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Research Article

SOME NOTIONS BASED UPON STRONGLY GENERALIZED STAR SEMI - CLOSED SETS

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ABSTRACT
The aim of this paper is to introduce and study the class of strongly generalized star semi – closed sets which is weaker than semi- closed sets (Crossly and Hildebrand, 1971) and stronger than strongly generalized semi-closed sets (El-Maghrabi and Nasef, 2008) . Also, some notions in terms of strongly generalized star semi-closed sets are introduced. Further, the concept of strongly semi star -$T_{1/2}$ space is studied.

Key Words: strongly generalized star semi - closed sets, strongly generalized star semi -open sets , strongly generalized star semi- closure , strongly generalized star semi -interior , strongly generalized star semi-boundary, strongly generalized star semi-exterior and strongly semi-star- $T_{1/2}$ spaces .

INTRODUCTION AND PRELIMENARIES

In 1970, Levine introduced the concept of generalized closed (briefly, g-closed) sets of a topological space. Bhattacharrya and Lahiri (1987) defined and studied the notion of sg- closed sets. In 1990, Arya and Nour introduced the concept of gs- closed sets. Veera Kumar (2001) defined and studied the notion of g^*-closed sets. The notion of g^*s - closed sets was defined by El-Maghrabi and Nasef (2008).

The purpose of the present paper is to define and investigate the concept of strongly generalized star semi-closed sets and the notion of strongly generalized star -open sets. Moreover, some of their properties are discussed. Further, we define strongly semi- star - $T_{1/2}$ spaces as the space in which every strongly generalized star semi - closed set is semi - closed.

Throughout this paper, spaces always mean topological spaces on which no separation axiom is assumed unless explicitly stated. Let X be a space and A be a subset of X. The closure of A and the interior of A are denoted by cl (A) and int (A) respectively. A subset A of X is said to be regular-open (Singal and Singal, 1968) (resp. semi – open (Levina, 1963), pre-open (MAshhour et al., 1982), Q-set (Levine, 1961)) if $A=\text{int}(cl(A))$ (resp. $A\subseteq cl(\text{int}(A))$, $A\subseteq \text{int}(cl(A))$, $\text{int}(cl(A))=cl(\text{int}(A))$). A subset A of X is said to be semi – closed if $X-A$ is semi – open or, equivalently, if $\text{int}(cl(A))\subseteq A$ (Crossly and Hildebrand, 1974). The family of all semi – open (resp. semi-closed) sets will be denoted by $SO(X,\tau)$ (resp. $SC(X,\tau)$). The intersection (resp. the union) of all semi- closed (resp. semi-open) sets containing (resp. contained in) A is called the semi – closure (resp. the semi - interior) of A and will be denoted by $s-cl(A)$ (resp. $s-\text{int}(A)$).

Definition 2.1. A subset of a space (X,τ) is called:

1- a generalized closed (briefly, g-closed) (Levine, 1970) set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open,

2- a semi generalized-closed (briefly, sg-closed) (Bhattactaryya and Lahiri, 1987) set if $s-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi- open, 3- a generalized semi-closed (briefly, gs-closed) (Arya and TM, 1990) set if $s-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open,

4- a strongly generalized semi-closed (briefly, $g^{*}s$-closed) (El-Maghrabi and Nasef, 2008) set if $s-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open,

5- a g^{*}-closed (Veera Kumar, 2001) set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open.

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Remark 2.1. The complement of g-closed (resp.sg- closed, gs-closed, $g^{*}s$ - closed, g^{*}-closed) is called g-open (resp. sg-open, gs-open, $g^{*}s$ - open, g^{*}-open).

Definition 2.2. For a subset E of (X,τ), we define the following:

(i) $s-cl_{*}(E) = \cap\{A: A \text{ is gs - closed set}, E\subseteq A\}$ (Dunham, 1982) ,

(ii) $s-cl^{*}(E) = \cap\{A: A \text{ is sg - closed set}, E\subseteq A\}$ (Sundaram *et al.*, 1991) .

Definition 2.3. A topological space (X,τ) is called:

(i) a T_d -space [6] if every gs- closed set is g-closed,

(ii) a semi – $T_{1/2}$ space (Bhattactaryya and Lahiri, 1987) if every sg – closed set is semi – closed,

(iii) a T_b –space (Devi, 1994) if every gs-closed set is closed,

(iv) a $T_{\frac{1}{2}}^{*}$-space (Veera Kumar,2001) if every g^{*}-closed set is closed,

(v) a strongly semi- $T_{1/2}$ (briefly, st. semi- $T_{1/2}$) (El-Maghrabi and Nasef, 2008) space if every gs-closed set is $g^{*}s$-closed,

(vi) a semi-T_b space (El-Maghrabi and Nasef, 2008) if every gs-closed set is semi-closed,

(vii) a semi- T_p space (El-Maghrabi and Nasef, 2008) if every $g^{*}s$-closed set is closed,

(viii) a $T_{1/2}$ space (Levina, 1970) if every g- closed set is closed.

