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Research Article

A TRANSIENT HEAT CONDUCTION PROBLEM OF SEMI-INFINITE SOLID CIRCULAR CYLINDER AND ITS THERMAL DEFLECTION BY QUASI-STATIC APPROACH

[*]G.D. Kedar[1] and K.C. Deshmukh[2]
[1]*Department of Mathematics, Kamla Nehru Mahavidyalaya, Nagpur, Maharashtra, India*
[2]*Department of Mathematics, RTM Nagpur University, Nagpur, Maharashtra, India*
**Author for Correspondence*

ABSTRACT
This paper deals with a transient heat conduction problem and determination of quasi-static thermal deflection of a semi-infinite solid circular cylinder subjected to arbitrary initial heat supply on the lower surface with the curved surface having zero heat flux. The numerical calculations have been carried out for a copper cylinder and illustrated graphically.

Key Words*: Thermal deflection, Heat Conduction Problem, Quasi-Static and Semi-Infinite Cylinder*

INTRODUCTION

Heating of the various parts of the modern machines takes place. Heating of these parts causes the development of the stresses in the body. Due to this body gets deformed under the thermal effects. The prediction of these measures is possible by solving heat conduction problem by prediction solution method and studying thermoelastic behavior of the metallic bodies of any shape.

Nowacki (1957) has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Further Roy Choudhuri (1972) has succeeded in determining the quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of a circle over the upper face with lower face at zero temperature and the fixed circular edge thermally insulated. Wankhede (1982) determined thermal stresses in a thin circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature and fixed circular surface thermally insulated. Roy Choudhuri (1973) discussed thermal deflection of a thin clamped circular plate due to ramp-type heating of a concentric circular region of the upper face when lower face of the plate was kept at zero temperature, while the circular edge is thermally insulated. Recently Deshmukh *et al.*, (2009) determined quasi-static thermal deflection of a thin clamped circular plate due to heat generation within it. Very recently Deshmukh *et al.*, (2010) solved inverse heat conduction problem in a semi-infinite circular plate and its thermal deflection by quasi-static approach.

Kedar *et al.*, (2012) determined thermal stresses in a semi-infinite solid circular cylinder subjected to arbitrary initial heat supply on the lower surface with the curved surface are insulated. In this present paper one modifies the problem of Kedar *et al.*, (2012) and deals with the determination of quasi-static thermal deflection when the outer edge of the cylinder is fixed and clamped (built-in edge). The solution of heat conduction equation is obtained in the series form in terms of Bessel's functions. Mathematical model has been constructed with the help of numerical illustration. This problem is applicable in the field of mechanical engineering for the heating of cylindrical rods in machines. No one has studied such type of problem previously and it is a new contribution to the field of thermoelasticity.

Formulation of the Problem

Consider a semi-infinite solid circular cylinder of radius $r = a$ and $0 \leq z < \infty$. Let the lower surface be subjected to arbitrary initial temperature and a curved boundary surface $r = a$ is at zero heat flux. Under these more realistic prescribed conditions the quasi-static thermal deflection in the cylinder are required to be determined.

The temperature $T(r,z,t)$ of the cylinder at time t satisfies the equation

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

with boundary conditions

$$T=0\text{, for } z=\infty \tag{2}$$

$$T=F(r)\text{, for } z=0\text{, at } t=0 \tag{3}$$

$$\frac{\partial T}{\partial r}=0\text{, for } r=a \tag{4}$$

Where, α is the thermal diffusivity of the material of the cylinder.

Following the procedure Roy Choudhuri (1973), the differential equation satisfying the deflection function $W(r,t)$ stated as

$$\nabla^4 W=-\frac{\nabla^2 M_T}{D(1-\nu)} \tag{5}$$

Where, $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$, D is the flexural rigidity and M_T is the thermal moment of the semi-infinite cylinder.

The thermal moment and flexural rigidity are defined as

$$M_T=a_t E\int_0^\infty T(r,z,t)\,z\,dz \tag{6}$$

$$D=\frac{Eh^3}{12(1-\nu^2)} \tag{7}$$

Where E, ν and a_t are Young's modulus, Poisson's ratio and linear coefficient of thermal expansion of the material.

