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Research Article

CALCULATION OF FUSION REACTION CROSS-SECTION AND ANGULAR MOMENTUM WINDOW OF ^{6}LI, ^{16}O, ^{56}FE AND ^{86}KR ON FUSION REACTION WITH ^{208}PB AT E_{LAB}=500MEV

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ABSTRACT
Fusion reaction cross-section and angular momentum values help in identifying the possibility of occurrence of a fusion reaction. Fusion cross-sections of heavy ion reactions have been calculated using the semi-classical approach with heavy ions as projectiles. In this model of calculation of fusion reaction cross section, three potentials have been used namely: Coulomb potential, nuclear potential and centrifugal potential. Fusion reactions between the pairs of heavy ions have been studied and their cross-section calculated in semi classical formulation using one-dimensional barrier penetration model, taking scattering potential as the sum of Coulomb, centrifugal and proximity potential. Ion-ion interaction potentials have been calculated and various quantities of interest obtained from their potential curves. The quantities of interest are then used in the calculation of fusion reaction cross-section and angular momentum window. The calculated theoretical values of interest have been found to agree with the experimental values with a small variation of less than ten percent. The calculated V_{1B} values of ^{6}Li+^{208}Pb, ^{16}O+^{208}Pb and ^{56}Fe+^{208}Pb reactions were found to be 32.92MeV, 80.49MeV and 234.99MeV respectively, while the experimental values are 30.10MeV, 74.90MeV and 233.0MeV respectively. The calculated fusion cross-sections are found to be 292.88mb for ^{6}Li+^{208}Pb, 314.00mb for ^{16}O+^{208}Pb and 182.60mb for ^{56}Fe+^{208}Pb reactions. It has also been found out from the results that heavy ions can undergo fusion reaction even though there is an enormous Coulomb repulsive force associated with the heavy ions.

INTRODUCTION
Due to the short deBroglie wavelength of heavy-ions compared to the size of the ions, classical approximations for low energy collisions are expected to be good at least for the macroscopic features of heavy-ion reactions such as fusion and deep inelastic collision. Therefore, classical macroscopic approaches have been widely used in which one chooses the relevant collective degrees of freedom and then invokes suitable mechanisms for transfer of energy from the collective degrees to the frozen internal degrees of freedom (Godre *et al.,* 1989; Bucham, 1988 and Zetili, 2007). The knowledge of a variety of accelerators in the last few decades has made it possible to accelerate not only protons, deuterons and alpha-particles, but also heavy ions like carbon, nitrogen and oxygen among others. During the last forty years very heavy-ion beams like those of krypton, xenon and uranium have been used as projectiles to study a variety of nuclear reactions between complex heavy ions. A new field of heavy ions physics has, therefore, opened up with a number of promising applications (Flerov and Barashenkov, 1975). The study of fusion of complex heavy nuclei and the structure of nuclei at high excitation energy and angular momentum has become the centre of attraction for theoretical as well as experimental nuclear physicists worldwide in recent years. The most interesting aspect of heavy ion physics lies in the fact that the classical and semi classical theories are capable of explaining many features of heavy ion elastic, inelastic scattering fusion and other reactions.

It has been shown (Burcham, 1988 and Marmier and Sheldon, 1970) that under appropriate conditions, heavy-ion reactions may clearly show a classical character. The criterion for heavy-ion reactions which qualify to be treated classically can be expressed as

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} > \text{unity} \qquad (1)$$

A condition satisfied for most of the cases in heavy-ion reactions (Burcham 1988). Here η is known as Sommerfeld parameter and is the ratio of distance of closest approach "d_{min}" and deBroglie wavelength $\frac{\lambda}{2}$ associated with the projectile. The total ion-ion interaction potential, $V_{total}(r)$ for the heavy-ion pairs is written as; (Dutt and Puri, 2010 and Santhoshi *et al.*, 2008).

$$V_{total}(r) = V_{Coulomb}(r) + V_{nuclear}(r) + V_{centrifugd}(r)$$

Or

$$V = \frac{Z_1 Z_2 e^2}{r} + V_N(z) + \frac{\hbar^2 l(l+1)}{2\mu r^2} \qquad (2)$$

Where Z_1 and Z_2 are the atomic numbers of projectile and target respectively, r is the distance between the centres of the projectile and target, z is the distance between the near surfaces of the projectile and target, l is the angular momentum, μ is the reduced mass of the target and projectile and $V_N(z)$ is the interaction potential between two surfaces of two colliding nuclei and is also called the proximity potential (Santhosh *et al.*, 2008) given as:

$$V_N(z) = 4\pi\gamma b \frac{C_1 C_2}{C_1 + C_2} \phi\left(\frac{z}{b}\right) \qquad (3)$$

With the nuclear surface tension coefficient (Santhoshi *et al.*, 2008)

$$\gamma = 0.9517[1 - 1.7826(N-Z)^2 / A^2] \qquad (4)$$

ϕ, the universal proximity potential is given as

$$\phi(\xi) = -4.41\exp\left(\frac{-\xi}{0.7176}\right), \quad \text{for } \xi \geq 1.9475, \qquad (5)$$

$$\phi(\xi) = 1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3, \quad \text{for } 0 \leq \xi \leq 1.9475 \qquad (6)$$

With $\xi = \frac{r}{b}$, where the width b (diffuseness) of nuclear surface, has been evaluated to be close to unity $(b = 1)$ and Siissmann central radii C_i is related to sharp radii as (Santhosh *et al.*, 2008):

$$C_i = R_i - \frac{b^2}{R_i} \qquad (7)$$

For R_i, the semi empirical mass formular in terms of mass number A_i is used as (Santhoshi *et al.*, 2008)

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \qquad (8)$$

During the last three decades several attempts have been made to improve the proximity potential. In these works an improved version of nuclear surface tension co-efficient is presented as (Dutt and Puri, 2010 and Santhoshi *et al.*, 2008);

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$$\gamma = 1.2496[1 - 2.3(N - Z)^2 / A^2].$$

The choice of the potential and its form to be adopted is one of the most challenging aspects, when one wants to compare the experimental fusion data with theory, both below and above the barrier V_{1B}. Other forms of proximity potentials which can be used in calculating the nuclear potential between ions are available in reference (Dutt and Puri, 2010).
The ion-ion potentials are then used in plotting ion-ion potential curves (Interaction Potential against the distance between the centers of the projectile and the target). From these interaction potential curves, various quantities of interest like radius of interaction R_{IB}, interaction barrier V_{IB}, the critical distance R_{cr} (distance below which fusion occurs) and critical potential V_{cr} have been obtained which are then used in the calculation of fusion reaction cross-section. The limits of angular momentum values l_{cr} defined as angular momentum below which no fusion takes place and $l_{\max}$ which is the maximum angular momentum projectile needed for fusion reaction have also been calculated. The compound nucleus formation is only possible when the angular momentum corresponding to relative motion of the two ions lies between l_{cr} and $l_{\max}$. Since l_{cr} is closely connected to the impact parameters, which is in turn, connected with the energy of the projectile, one can control the value of l by allowing the incident beam of precise energy values.