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Lemma 2.1 [4]. If A and B are two subsets of X , then the following statements are hold:

(i) s-cl (A) (resp. s- int (A)) is semi – closed (resp. semi- open),

(ii) A is semi – closed (resp. semi – open) iff $A=s-cl(A)$ (resp. $A=s-\text{int}(A)$),

(iii) $s-cl(X-A)=X-s-\text{int}(A)$ and $s-\text{int}(X-A)=X-s-cl(A)$,

(iv) $x\in s-cl(A)$ Iff for each $G\in SO(X,\tau)$ containing x, $G\cap A\neq\phi$.

Lemma 2.2 Andrijevic, 1986 Let A be a subset of a space X. Then, $s-cl(A)=A\cup\text{int}(cl(A))$.

Definition 2.4. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called:

(i) semi-continuous [10] if f^{-1}(U) is semi-open in (X,τ), for every open U of (Y,σ),

(ii) irresolute [5] if f^{-1} (U) is semi-open in (X,τ), for every semi-open U of (Y,σ),

(iii) pre- semi – closed [5] if f (V) is semi- closed in (Y,σ) ,for every semi -closed set V of (X,τ).

STRONGLY GENERALIZED STAR SEMI-CLOSED SETS

Definition 3.1. A subset A of a space X is called a strongly generalized star semi – closed (briefly, strongly $g^{*}s$ – closed) set if $s-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is gs – open.

Remark 3.1. The family of all strongly $g^{*}s$ - closed subsets of a space (X,τ) is denoted by *strongly* $G^{*}SC(X,\tau)$.

Remark 3.2. The concepts of g-closed (resp. g^{*}-closed) and strongly $g^{*}s$-closed sets are independent.

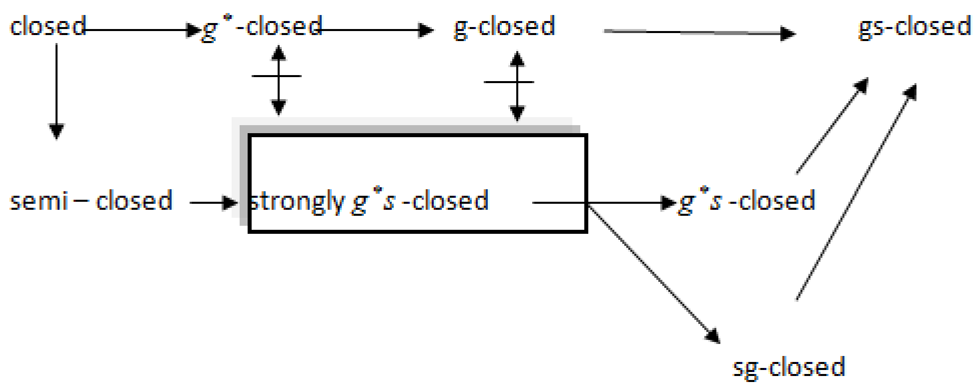
Example 3.1. If X={a,b,c,d} with two topologies τ_1, τ_2 on X such that : τ_1={X, ϕ,{a},{a,b}}, τ_2 ={X, ϕ, {a},{b,c},{a,b,c}} , then:

(1) a subset A={b} of X on τ_1 is strongly $g^{*}s$ - closed but not g-closed and

a subset B={a,b,d}of X on τ_1 is g-closed but not strongly $g^{*}s$ -closed.

(2) a subset C={a}of X on τ_2 is strongly $g^{*}s$ -closed but not g^{*}-closed and a subset D={b,d} of X on τ_2 is g^{*}-closed but not strongly $g^{*}s$ -closed.

Remark 3.3. By Definition 3.1 and Remark 3.2, we obtain the following diagram.



However, the converses are not true in general by (Arya and TM, 1990; Crossly and Hildebrand; 1974; Maghrabi and Naesf, 2008; Veera Kumar, 2001) and the following examples .

Example 3.2. If X = {a, b, c, d} with topologies τ_1, τ_2 on X such that:

τ_1 ={X, ϕ,{c,d}}, τ_2 ={X, ϕ,{c},{c,b},{b,c,d}}, then a subset A= {a,b,c} of X on τ_1 is strongly $g^{*}s$ - closed but not semi– closed. While, a subset B= {a, c} of X on τ_2 is $g^{*}s$ -closed but not strongly $g^{*}s$ - closed.