For built-in edge, the deflection W and its first derivative with respect to the radius r on the boundary surface must be zero.

$$W=\frac{\partial W}{\partial r}=0\text{, at } r=a \tag{8}$$

Initially,

$$W=0\text{ at } t=0 \tag{9}$$

Solution

To obtain the expression for temperature, $T(r,z,t)$ one assumes that,

$$T(r,z,t)=e^{-z}\sum_{n=1}^{\infty} f_n(t)J_0(\lambda_n r) \tag{10}$$

Where, $\lambda_1,\lambda_2,...$ are the positive roots of the transcendental equation $J_1(\lambda_n a)=0$, and $J_n(x)$is the Bessel's function of first kind and of order n and the function can be determined.

Equations (4) and (10) give

$$f_n(t)=A_n\ e^{-\alpha(\lambda_n{}^2-1)t} \tag{11}$$

Where, A_n is a constant which can be found from the nature of the temperature at the lower surface of the cylinder. Thus, the expression of the temperature becomes,

$$T(r,z,t) = e^{-z} \sum_{n=1}^{\infty} A_n \; e^{-\alpha\left(\lambda_n{}^2-1\right)t} J_0(\lambda_n r) \tag{12}$$

Using condition (6) in equation (12) one obtains

$$F(r) = \sum_{n=1}^{\infty} A_n \; J_0(\lambda_n r) \tag{13}$$

Using the orthogonal property of Bessel's functions, one gets

$$A_n = \frac{2}{a^2 J_0{}^2(\lambda_n a)} \int_0^a r' F(r') J_0(\lambda_n r') dr' \tag{14}$$

Hence, the temperature distribution in the cylinder is obtained as,

$$T(r,z,t) = e^{-z} \sum_{n=1}^{\infty} \left[\frac{2}{a^2 \; J_0{}^2(\lambda_n a)} \int_0^a r' \; F(r') \; J_0(\lambda_n r') \; dr' \right] e^{-\alpha\left(\lambda_n{}^2-1\right)t} J_0(\lambda_n r) \tag{15}$$

Using temperature distribution from equation (15) in equation (6) the expression for thermal moment can be obtained as

$$M_T = e^{-z} \sum_{n=1}^{\infty} \left[\frac{2}{a^2 \; J_0{}^2(\lambda_n a)} \int_0^a r' \; F(r') \; J_0(\lambda_n r') \; dr' \right] e^{-\alpha\left(\lambda_n{}^2-1\right)t} \; J_0(\lambda_n r) \tag{16}$$

Hence,

$$\nabla^2 M_T = e^{-z} \sum_{n=1}^{\infty} \lambda_n{}^2 \left[\frac{2}{a^2 J_0{}^2(\lambda_n a)} \int_0^a r' \; F(r') \; J_0(\lambda_n r') \; dr' \right] e^{-\alpha\left(\lambda_n{}^2-1\right)t} J_0(\lambda_n r) \tag{17}$$

According to the boundary conditions as mentioned in equation (8) one can assume that

$$W(r,t) = \sum_{n=1}^{\infty} C_n(t) \left[J_0(\lambda_n r) - J_0(\lambda_n a) \right] \tag{18}$$

$$\nabla^4 W(r,t) = \sum_{n=1}^{\infty} C_n(t) \, (\lambda_n)^4 \; J_0(\lambda_n r) \tag{19}$$

Hence, taking the expression for thermal moment in consideration (16) one obtains thermal deflection using relation (5) as

$$\frac{W(r,t)}{A} = \sum_{n=1}^{\infty} \left[\frac{1}{\left(\lambda_n{}^2\right) J_0{}^2(\lambda_n a)} \int_0^a r' \; F(r') \; J_0(\lambda_n r') \; dr' e^{-\alpha\left(\lambda_n{}^2-1\right)t} \left[J_0(\lambda_n r) - J_0(\lambda_n a) \right] \right] \tag{20}$$

Where, $A = \dfrac{24(1+\nu)a_t}{a^2 \; h^3}$ (21)

NUMERICAL CALCULATIONS

The numerical calculations have been carried out for a copper cylinder with the following material properties.