HEAVY –ION FUSION REACTION

When a heavy ion undergoes fusion, a large amount of material, energy and momentum is added to the target nucleus. This can lead to formation of a new nuclei and nuclear matter under extreme conditions of temperature and stress. Different combinations of colliding particles and energies produce different results. For many years, heavy ion fusion has been used to create nuclei much heavier than any that occur naturally on the earth. J. S. Lilley in his book Nuclear Physics, Principles and Applications, page 120, writes of a discovery of an element $_{112}X^{277}$ by a fusion reaction. The nucleus was synthesized in February 1966 during an experiment in which ^{208}Pb was bombarded with a beam of ^{70}Zn ions in the fusion reaction (Lilley, 2008). The tendency for these very heavy nuclei to undergo fission increases with mass and, if the compound nucleus is to decay by nucleon evaporation and not by fission, it is important to produce it with minimum amount of excitation energy. This is best achieved using target and projectile nuclei that are tightly bound (Wong, 1973). In general, the fusion of two heavy ions produces a nucleus, which is not only much heavier than either the original target or projectile, but is also much more proton rich. The high amount of an incoming heavy ion means that the fused system can be treated with very high angular momentum. For example, in the reaction: $^{40}Ca + ^{90}Z_r = ^{130}Nd$, at a centre of mass bombarding energy about 50% above the Coulomb barrier of 100MeV, a compound nucleus may be formed with angular momentum up to about $60\hbar$ (Lilley, 2008).
To develop a reliable theoretical model for describing fusion reaction, interpretation of directly measured fusion excitation functions require an interaction that is a function of the distance between the centre-of –mass of the target and projectile and consists of a repulsive Coulomb and, of course, a short ranged attractive nuclear component. The total potential attains a maximum value at a distance where the repulsive and attractive forces balance each other, referred to as Coulomb barrier and the energy of relative motion must overcome this barrier in order for the nuclei to be captured and fused (Santhosh *et al.,* 2008).
In the fusion processes and more specifically, in the fusion of weakly bound nuclei, two different and independent processes can be distinguished both experimentally and theoretically. One denoted as complete fusion (CF) is associated with the capture of all of the projectile constituents by the target. The other denoted as incomplete fusion (ICF) or partial fusion occurs when part of the projectile is captured

by the target and the remaining part escapes. Total fusion (TF) is understood as the sum of these two processes (CF+ICF) (Santhoshi *et al*., 2008).
it is well known that the simple one-dimentional barrier penetration model explains the fusion reactions of heavy ions when the energy of the projectile is above the barrier, whereas when the incident energy is not so large and the system is not so light, the reaction process can be predominantly governed by quantum tunneling over the Coulomb barrier created by the strong cancellation between the repulsive Coulomb interaction and the interactive nuclear interaction. At energies near and below Coulomb barrier, the different reactions mechanisms have strong couplings and the intrinsic degrees of freedom (such as rotation, vibration etc.) are taken into account, whose coupling with the relative motion effectively causes a splitting in energy of the single uncoupled fusion barrier.

The Fusion Cross Section

To describe the fusion reactions at energies not too much above the barrier and at higher energies, the barrier penetration model (Santhosh *et al.,* 2008 and Wong, 1973) developed by Wong has been widely used which explains the experimental results properly. Wong gave the total cross-section for the fusion of two nuclei by quantum mechanical penetration of simple one-dimensional potential barrier as

$$\sigma = \frac{\pi}{k^2}\sum_l \frac{2l+1}{1+\exp[2\pi(E_l - E)/\hbar\omega_i]} \quad \text{...(9)}$$

Where $k = \sqrt{\frac{2\mu E}{\hbar^2}}$ (propagation vector)

Here $\hbar\omega_l$ is the curvature of the inverted parabola. Using some parameterization in the region $l = 0$ and replacing the sum in Eq. (9) by an integral, Wong gave the reaction cross section as

$$\sigma = \frac{R_B^2\hbar\omega_0}{2E}\ln\left\{1-\exp\left[\frac{2\pi(E-E_B)}{\hbar\omega_0}\right]\right\} \text{...(10)}$$

For relatively larger values of E, the above equation reduces to the well known formula

$$\sigma = \pi R_B^2\left[1-\frac{E_B}{E}\right] \text{...(11)}$$

This shows that quantum mechanical equation for fusion reaction cross section reduces to classical estimates for $E >> E_B$ where E_B is the barrier potential of the two interacting nuclei. For relatively small values of E, such that $E << E_B$,

$$\sigma = \frac{R_B^2\hbar\omega_0}{2E}\exp[2\pi(E-E_B)/\hbar\omega_0] \text{...(12)}$$

Lefort and his collaborators have shown that a critical distance of approach may be the relevant quantity limiting complete fusion during a collision between two complex nuclei (Burcham, 1988 and Santhosh *et al.,* 2008). In order to substantiate the finding of a critical distance of approach, it is necessary to check the linear dependence of σ on $1/E$ in the region of high energy. The value of critical distance was found to be:

$$R_C = r_C\left(A_1^{1/3} + A_2^{1/3}\right) \text{...(13)}$$

where $r_C = 1.0 \pm 0.07$ fm, A_1 is projectile mass number and A_2 is the target mass number.