Example 3.3. Let X= { a, b, c} with topologies τ_1, τ_2 on X such that

τ_1 = { X, ϕ , {a,b},{c}}, τ_2 ={X, ϕ,{a},{a,b}}. Then, a subset C= {a} of X on τ_1 is sg-closed but not strongly $g^{*}s$ -closed. But a subset D = {a,c} of X on τ_2 is gs-closed but not strongly $g^{*}s$ -closed.

Remark 3.4. The union of two strongly g^*s-closed sets need not be strongly g^*s-closed. Let X ={a,b,c,d} with topology τ ={X, ϕ,{a},{b},{a,b}}. Then, the subsets A={a} and B={b} are strongly g^*s-closed but their union is not strongly g^*s-closed.

Theorem 3.1. A subset A of a space (X,τ) is strongly g^*s-closed if and only if every gs - open set G containing A, there exists a semi – closed set F such that $A \subseteq F \subseteq G$.

Proof **.** Necessity. Let A be a strongly g^*s-closed set , $A \subseteq G$ and G be gs – open. Then $s-cl(A) \subseteq G$. Set, $s-cl(A)=F$. Hence, there exists a semi – closed set F such that $A \subseteq F \subseteq G$.

Sufficiency . Assume that A $\subseteq$ G and G is a gs – open set of X. Then by hypothesis, there exists a semi – closed set F such that $A \subseteq F \subseteq G$, therefore, $s-cl(A) \subseteq G$. So, A is strongly g^*s-closed.

Theorem 3.2. Let A be a strongly g^*s-closed set of X. Then $(s-cl(A))-A$ does not contain any non empty g - closed set.

Proof. Let F be a g - closed set such that $F \subseteq (s-cl(A))-A$. Then $F \subseteq X-A$ this implies that $A \subseteq X-F$. Since, A is strongly g^*s- closed and $X-F$ is g - open, then $s-cl(A) \subseteq X-F$, that is $F \subseteq X-(s-cl(A))$, hence $F \subseteq s-cl(A) \bigcap (X-(s-cl(A))) = \phi$. This shows that $F=\varphi$.

The converse of the above theorem may not be true as is shown by the following example.

Example 3.4. In Example 3.1, if A= {a, b, d} is a subset of X on a topology τ_2 , then $(s-cl(A))-A=\{c\}$ does not contain any non empty g-closed set.

Corollary 3.1. Let A be a strongly g^*s-closed set of X. Then $(s-cl(A))-A$ does not contain any non empty gs - closed set .

Proof. Obvious.

Corollary 3.2. Let A be a strongly g^*s - closed set. Then A is semi - closed if and only if $(s-cl(A))-A$ is gs - closed.

Proof **.** Necessity. Assume that A is strongly g^*s - closed and semi - closed sets. Then $s-cl(A)=A$ and hence $(s-cl(A))-A=\varphi$ which is gs - closed.

Sufficiency. Suppose that $s-cl(A)-A$ is gs - closed and A is strongly g^*s -closed. Then by Corollary 3.1 , $s-cl(A)-A$ does not contain any non empty gs - closed subset of X. Hence A is semi - closed.

Theorem 3.3. For each x $\in$ X, then {x} is gs- closed or its complement $X-\{x\}$ is strongly g^*s - closed .

Proof. Suppose that {x} is not gs- closed. Then its complement is not gs- open . Since, X is the only gs- open set containing $X-\{x\}$, that is , $s-cl(X-\{x\}) \subseteq X$ holds . This implies that $X-\{x\}$ is strongly g^*s - closed.

Proposition 3. 1. If A is a strongly g^*s -closed set and $A \subseteq B \subseteq s-cl(A)$, then B is strongly g^*s - closed.

Proof **.** Let B $\subseteq$U and U be a gs- open set of X. Then A$\subseteq$U. Since, A is strongly g^*s - closed, hence $s-cl(A) \subseteq U$, but $B \subseteq s-cl(A)$. Then $s-cl(B) \subseteq U$. Hence, B is strongly g^*s - closed.

Proposition 3. 2. If (X,τ) is a topology space and $A \subseteq X$, then A is semi – closed, if one of the following two cases holds:

(1) If A is strongly g^*s -closed and gs-open.

(2) If A is strongly g^*s -closed and open.

Theorem 3.4. Let A be a subset of a space X, the following are equivalent:

(i) A is regular – open,

(ii) A is open and strongly g^*s -closed.

Proof. (i) →(ii). Let U be a gs-open set containing A and A be a regular-open set. Then, $A \bigcup \text{int}(cl(A)) = A \subseteq U$. So, $s - cl(A) \subseteq U$ and therefore A is strongly g^*s -closed.

(ii)→ (i). Since, A is an open and a strongly g^*s -closed sets, then by Proposition 3.2(2), A is semi-closed. But, A is pre-open. Therefore, A is regular-open.

