}\left[\left(560+\frac{20}{2}\right).\left(3+\frac{1}{2}\right)+\left(640-\frac{20}{2}\right)\left(8+\frac{1}{2}\right)\left(7-\frac{1}{2}\right)\right]$$

$$= \frac{1}{8}[(570)(3.5) + (630)(6.5)]$$

$= 8006.25$

$K_4(\alpha_\infty, \alpha_A)$

$$= \frac{1}{4}\left[(560+10)(90+5) + (640-10)(13-5)\right]$$

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$$= \frac{1}{4}\left[(570)(95) + (630)(125)\right]$$

$= 33225$

$K_5(\alpha_r, \alpha_{C_5})$

$$= \frac{1}{4}\left[(8 + 0.5)(2 + 0.5) + (12 - 0.5)(6 - 0.5)\right]$$

$$= \frac{1}{4}\left[(8.5)(2.5) + (11.5)(5.5)\right]$$

$= 21.125$

$K_6(\alpha_{\pi b})$

$$= \frac{1}{2}\left[(2 + 0.5) + (6 - 0.5)\right]$$

$$= \frac{2.5 + 5.5}{2} = 4$$

$$Q = \sqrt{\frac{2 \times 700[30225 + 33225]}{8006.25 + 700\,[(0.81)(21.125) + 0.01 \times 4]}}$$

$Q = 66.18$

CONCLUSION

The purpose of this paper is to study the integrated models under fuzzy environment. This fuzzy model assists in determining the optimal order quantity amidst the existing fluctuations. This model benefits the enterprise manager in decision making. In this paper the yager ranking method is employed as this does not require the explicit form of the membership function.

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