Validity of Classical Approach in Describing Heavy Ion Collisions

The heavy-ion reactions to a good approximation have been explained on the basis of classical theories already available as given in deriving $\sigma = \pi R_B^2\left[1-\frac{E_B}{E}\right]$. The quantum mechanical modifications over

these classical theories have taken care of these finer microscopic points for which classical theories are considered to be adequate. This semi-classical or semi-quantal approach, as is popularly known, has become a very powerful tool to explain majority of results in heavy-ion reactions (Malfiet, 1974). It is an important point to note that even the most sophisticated quantum mechanical calculations have yielded results very similar to those obtained through classical or semi-classical approach.
In the following expressions, a criterion for the heavy-ion reactions which qualify to be treated classically is set up. For classical treatment of heavy-ion reaction it is desired "that the extension of the wave packet associated with the heavy-ion should be small in comparison with some appropriate length, such as the distance of closest approach in a head-on encounter with a target nucleus". This distance is (Malfiet, 1974);

$$d_{\min} = \frac{Z_1 Z_2 e^2}{E} \qquad (14)$$

This expression (14) can be derived from the fact that Coulomb potential energy E between two ions of charges Z_1 and Z_2 separated by distance r is given by,

$$E = \frac{Z_1 Z_2 e^2}{r} \qquad (15)$$

From equation (15) we get;

$$r = \frac{Z_1 Z_2 e^2}{E} \qquad (16)$$

For the minimum distance $d_{\min}$ between ions, equation (16) gives, $d_{\min} = \frac{Z_1 Z_2 e^2}{E}$ where $Z_1 e$ and $Z_2 e$ are the ion-target charges and E is the ion's kinetic energy (C-M system). The wave packet extension can be expressed through the reduced deBroglie wave-length,

$$\lambda = \frac{\hbar}{\mu v} \qquad (17)$$

Where μ is the reduced mass of the ion-target system and v is ion's velocity corresponding to its energy E (Ford and Wheeler, 1959).
When half the ratio of $d_{\min}$ to reduced deBroglie wavelength, designated as Sommerfeld parameter η, exceeds unity, the collision can be considered to be classical in nature and the particle orbits can be taken to be purely Rutherford orbits. In other words trajectory picture of scattering theory becomes valid to describe such heavy-ion reactions. The criterion therefore is (Burcham, 1988).

$$\eta = \frac{1}{2}\frac{d_{\min}}{\lambda} = \frac{1}{2}\left(\frac{Z_1 Z_2 e^2}{E}\right)\left(\frac{\mu v}{\hbar}\right) \qquad (18)$$

Or $\eta = \frac{Z_1 Z_2 e^2}{\hbar v} > 1$

In practice for the most of the heavy-ion reactions, η is considerably higher than unity. For instance, in a central collision between an 80MeV ^{16}O ions and a stationary ^{16}O target nucleus, $\eta = 4.5$. η for the reactions picked have been calculated and all the values of η are considerably higher than unity as tabulated in table 2. This means that the classical approach used is valid for the study of picked heavy ions (Frahn and Venter, 1963).

Theory of Compound Nucleus Formation

The heavy-ion reaction to a good approximation have been explained on the basis of classical theories available in the literature to explain certain phenomenon in mechanics and optics (Frahn and Venter, 1963 and Malfiet, 1974). The quantum mechanical modifications over these classical theories have taken care of these finer microscopic points for which classical theories are considered inadequate. The semi-classical or semi-quantal approach, as it is popularly known, has become a very powerful tool to explain majority of results in heavy-ion reactions (Gross, 1973 and Mac Donald, 1962). It is important to note that most sophisticated quantum mechanical calculations have yielded results most similar to those obtained through semi-classical approach (eqs. 9-11).

There can be many reaction channels when two heavy ions approach each other depending on their initial energies of motion. The main interest here is in the compound nucleus formation aspect which is responsible for the nuclear fusion reaction in the system considered in this work. Although it might seem improbable that at high collision energies, typical of heavy ion interactions, even a head on collision could lead to compound nucleus formation. There is evidence (McGee, 1966 and Jensen *et al.*, 1979) to suggest that compound nucleus system can indeed be formed in a highly excited state ($E^* = 40$ MeV). In nuclear fusion, the initial system is strictly defined as two complex nuclei in their ground states, accelerated to a precise kinetic energy above the Interaction barrier V_{1B} (which is essentially the coulomb repulsive barrier). The result is a population of more or less deformed systems consisting of an assembly of all the impinging nucleons. For a given projectile-target system, the same partial l - wave corresponds to a smaller and smaller impact parameter with increased bombarding energy, and therefore constant distance between the two colliding centers implies larger l-values for higher energies. It is then considered that the possibility for a large number of intrinsic excitations ending up as compound nucleus formation depends upon two conflicting tendencies:

i. Attractive nuclear forces which are more effective when the distance between the two ions diminishes, and is within the range of the nuclear force.

ii. The centrifugal forces which prevent the two nuclei from fusing into a single composite system whose shape may be spherical.

A simplified representation which might, nevertheless, be useful, is to consider potential energy as a function of the distance between the two centers and to keep all other degrees of freedom frozen (e.g. rotation and vibration of ions) frozen except the relative motion. This implies the sudden approximation, which is certainly justified for relativistic systems. At low velocities the justification comes from the assumption of a strong interaction (Huizenger *et al.,* 1976) in the entrance channel. It is, therefore, worthwhile to build a potential with all degrees of freedom frozen except the distance and the orbital angular momentum and to study at which point one should unfreeze other degrees of freedom and abandon the two-body potential in order to proceed to an attractive potential well representing the compound nucleus. If such a distance cannot be reached, then a complete fusion does not take place. On the contrary, if the two nuclei approach each other up to this point-of-no-return, they stick together due to loss of energy, which is large enough to establish a common nuclear structure. Through the nuclear potential $V_N(r)$ and total potential $V(l,r)$, it can be useful at least in order to illustrate qualitatively how the critical angular momentum l_{cr} and the constant critical distance of approach R_{cr} are related. A consequence of the concept of critical distance is the deduction of a simple formula (20), for calculating the compound nucleus cross-section σ_{CF} (Wong, 1973 and Huizenger *et al.*, 1976). There is obviously a relation between R_{cr} and l_{cr} which expresses conservation of angular momentum.