Theorem 3.5. If A is a subset of a space X, the following are equivalent:

(i) A is clopen,

(ii) A is open, a Q-set and strongly g^*s -closed.

(iii)

Proof. (i) → (ii). Since, A is clopen, hence A is both open and a Q- set. Let U be a gs-open set containing A. Then, $A \bigcup \text{int}(cl(A)) \subseteq U$ and so $s - cl(A) \subseteq U$. Hence, A is strongly g^*s -closed.

(ii) → (i). Hence by Theorem 3.4, A is regular-open. Since, every regular-open set is open, then A is a Q-set, hence A is closed. Therefore, A is clopen.

STRONGLY GENERALIZED STAR SEMI-OPEN SETS

Definition 4.1. A subset A of a space X is called a strongly generalized star semi-open (briefly, strongly g^*s - open) set iff $X - A$ is strongly g^*s - closed.

Remark 4.1. The family of all strongly g^*s - open subsets of X is denoted by $\textit{strongly } G^*SO(X, \tau)$.

Theorem 4.1. For a subset A of a space X, the following statements are equivalent:

(i) A is strongly g^*s - open,

(ii) For each gs-closed set $F \subseteq X$ contained in A, $F \subseteq s - \text{int}(A)$,

(iii) For each gs-closed set $F \subseteq X$ contained in A, there exists a semi -open set $G \subseteq X$ such that $F \subseteq G \subseteq A$.

Proof. (i) →(ii). Let $F \subseteq A$ and F be a gs- closed set. Then $X - A \subseteq X - F$ which is gs-open. Hence, $s - cl\ (X - A) \subseteq X - F$. Therefore by Lemma 2.1, (iii), $F \subseteq s - \text{int}(A)$.

(ii) → (iii). Let $F \subseteq A$ and F be a gs-closed set. Then by hypothesis, $F \subseteq s - \text{int}(A)$. Set $s - \text{int}(A) = G$, hence $F \subseteq G \subseteq A$.

(iii) →(i). Let $X - A \subseteq U$ and U be a gs-open set .Then $X - U \subseteq A$ and by hypothesis, there exists a semi-open set G such that $X - U \subseteq G \subseteq A$, that is, $X - A \subseteq X - G \subseteq U$. Therefore, by Theorem 3.1, $X - A$ is strongly g^*s - closed. Hence, A is strongly g^*s -open.

Remark 4.2. Every semi-open set is strongly g^*s -open but, the converse is not true as is shown by the following example.

Example 4.1. In Example 3.2, a subset A= {d} of X on τ_1 is strongly g^*s -open but not semi-open.

Remark 4.3. The intersection of two strongly g^*s -open sets need not be strongly g^*s -open, as is illustrated by the following example.

Example 4.2. Let X= {a, b, c, d} and τ ={X, ϕ ,{a},{b},{a,b}}. Then, the subsets A= {b,c,d} and B={a,c,d} are strongly g^*s-open but their intersection is not strongly g^*s-open.

Theorem 4.2. If A is a strongly g^*s-open subset of X, then U = X, whenever U is gs- open and $s-\text{int}(A)\bigcup(X-A)\subseteq U$.

Proof. Suppose that U is a gs- open set and $s-\text{int}(A) \bigcup (X-A) \subseteq U$. Then, $X-U \subseteq (X-(s-\text{int}(A))) \bigcap A$ and by Lemma 2.1(iii), $X-U\subseteq s-cl(X-A)-(X-A)$. Hence, by Corollary 3.1, $X-U=\phi$ which implies that $X=U$.

Proposition 4.1. If $s-\text{int}(A)\subseteq B\subseteq A$ and A is strongly g^*s -open, then B is strongly g^*s -open.

Proof. Since, $s-\text{int}(A)\subseteq B\subseteq A$, then $X-A\subseteq X-B\subseteq X-s-\text{int}(A)$,hence by Lemma 2.1, (iii), $X-A\subseteq X-B\subseteq s-cl\,(X-A)$, then by Theorem 3.4, $X-B$ is strongly g^*s -closed. Therefore, B is strongly g^*s-open.

Lemma 4.1. Let A $\subseteq$ X be a strongly g^*s -closed set. Then $s-cl(A)-A$ is strongly g^*s - open.

Proof. Let F be a gs- closed set such that $F\subseteq(s-cl(A))-A$. Since A is strongly g^*s-closed, then by Corollary 3.1, $F=\phi$. Therefore, $\phi \subseteq s-\text{int}\,(s-cl(A)-A)$. Hence, by Theorem 4.1, $s-cl(A)-A$ is strongly g^*s - open.