Material properties

Thermal diffusivity	$\alpha = 112.34 \times 10^{-6} m^2 s^{-1}$
Poisson ratio	$\nu = 0.35$
Coefficient of linear thermal expansion	$a_t = 16.5 \times 10^{-6} K^{-1}$
Lamé constant	$\mu = 26.67$
Radius of the cylinder	$a = 2m$

Height of cylinder $z = h = 10\,m$

Roots of transcendental equation

The first ten roots of transcendental equation $J_1(\lambda_n a) = 0$, are

$\lambda_1 = 1.9158$, $\lambda_2 = 3.5078$, $\lambda_3 = 5.0867$, $\lambda_4 = 6.6618$,
$\lambda_5 = 8.2363$, $\lambda_6 = 9.8079$, $\lambda_7 = 11.38005$,
$\lambda_8 = 12.95185$, $\lambda_9 = 14.5234$ and $\lambda_{10} = 16.09485$.

To construct the mathematical thermoelastic behavior of a semi-infinite circular cylinder, we set the function

$F(r) = (r - a)^2$

The maximum deflection is observed at the centre of the cylinder and it goes on decreasing towards outer surface and becomes negligible after radius $r = 1.8\ m$ as shown in Figure 1. Also, the maximum central deflection is observed for the time $t = 0$ and it is observed that the deflection monotonically decreases with respect to time as shown in Figure 2.

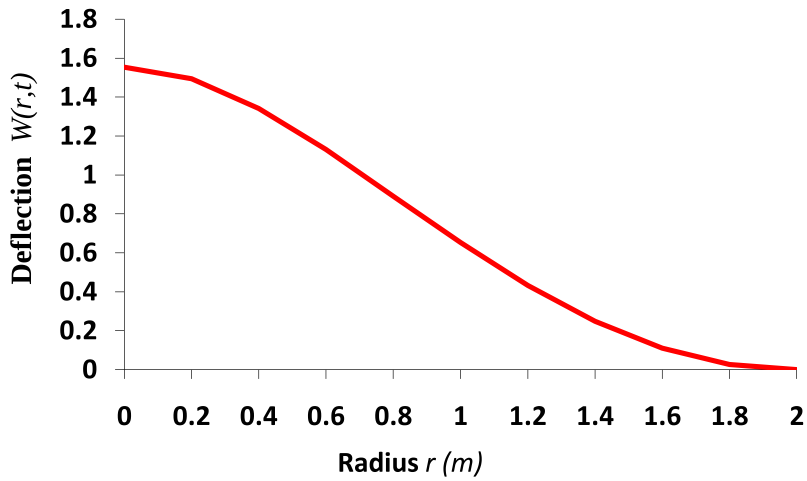

Figure 1: Deflection along radius at $t = 2\,s$.

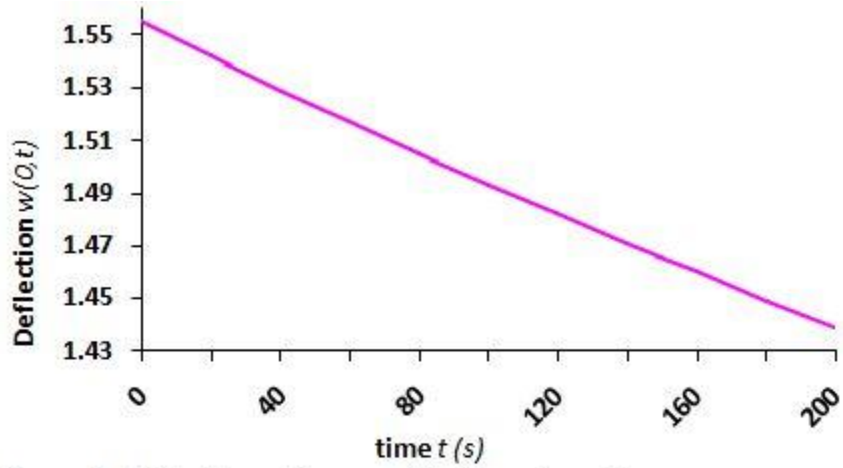

Figure 2: Deflection with respect to time at $r = 0$.