Since at the contact point, the distance between the two centers is R_{cr} ,

$$l_{cr}^2 = R_{cr}^2 \cdot 2\mu(E_{cm} - V_{cr}) \qquad (19)$$

If V_{cr} is the potential taken at the distance R_{cr} for $l = 0$ wave as shown in the potential curves (fig 7) the expression for the compound nucleus cross-section becomes (Godre and Waghmare, 1989).

$$\sigma_{CF} = \pi R_{cr}^2\left(1 - \frac{V_{cr}}{E_{cm}}\right) \qquad (20)$$

Which is similar to the classical reaction cross-section given in Eq. (11) and

$$\sigma_R = \pi R_{lB}^2\left(1 - \frac{V_{lB}}{E_{cm}}\right) \qquad (21)$$

All these quantities, i.e. R_{cr}, V_{cr}, R_{lB} and V_{lB} are obtained from ion-ion potential curve drawn in figure (1) and tabulated in table (8). It is also possible to obtain an expression for the upper limit of orbital angular momentum $l_{\max}$ after which compound nucleus formation is not possible. Thus fusion takes place only for l-values between l_{cr} and $l_{\max}$ (Malfiet, 1974),

$$\text{where } l_{\max} = \frac{R_{\text{int}}}{\lambda}\left(1 - \frac{2\eta\lambda}{R_{\text{int}}}\right) \qquad (22)$$

λ is the reduced deBroglie wave lengtgh, $R_{\text{int}} = r_0\left(A_1^{1/3} + A_2^{1/3}\right)$ with $r_0 = 1.44\,\text{fm}$, η is the sommerfeld parameter (Burcham, 1988) and A_1 and A_2 are the projectile and target atomic masses, respectively.

Ion-Ion Interaction Potentials

Ion –ion interaction potential to be used in the calculation is composed of nuclear potential, Coulomb potential and centrifugal potential terms. The nuclei of the two ions are considered to move in classical trajectories as long as the identity of the projectile and the target is not completely lost during the reaction process. The potential responsible for the nuclear interaction along the grazing trajectory and close trajectory is the nuclear potential and it is experienced by the two participating ions at an impact parameter equal to or less than the grazing impact, the interaction is completely due to Coulomb field and this is where nuclear potential takes over.

Nuclear Potential

The nuclear potential used here is the proximity (Dutt and Puri, 2010) given as:

$$V_N = -1.9898\frac{R_1R_2}{R_1+R_2}\phi(r - R_1 - R_2 - 2.65)\times\left[1 + 0.003525\left(\frac{A_1}{A_2} + \frac{A_2}{A_1}\right)^{\frac{3}{2}} - 0.41133(I_1 + I_2)\right] \qquad (23)$$

With $I_i = \dfrac{N_i - Z_i}{A_i}$ where $i = 1,2$, I_i is the neutron excess parameter. The effective nuclear radius R_i (Dutt and Puri, 2010) is given as:

$$R_i = R_{ip}\left(1 - \frac{3.4138}{R_{ip}^2}\right) + 1.2846\left(I_i - \frac{0.4A_i}{A_i + 200}\right)$$

where proton radius R_{ip} [6] is given by

$R_{ip} = 1.24A_i^{\frac{1}{3}}\left\{1 + \dfrac{1.646}{A_i} - 0.191\left(\dfrac{A_i - 2Z_i}{A_i}\right)\right\}$ and $\phi(S)$ (where $S = r - R_1 - R_2 - 2.65$) is given by the following form:

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$$\phi(S)=\begin{cases}1-\dfrac{S}{0.7882}+1.229S^2-0.2234S^3-0.1039S^4-\dfrac{R_1R_2}{R_1+R_2}\left(0.1845S^2+0.0757S^3\right)\\ \qquad +\left(I_1+I_2\right)\left(0.0447S^2+0.0335S^3\right),\ -5.65\le S\le 0\\ \left\{1-S^2\left[0.0541\dfrac{R_1R_2}{R_1+R_2}\exp\left(-\dfrac{S}{1.761}\right)-0.53954\left(I_1+I_2\right)\exp\left(-\dfrac{S}{2.4244}\right)\right]\right\}\\ \times\exp\left(-\dfrac{S}{0.7882}\right),\quad S\ge 0\end{cases} \qquad \text{..........(24)}$$

Here A_i, N_i, Z_i, R_i and R_{ip} are respectively, the mass number, the number of neutrons, the number of protons, the effective nuclear radius, and the proton radius of the target and projectile.

Coulomb Potential

The Coulomb potential plays a very important role in the interaction between heavy ions. This potential (Burcham, 1988) can be expressed as:

$$V_C(r)=\begin{cases}\dfrac{Z_1Z_2e^2\left(3R_c^2-r^2\right)}{2R_c^3} & r<R_c\\ \dfrac{Z_1Z_2e^2}{r} & r>R_c\end{cases} \qquad \text{.........(25)}$$

Where Z_1 is the charge of the projectile, Z_2 is the charge of the target, e is an electron charge, r is the distance between the ions and $R_c = r_c\left(A_1^{1/3}+A_2^{1/3}\right)$, $r_c = 1.0 \pm 0.07$ fm for a centre of mass frame of reference with A_1 and A_2 as projectile and target mass numbers respectively, or $R_c = r_c A_T^{1/3}$ for laboratory frame of reference with A_T as the mass number of the target.

The Centrifugal Potential

The centrifugal potential (Burcham, 1988 and Dutt and Puri, 2010) term was expressed as

$$V_L(r)=\frac{l(l+1)\hbar^2}{2mr^2} \qquad \text{.........(26)}$$

Where l is the angular momentum quantum number for a given partial wave participating in the scattering. The effect of centrifugal potential is to augment the potential barrier of the nucleus whenever there is mutual orbital angular momentum present.

The Barrier Heights, Critical Radii and Angular Momentum

If nuclear forces had zero range, that would mean that nuclear forces have reached the point when they cannot interact, and if the density distributions were homogenous with sharp surface, then the interaction Barrier, V_{IB} would be equal to the Coulomb Barrier $V_C(R_c)$ that is;

$$V_{IB}=V_C(R_c) \qquad \text{.........(27)}$$

Where,

$$V_C(R_c)=\frac{Z_1Z_2e^2}{r_c\left(A_1^{1/3}+A_2^{1/3}\right)} \qquad \text{.........(28)}$$

and $r_c = 1.0 \pm 0.07$ fm is a constant parameter. The interaction barrier or the barrier height is then defined by the maximum of the potential $V = V_C(r)+V_N(r)+V_l(r)$.