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STRONGLY GENERALIZED STAR SEMI-TOPOLOGICAL OPERATIONS

Definition 5.1. In a space (X,τ), if $A\subseteq X$, then the strongly generalized star semi- closure (briefly, $strongly\ g^*s-cl\,(A)$) and the strongly generalized star semi- interior (briefly, $strongly\ g^*s-\text{int}\,(A)$) of A are defined by respectively.

$$strongly\ g^*s-cl\,(A)=\bigcap_{i=1}\{F_i : A\subseteq F_i, F_i\in strongly\ G^*SC(X,\tau)\},$$

$$strongly\ g^*s-\text{int}\,(A)=\bigcup_{i=1}\{G_i : G_i\subseteq A, G_i\in strongly\ G^*SO(X,\tau)\}.$$

According to the above definition, it is easy to see that A$\subseteq strongly\ g^*s-cl\,(A)$ and $strongly\ g^*s-\text{int}\,(A)\subseteq A$.

Proposition 5.1. For a topological space (X,τ) , then:

(i) If A $\subseteq$ F and F is strongly g^*s-closed, then $A\subseteq strongly\ g^*s-cl(A)\subseteq F$.

(ii) If G$\subseteq$A and G is strongly g^*s-open, then $G\subseteq strongly\ g^*s-\text{int}(A)\subseteq A$.

Proof. Obvious.

Proposition 5.2. For a space (X,τ) , if A is a subset of X, then the following statements are hold:

(i) If A is strongly g^*s-closed, then A= $strongly\ g^*s-cl\,(A)$.

(ii) If A is strongly g^*s-open, then A= $strongly\ g^*s-\text{int}\,(A)$.

Proof. Obvious.

Lemma 5.1. For a space (X,τ) , if A is a subset of X, then

(i) $strongly\ g^*s-cl\,(X-A)=X-strongly\ g^*s-\text{int}(A)$,

(ii) $strongly\ g^*s-\text{int}\,(X-A)=X-strongly\ g^*s-cl(A)$.

Proof. Obvious from Definition 5.1.

Lemma 5.2. If A is a subset of a space (X,τ), then $x \in \textit{strongly } g^*s - \text{int}(A)$ if and only if there exists a strongly g^*s-open set W such that $x \in W \subseteq A$.

Proof. The necessity. Let $x \in \textit{strongly } g^*s - \text{int}(A)$. Then, $x \in \bigcup_{i=1} W_i, W_i$ is strongly g^*s-open, $W_i \subseteq A$. Therefore there exists at least W contains x such that $x \in W \subseteq \bigcup_{i=1} W_i = \textit{strongly } g^* - \text{int}(A) \subseteq A$. Hence, $x \in W \subseteq A$.

The sufficiency. Assume that there exists a strongly g^*s- open set W such that $x \in W \subseteq A$. Then, $X - A \subseteq X - W$. By Lemma 5.1, $x \notin X - \textit{strongly } g^*s - cl\,(X - A)$. Hence, by Lemma 5.1, $x \notin \textit{strongly } g^*s - \text{int}(A)$. Therefore, $x \in \textit{strongly } g^*s - \text{int}(A)$.

Lemma 5.3. If A is a subset of a space (X,τ), then x$\in$ $\textit{strongly } g^*s - cl(A)$ if and only if for each G$\in$ $\textit{strongly } G^*SO(X,\tau)$ containing x, G$\bigcap$A$\neq \phi$.

Proof. The necessity. Let $x \notin \textit{strongly } g^*s - cl\,(A)$. Then, $x \in X - \textit{strongly } g^*s - cl(A)$, hence by Lemma 5.1, $x \in \textit{strongly } g^*s - \text{int}(X - A)$. Then, by Lemma 5.2, there exists a strongly g^*s-open set G such that $x \in G \subseteq X - A$. So, $A \bigcap G = \phi$.

The sufficiency. Assume that there exists a strongly g^*s- open set G containing x such that A $\bigcap$ G=ϕ. Then, A $\subseteq$ X −G which is a strongly g^*s-closed set. Therefore, by Proposition 5.1, $A \subseteq \textit{strongly } g^*s - cl\,(A) \subseteq X - G$, but $x \notin X - G$. Hence, $x \notin \textit{strongly } g^*s - cl\,(A)$.

Remark 5.1. For a space (X,τ), if A$\subseteq$X, we have:

int(A)$\subseteq$s-int(A)$\subseteq$ $\textit{strongly } g^*s - \text{int}(A) \subseteq$ A$\subseteq$ $s - cl_*(A) \subseteq s - cl^*(A) \subseteq$ $\textit{strongly } g^*s - cl(A)$ $\subseteq$ s-cl(A) $\subseteq$ cl(A).