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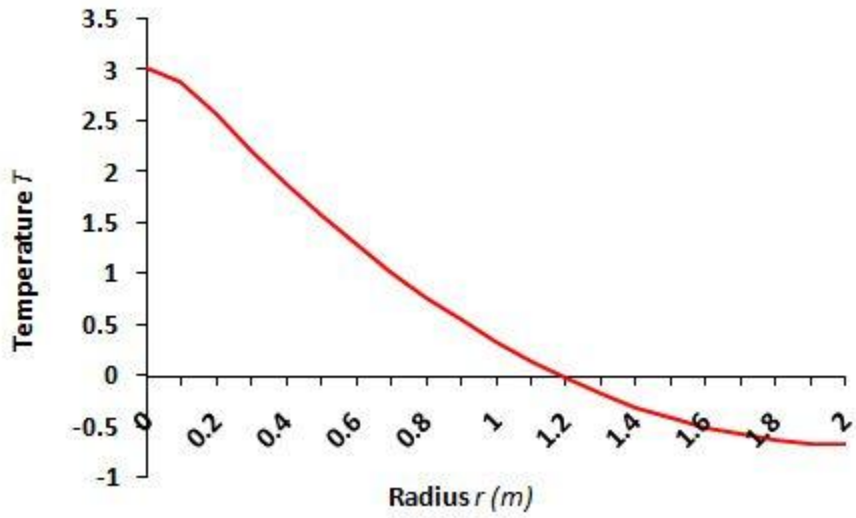

Figure 3: Temperature Distribution at $z = 0$.

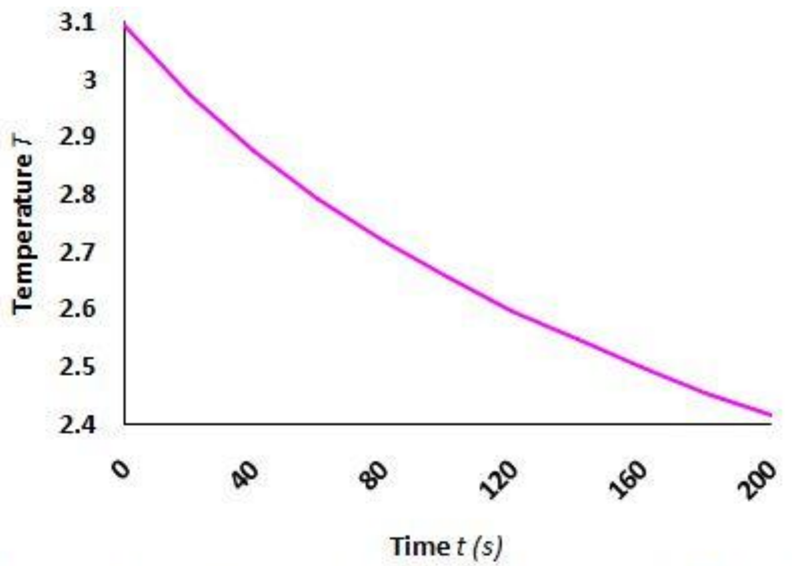

Figure 4: Temperature with respect to time at $z = 0$ **and** $r = 0$

Figure 4 shows the temperature distribution at circular level for $z = 0$, $r = 0$ with respect to time. The temperature is high at the centre and monotonically decreases along radial direction towards outer surface of the cylinder as shown in Figure 3. Since, the upper surface is maintained at zero temperature, the heat flux can be observed below the level of $z = 4$ only and there after it remains as such flat zero.

CONCLUSION

Wankhede (1982) determined temperature distribution in a thin circular plate subjected to arbitrary initial heat supply on the upper face with lower face at zero temperature and fixed circular surface thermally insulated. In this paper, we extended the work of Wankhede (1982) for a semi-infinite circular cylinder subjected to arbitrary initial heat supply in the form of $(r-a)^2$ on the lower surface and determined the thermal deflection and observed that it increases rapidly to high at the centre and gradually decreases towards outer surface of the cylinder as it is built-in edge. We consider arbitrary initial heat supply just like a burner at lower surface of solid which is more realistic, due to which central deflection is high at $t = 0$ and gradually it go on decreasing with time. Due to arbitrary initial heat supply temperature goes down with respect to time but its rate of cooling is slow and it is observed the deflection decreases linearly with time.

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