Thus the barrier height V_{IB} can be found from the potential energy curve $V(l,r)$ which is plotted on Figure 7 when the value of l is zero. Then for $E_{cm} >> V_{IB}$ where E_{cm} is the energy of the projectile in the centre of mass frame and V_{IB} is the Barrier potential at the Barrier radius R_{IB}, whereby the two heavy ions penetrated deeply into the region of nuclear interaction. Galin *et al.*, 1974 have introduced the concept of a critical distance for fusion. They have defined an attractive potential $V_{eff}(l,r)$[7] as

$$V_{eff}(l,r) = V_C(r) + V_N(r) + \frac{l(l+1)\hbar^2}{2mr^2} \quad \text{.......}(29)$$

at $l = l(E)$ where E is the energy of the projectile. This happens at the distance of closest approach where the nuclear potential has a minimum value; this distance of closest approach defines a critical radius. $R_{cr} = 1.0\left(A_T^{1/3} + A_P^{1/3}\right)$fm which is independent of energy.

If the potential barrier $V_{IB}(R_{IB})$=maximum, $V(l=0,r)$ is less than $V_{cr}(R_{cr})$, the critical radius R_{cr} becomes the relevant parameter and V_{cr} is then fusion threshold. It is, therefore, very important to note that these ion-ion interaction potential curves provide very vital information regarding heavy ion nuclear reactions. Here, $V(l=0,r)$ is the potential for $l=0$ while $V(l,r)$ is the potential for a given value of l.

RESULTS

Coulomb Potential for the Ions

The graph of Coulomb potential (MeV) against radius (separation distance between nuclei) of the ions is as plotted in figure (1) using Eq. (25).

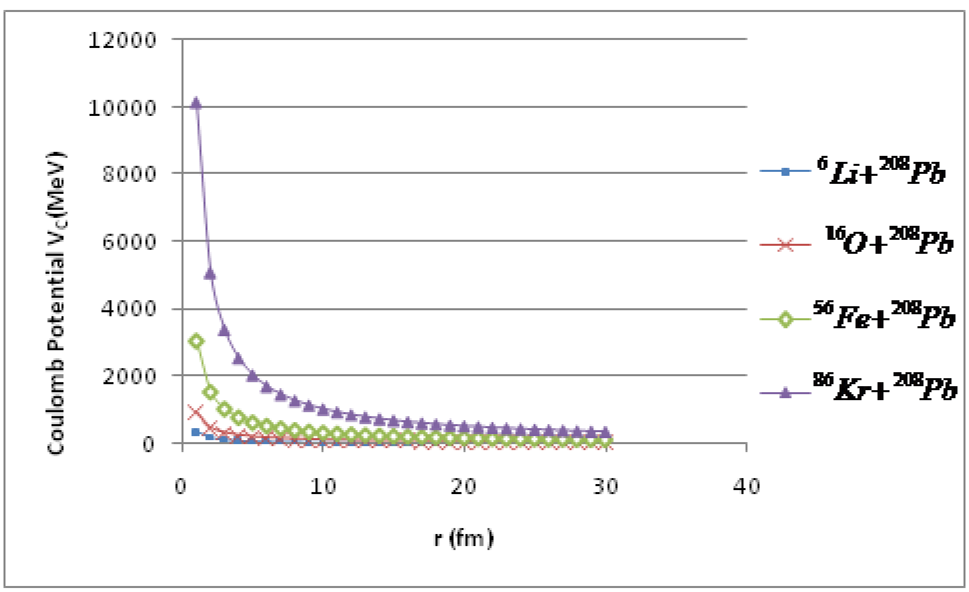

Figure 1: Variation of Coulomb potential V_C (MeV) against distance between ion centers r (fm)

Figure (1) shows that as the charge of the interacting ions increases, the values of potential increase. This means that at a given r, the value of V_C for interacting ions of low charge is low compared to ions with higher charge values. Furthermore, the Coulomb potential decreases exponentially with r. For small values of r, the reaction ^{6}Li+^{208}Pb has the lowest value of V_C while the reaction ^{86}Kr+^{208}Pb has the highest value of 1000MeV. This is due to the difference in the nuclear charge of the projectile.

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Centrifugal Potential of the ions

Figure 2 shows that centrifugal potential between nuclei increases with angular momentum l but decreases with increase in separation distance r.

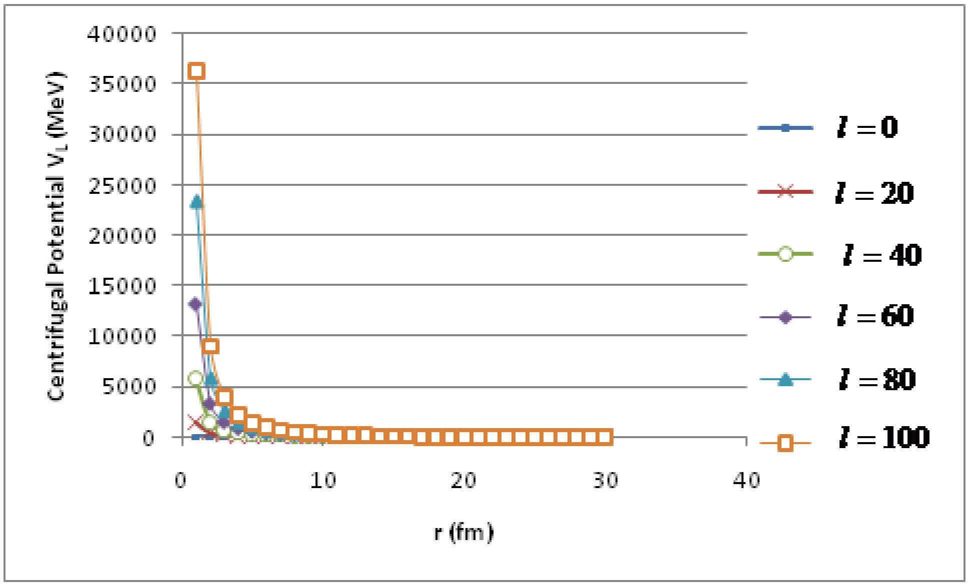

Figure 2: Centrifugal Potential V_L (MeV) against ion-ion distance r (fm) in the reaction ^{6}Li+^{208}Pb