The converse of the above remark is not true as is shown by (Crossly and Hildebrand, 1971; Maki *et al.*, 1996) and the following example.

Example 5.1. Let X = {a,b,c,d} with topologies τ_1 = { X, ϕ, {c,d}}, τ_2 ={X, ϕ,{a},{b,c},{a,b,c}}.

(i) If A={b,c} of X on τ_1, then s-cl(A)=X but strongly g^*s-cl(A) = {a,b,c}. Therefore, s-cl(A) $\not\subset$ strongly g^*s-cl(A).

(ii) Also, if B= {b} of X on τ_2, then $s - cl^*(B)$=B but strongly g^*s-cl (B) = {b, c}. Therefore, strongly g^*s-cl (B) $\not\subset$ $s - cl^*(B)$.

(iii) Further, if C={c} of X on τ_1, then s-int (C)= ϕ but strongly- g^*s-int(C)=C. Therefore, strongly-g^*s-int(C) $\not\subset$ s-int (C).

Definition 5.2. Let (X,τ) be a space and A$\subseteq$X. Then, the strongly generalized star semi-boundary of A (briefly, $\textit{strongly } g^*s - b(A)$) is defined by

$$\textit{strongly } g^*s - b(A) = \textit{strongly } g^*s - cl\,(A) \bigcap \textit{strongly } g^*s - cl\,(X - A).$$

Theorem 5.1. In a space (X,τ), if A and B are two subsets of X, then the following statements are hold:

(i) $\textit{strongly } g^*s - b(A) = \textit{strongly } g^*s - b(X - A)$,

(ii) $strongly\ g^*s-b(A)=strongly\ g^*s-cl(A)-strongly\ g^*s-\text{int}(A)$,

(iii) $strongly\ g^*s-b(A)\bigcap strongly\ g^*s-\text{int}(A)=\phi$,

(iv) $strongly\ g^*s-b(A)\bigcup strongly\ g^*s-\text{int}(A)=strongly\ g^*s-cl(A)$.

Definition 5.3. If (X,τ) is a space and $A\subseteq X$, then the set $X-strongly\ g^*s-cl(A)$ is called the strongly generalized star semi-exterior of A and is denoted by $strongly\ g^*s-ext(A)$.

Each point $P\in X$ is called a strongly g^*s-exterior point of A, if it is strongly g^*s-interior point of X−A.

Theorem 5.2. For a space (X,τ) and A be a subset of X, the following statements are hold:

(i) $strongly\ g^*s-ext(A)=strongly\ g^*s-ext(X-A)$,

(ii) $strongly\ g^*s-ext(A)\bigcap strongly\ g^*s-b(A)=\phi$,

(iii) $strongly\ g^*s-ext(A)\bigcup strongly\ g^*s-b(A)=strongly\ g^*s-cl(X-A)$.

Proof. (i) By Definition 5.3, $strongly\ g^*s-ext(A)=X-strongly\ g^*s-cl(A)$,hence by Lemma 5.1, $strongly\ g^*s-ext(A)=strongly\ g^*s-\text{int}(X-A)$.

(ii) Obvious.

Since, $strongly\ g^*s-ext(A)\bigcup strongly\ g^*s-b(A)=$

$strongly\ g^*s-\text{int}(X-A)\bigcup strongly\ g^*s-b(X-A)=strongly\ g^*s-cl(X-A)$.

STRONGLY SEMI STAR $T_{1/2}$ SPACES

Definition 6.1. A topological space X is said to be:

(1) strongly semi -star-$T_{1/2}$ (briefly, $st.semi^*-T_{1/2}$) if every strongly g^*s -closed set in X is semi -closed,

(2) strongly semi- star-T_p (briefly, $st.semi^*-T_p$) if every strongly g^*s -closed set in X is closed.

Example 6.1. Let $X=\{a,b,c\}$ with topologies $\tau_1=\{X,\phi,\{a\}\}$ and $\tau_2=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$. Then (X,τ_1) is $st.semi^*-T_{1/2}$ and (X,τ_2) is $st.semi^*-T_p$.

Lemma 6.1. For a space (X,τ), the following are hold:

(1) Every $st.semi^*-T_p$ (resp. semi-T_p, semi-T_b, T_b) space is $st.semi^*-T_{1/2}$.

(2) Every T_b- space is $st.semi^*-T_p$.

Proof. (1) Let A be a strongly g^*s-closed subset of X. Since, (X,τ) is $st.semi^*-T_p$, then A is closed. Therefore, A is semi-closed. Hence (X,τ) is $st.semi^*-T_{1/2}$.

(2) Let A be a strongly g^*s-closed subset of X. Then, A is gs-closed. Since, (X,τ) is T_b, then A is closed. Hence, (X,τ) is $st.semi^*-T_p$.