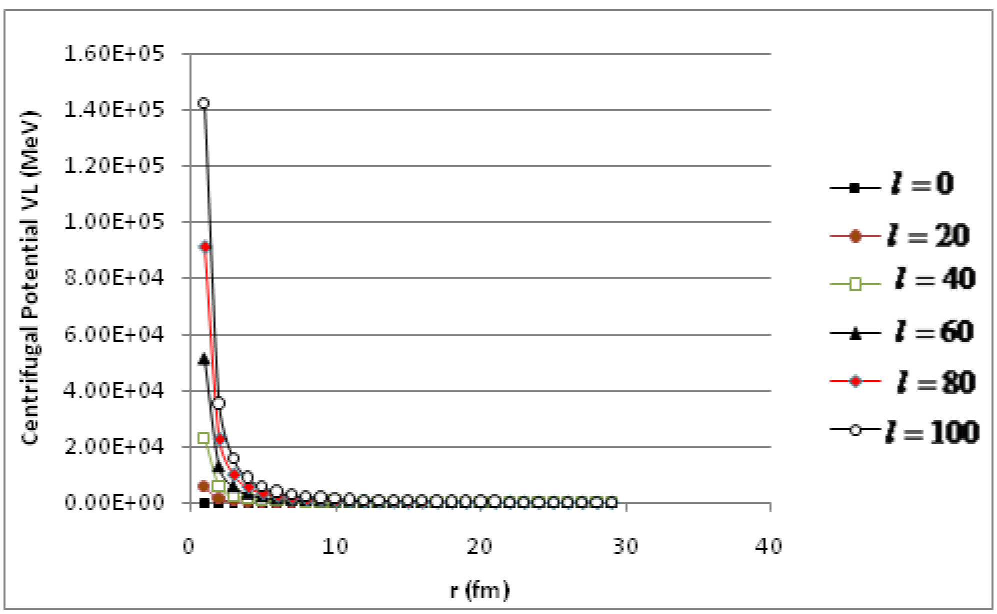

Figure 3: Centrifugal Potential V_L (MeV) against ion-ion distance r (fm) in the reaction ^{16}O+^{208}Pb

Figure 3 shows the increased potential with increase in projectile mass number when compared to Fig (2) at high l values and the increase of potential between nuclei with increase in angular momentum and its decrease with increase in separation distance r.

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Figure 4 shows that centrifugal potential between the nuclei increases with angular momentum, but l decreases with increase in separation distance r.

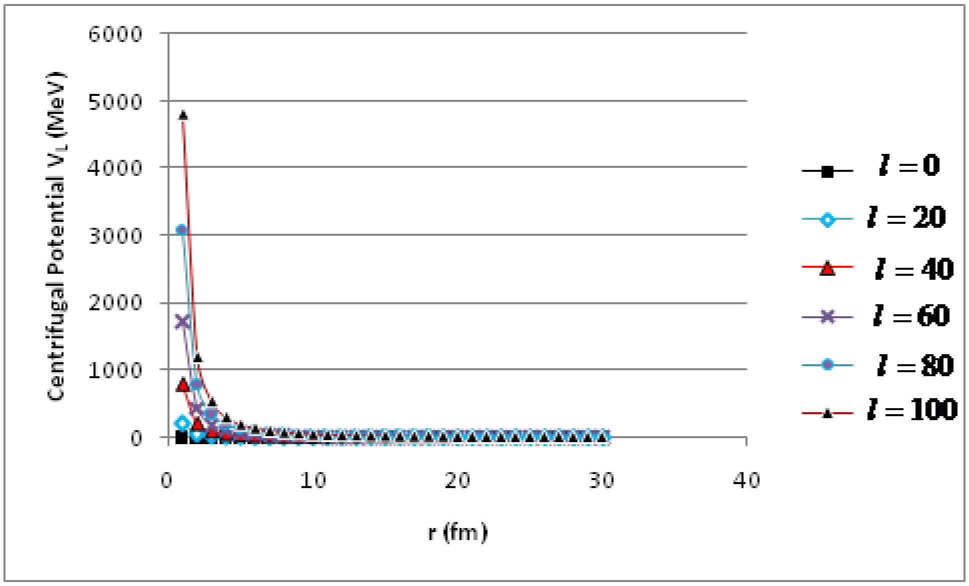

Figure 4: Centrifugal Potential V_L (MeV) against ion-ion distance r (fm) in the reaction $^{56}Fe+^{208}Pb$

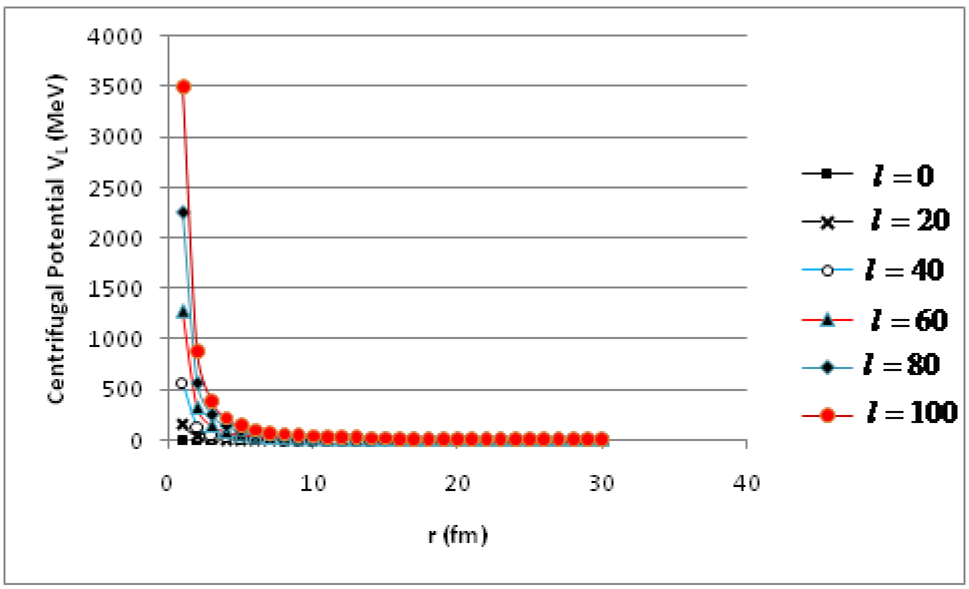

Figure 5: Centrifugal Potential V_L (MeV) against ion-ion distance r (fm) in the reaction $^{86}Kr+^{208}Pb$

Figure 5 shows that the reduced centrifugal potential between the nuclei increases with angular momentum, but l decreases with increase in separation distance r. Fig (2) to (5) show that the centrifugal potential decreases with increase in mass number of projectile.