The converses of the above theorem need not be true as may be seen by the following example.

Example 6.2. Let $X=\{a,b,c\}$ with the following topologies:

(1) $\tau_2 = \{X, \phi, \{a\}, \{a,b\}\}$,

(2) $\tau_3 = \{X, \phi, \{a,b\}, \{c\}\}$.

Then, from Example 6.1, (X,τ_1) is a $st.semi^* - T_{\frac{1}{2}}$ space but it is not $st.semi^* - T_p$ (resp. semi-T_p , T_b). Further, (X,τ_2) is $st.semi^* - T_{\frac{1}{2}}$ but it is not semi-T_b. Also, (X,τ_3) is a $st.semi^* - T_p$ - space but it is not T_b .

Remark 6.1. (1) st. semi- $T_{\frac{1}{2}}$ and $st.semi^* - T_p$ spaces are independent,

(2) semi – $T_{\frac{1}{2}}$ and $st.semi^* - T_p$ spaces are independent,

(3) $st.semi^* - T_{\frac{1}{2}}$ and T_d spaces are independent,

(4) st. semi- $T_{\frac{1}{2}}$ and $st.semi^* - T_{\frac{1}{2}}$ spaces are independent.

Example 6.3 Let $X = \{a,b,c\}$ with the following topologies:

(1) $\tau_4 = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$,

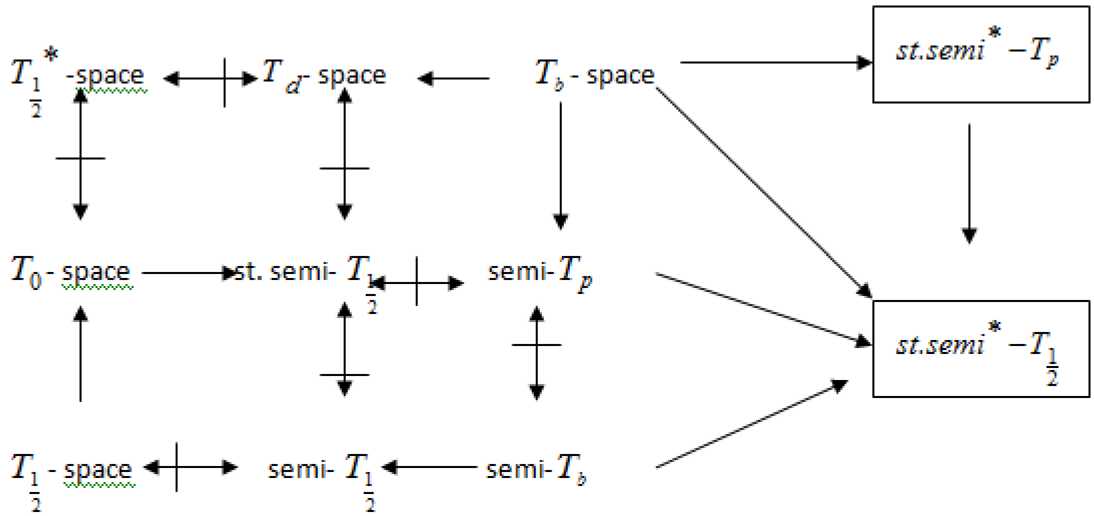
(2) $\tau_5 = \{X, \phi, \{a,b\}\}$. Then (1) From Example 6.2, (X,τ_3) is a $st.semi^* - T_p$ space but it is not st. semi- $T_{\frac{1}{2}}$ and (X,τ_2) is st. semi- $T_{\frac{1}{2}}$ but it is not $st.semi^* - T_p$.

(2) Also, from Example 6.2, (X,τ_3) is $st.semi^* - T_p$ but it is not semi $-T_{\frac{1}{2}}$ and (X,τ_4) is semi – $T_{\frac{1}{2}}$ but it is not $st.semi^* - T_p$.

(3) Further, from Example 6.2, (X,τ_2) is $st.semi^* - T_{\frac{1}{2}}$ but it is not T_d and (X,τ_5) is T_d but it is not $st.semi^* - T_{\frac{1}{2}}$.

(4) Furthermore, from Example 6.2, (X,τ_3) is $st.semi^* - T_{\frac{1}{2}}$ but it is not st. semi- $T_{\frac{1}{2}}$ and (X,τ_5) is st. semi- $T_{\frac{1}{2}}$ but it is not $st.semi^* - T_{\frac{1}{2}}$.

Remark 6.2. We can summarize the following diagram by using (Devi, 1994; El-Maghrabi and Nasef, 2008; Levine, 1970; Veera Kumar, 2001) and the above results.