Nuclear Potential for the Given Ion Reactions

Nuclear potential has been calculated from Eq. (23). The variation of the potential with the separation distance, r between the nuclei centers is as shown in Fig. (6).

At high values of r ($r > 15$ fm), nuclear potential is almost zero. The potential is mainly experienced between $r = 5$ fm to $r = 15$ fm.

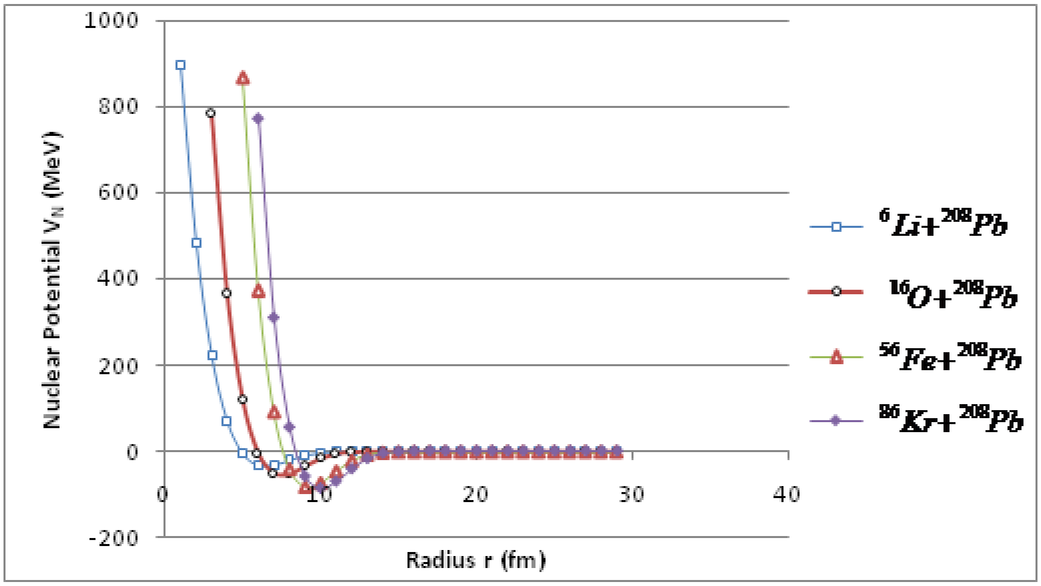

Figure 6: Variation of nuclear potential V_N (MeV)with separation distance r (fm) between nuclei centers

This means that it is of short range. As the particles come closer, Coulomb potential dominates and the fused particles are likely to split again.

Total Ion-Ion Potential for the Ions

This total ion-ion potential is obtained from the summing up of the three calculated potentials and is given by

$$V(r,l) = V_N + V_C + V_L$$

as expressed in Eq. (29)

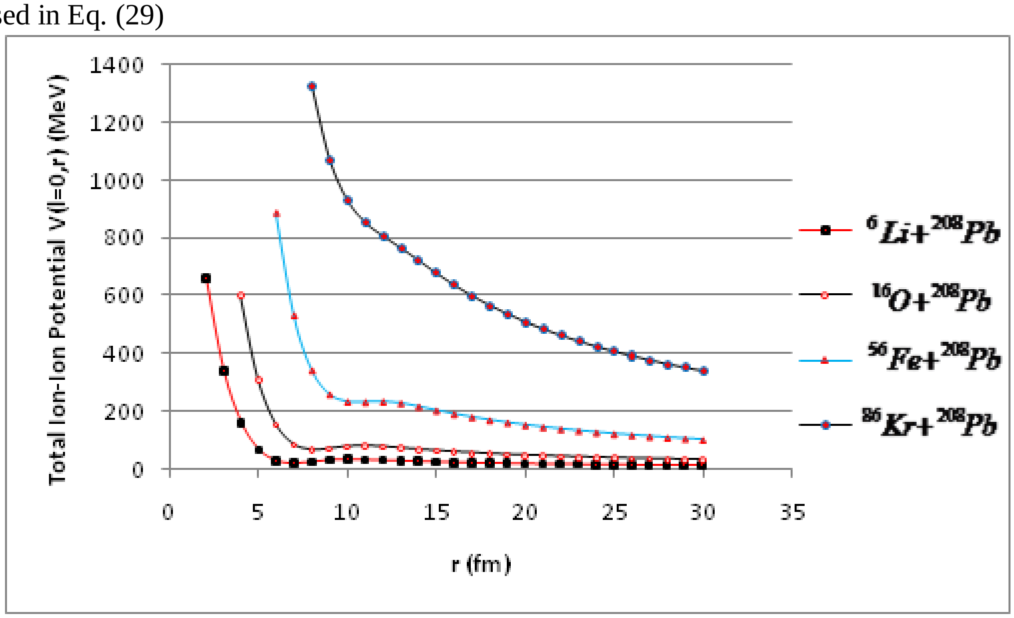

Figure 7: Variation of total ion-ion potential V(l=0,r) against ion separation distance r (fm)

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The curves in Fig (7) have two points of interest; the turning points, the one close to the origin gives R_{cr} (r-axis) and its corresponding potential, V_{CR} (y-axis) which helps in calculating the fusion cross-section. The second turning point gives R_{IB} (x-axis) and its corresponding potential V_{IB} (y-axis) which is instrumental in calculating the reaction cross-section. From fig (7), quantities of interest are such as: Barrier radius, R_{IB} (fm) and its corresponding potential V_{IB} (MeV), Critical radius R_{cr} (fm) and corresponding potential, Fusion Threshold V_{cr} (MeV) are read. E_{cm} has been calculated from Eq. (19) and these quantities are then used to obtain Table 2.

Table 1: Fusion Reaction Energy E_{cm}, Barrier Radius R_{IB}, Barrier Height V_{IB}, Critical Radius R_{cr} and Fusion Threshold values V_{cr} value

Reaction	Reaction Energy E_{cm} (MeV)	Barrier Radius R_{IB} (fm)	Barrier Height V_{IB} (MeV)	Critical Radius R_{cr} (fm)	Fusion Threshold V_{cr} (MeV)	E_{cm} - V_{cr} (MeV)
$^{6}Li+^{208}Pb$	485.98	10.00	32.92	7.00	19.26	466.72
$^{16}0+^{208}Pb$	462.50	11.00	80.49	8.00	66.06	396.44
$^{56}Fe+^{208}Pb$	393.9	12.00	234.99	10.00	234.06	141.98
$^{86}Kr+^{208}Pb$	353.50	....	...	....	...	...

The values obtained from Table (1) are finally used in calculating the desired results which are given in Table (2). The desired quantities have been calculated from Eqns. (19) for E_{cm} (MeV), (22)for R_{int} (fm), (1) for η, (19) for l_{cr}, (19) for $l_{\max}$, (21) for σ_R (mb) and (20) for σ_{CF} (mb).

Table 2: Showing R_{int} (fm), η, l_{cr}, $l_{\max}$, σ_R (mb) and σ_{CF} (mb) values

Reaction	Reaction Energy E_{cm} (MeV)	R_{int} (fm)	η	l_{cr}	$l_{\max}$	σ_R (mb)	σ_{CF} (mb)
$^{6}Li+^{208}Pb$	485.98	11.15	4.24	79.82	121.2	292.88	147.83
$^{16}0+^{208}Pb$	462.50	12.16	18.47	134.15	183.4	314.00	173.73
$^{56}Fe+^{208}Pb$	393.90	14.04	112.1	172.90	180.2	182.60	162.85
$^{86}Kr+^{208}Pb$	353.50	14.89	191.5	......	92.26	....	....

Table 2 shows that all the values of η are greater than 1 meaning that our reaction cross-sections qualify to be calculated classically as described by equation (1). From the values of σ_R (mb) and σ_{CF} (mb) the probabilities of the reactions can be obtained, and it reveals that it is possible for heavy ions to undergo fusion reaction.

DISCUSSION

The work involves calculations regarding one of the very important modes of nuclear reactions among heavy ions i.e. fusion reaction (Agarwalla *et al.*, 2006). In the reactions between heavy ions, there is a transfer of high energy, large mass and large angular momentum; therefore, it was thought that the probability of compound nucleus being formed might be very small, and therefore fusion of heavy ions might not take place. But currently there is ample experimental evidence (Dutt and Puri, 2010) that many heavy-ion reactions do take place via this mode provided the energy of the incident projectile can be controlled precisely. There is a well defined region of orbital angular momentum l between l_{cr} and $l_{\max}$, where the compound nucleus formation takes place since l_{cr} is intimately connected with impact parameter s, which is in turn connected to the energy of the incident projectile (Wong *et al.,* 1989). One can control the l values by allowing the incident beam of precise energy values. This study has revealed that as the charge of the interacting ions increases (see Figure 1), the potential values increase too. This suggests that at a given value r, the value of the potential V_C for interacting ions of low charge is low compared to ions with higher charge values (Satchler, 1983). Similarly, a plot of the centrifugal potential with distance r as the value of angular momentum is varied (see Figures 2-5) showed that the centrifugal potential, between the nuclei increases with angular momentum l, but decreases with increase in separation distance. When the nuclear potential was plotted against the separation distance between the nuclei centers, it was found that a positive value of the potential was obtained for a distance less than 5fm for the ^{6}Li+^{208}Pb reaction, ^{58}Fe+^{208}Pb and ^{86}Kr+^{208}Pb. The potential became negative at r values ranging from 6fm, 7.7fm and 8.8fm indicating that onset of the negative potential was determined by the size of the nuclei. The depth of the potential also increases as the size of the nuclei increases (Agarwalla *et al.*, 2006). Beyond 15fm, all the nuclear reactions considered in this study did not present any nuclear potential. This indicates that nuclear potential is of short range and therefore acts within a small radius with $r < 15$ fm in cases considered here. It can also be noted that when $r \le 2$ fm (see Figure 7), nuclear potential is infinite in all cases explaining the high repulsive nature of the nuclear core.

The graph of total ion-ion potential, $V(l,r)$ against the distance of separation between the nuclei centres, r (see Figure 7) has been used to find the quantities like; Critical radius of approach, R_{cr}, Critical potential of approach, V_{cr}, the interaction potential, V_{IB} and barrier radius R_{IB}, which have been used in calculation of fusion cross-section σ_{CF} and reaction cross-section σ_R using equations (20) and (21) respectively. The values of interaction radius R_{int}, Sommerfeld parameter η and reduced de Broglie wavelength λ for the pair of heavy ions have been calculated and then used to determine the value of $l_{\max}$ using equation (22) to find the upper limit of angular momentum value and consequently the precise value of the energy of the relative motion between the two heavy ions above which fusion cannot take place. The value of l_{cr} can be calculated from equation (19). All the above mentioned calculations have been tabulated in Table 2 and the calculated values of reaction cross sections compared with the experimental results from the internet site named NVR-Experimental on HI fusion reaction [www.NRM.comm 1/4/2010]. For example, the site gave the experimental value of σ_{CF} as 50mb for ^{16}O+^{208}Pb at $E_{CM} = 83.90$ MeV. Calculating the same quantity at $E_{cm} = 83.90$ MeV using the potential method gave the value as 42.75mb. This means that the percentage change in values is 14.5% calculated as $\left(\frac{\sigma_{Expt} - \sigma_{potential}}{\sigma_{Expt}}\right) \times 100\%$. The calculated V_{IB} and R_{IB} values were also compared with experimental results from (Dutt and Puri, 2010) and gave variation of below 10% as can be seen from the comparison Table 3.

The percentage changes have been calculated from the relation:

$$\left(\frac{\sigma_{Expt} - \sigma_{potential}}{\sigma_{Expt}}\right) \times 100\%$$

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Since the variation is small, this method is reliable for reaction cross-section estimation. In these calculations it has been found out that some of the quantities of interest of $^{86}Kr+^{208}Pb$ reaction could not be obtained from the graph because of the depth of its potential well (Wiczek, 2007).

Table 3: Comparison of Experimental interaction potential V_{IB} (MeV) with calculated potential values for the three reactions $^{6}Li+^{208}Pb$, $^{16}O+^{208}Pb$ and $^{56}Fe+^{208}Pb$

Reaction	Experimental Value-V_{IB} (MeV)	Calculated value -V_{IB} (MeV)	Percentage In variation
$^{6}Li+^{208}Pb$	30.10	32.92	9.369%
$^{16}O+^{208}Pb$	74.90	80.49	7.463%
$^{56}Fe+^{208}Pb$	223	234.99	5.377%

The depth could not allow the readings of values of interest tabulated in Table (1) because it is too shallow (see Figure 7). This made it difficult to point out the specific points to give the readings.

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