Theorem 6.1. For a topological space (X,τ), the following statements are equivalent:

(i) (X,τ) is $st.semi^{*}-T_{1/2}$,

(ii) Every singleton of X is either gs-closed or semi -open .

Proof .(i)→(ii) . Let x ∈ X and {x} be not gs- closed. Then by Theorem 3.3, $X-\{x\}$ is strongly $g^{*}s$ -closed. Hence by hypothesis, $X-\{x\}$ is semi - closed. Therefore, {x} is semi - open.

(ii)→(i). Let A ⊆ X be a strongly $g^{*}s$ -closed set and x ∈ $s-cl(A)$, we need to show that $x\in A$. For consider the following two cases:

case (1) . The set {x} is gs- closed. Then, if x ∉ A, there exists a gs- closed set in $(s-cl(A))-A$. So, by Corollary 3.1, x ∈ A.

case (2) . The set {x} is semi - open. Since, x ∈ s-cl (A), then $\{x\}\cap A\neq\phi$. Thus, $x\in A$. Hence, in both cases , x ∈ A. This show that s-cl (A) ⊆ A. So, A is semi -closed.

Theorem 6.2. For a topological space (X,τ) , the following statements are equivalent:

(i) (X,τ) is $st.semi^{*}-T_{1/2}$,

(ii) Every residual singleton of X is gs- closed.

(iii)

Proof. (i) → (ii). By Theorem 6.1, every singleton set of X is semi - open or gs- closed and since , a non void residual set can not be semi - open at the same time ,then every residual singleton set of X is gs-closed.

(ii) → (i). Since, every singleton set is either semi-open or residual, then by hypothesis, every singleton set of X is either semi-open or gs- closed. Hence by Theorem 6.1, X is $st.semi^{*}-T_{1/2}$.

Theorem 6.3. A space (X,τ) is $st.semi^{*}-T_{1/2}$ if and only if $SO(X,\tau)=strongly\ G^{*}SO(X,\tau)$.

Proof. Necessity. Let (X,τ) be a $st.semi^{*}-T_{1/2}$ space and $A\in strongly\ G^{*}SO(X,\tau)$. Then, $X-A$ is strongly $g^{*}s$ -closed. Hence by hypothesis, $X-A$ is semi -closed. Hence, A ∈ $SO(X,\tau)$.

Therefore, $strongly\ \ G^*SO\ (X,\tau) \subseteq SO(X,\tau)$. Also, By Remark 4.2, $SO(X,\tau) = strongly\ \ G^*SO(X,\tau)$.

Sufficiency. Let $SO(X,\tau) = strongly\ \ G^*SO(X,\tau)$ and A be a strongly g^*s-closed set. Then $X - A \in strongly\ \ G^*SO(X,\tau)$, hence by hypothesis, $X - A \in SO(X,\tau)$. Therefore, A is semi-closed.

Corollary 6.1. A space (X,τ) is $st.semi^* - T_p$ if and only if $\tau = strongly\ \ G^*SO(X,\tau)$.

Definition 6.2. A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be:

(1) strongly generalized star semi -continuous (briefly , strongly g^*s- continuous) if $f^{-1}(V)$ is strongly g^*s-closed in (X,τ), for every closed set V of (Y,σ),

(2) strongly generalized star semi-irresolute (briefly, strongly g^*s-irresolute) if $f^{-1}(V)$ is strongly g^*s- closed in (X,τ), for every strongly g^*s-closed V of (Y,σ).

Theorem 6.4. Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be strongly g^*s- continuous. Then, f is semi - continuous, if (X,τ) is $st.semi^* - T_{1/2}$.

Proof. Let V$\subseteq$Y be a closed set. Then f^{-1}(V) is strongly g^*s- closed in X. But (X,τ) is $st.semi^* - T_{1/2}$, hence, f^{-1}(V) is semi - closed. Therefore, f is semi-continuous.

Theorem 6.5. Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be strongly g^*s-irresolute . Then, f is irresolute, if (X,τ) is $st.semi^* - T_{1/2}$.

Proof. Obvious.

Theorem 6.6. Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be onto , pre semi - closed and strongly g^*s-irresolute mappings . Then (Y,σ) is $st.semi^* - T_{1/2}$, if (X,τ) is $st.semi^* - T_{1/2}$.

Proof. Let V $\subseteq$Y be a strongly g^*s-closed set. Then f^{-1}(V) is strongly g^*s-closed in X. But (X,τ) is $st.semi^* - T_{1/2}$, hence f^{-1}(V) is semi-closed. Therefore $f(f^{-1}(V)) = V$ is semi - closed in Y.

Corollary 6.2. Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be onto, closed and strongly g^*s-irresolute mappings. Then (Y,σ) is $st.semi^* - T_p$, if (X,τ) is $st.semi^* - T_p$.

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