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Research Article

COMPLEX DYNAMICS OF CONTINUOUS-TIME AUTONOMOUS CHAOTIC SYSTEM

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ABSTRACT
This paper reports a continuous-time autonomous dynamical system. Some basic dynamical properties, such as Lyapunov exponents, Poincare mapping, fractal dimension and chaotic behavior of this system are studies. Furthermore, we will investigate an external force that is applied on Stretch-Twist-Fold Flow and analyze the behavior of control parameter with theoretical details.

Key Words: *Stretch-Twist-Fold Flow, Jacobian Matrix, Lyapunov Exponent, Poincare Mapping, Fractal Dimension, Control Parameter*

INTRODUCTION
The distinction between slow and fast dynamos was first drawn by Vainshtein & Zeldovich (1972) in this research; we describe the stretch-twist-fold (STF) fast dynamo, which is the archetype of the elementary models of the process. Basically, stretch-twist-fold is applied in fluid mechanics in aerospace. In space, any fluid can be de tracked easily so a magnetic field is required to compel the fluid to be in the same orbit and this method is called STF system. A modified chaotic system is proposed in this paper. It is a three-dimensional autonomous system which relies on two multipliers and one quadratic term to introduce the nonlinearity necessary for folding trajectories. In this paper, we will investigate an external force that is applied on this system. Jacobian matrix will be able to show saddle point and saddle-focus point. The Lyapunov exponent shows the stable and unstable position of this system. Furthermore, we will describe the fractal dimension and add the one controlling parameter that shows the chaotic behavior of this system.

STRETCH-TWIST-FOLD-FLOW (STF)
The STF flow is defined as (Bao and Yang, 2011):

$$\dot{x}(t) = \alpha z - 8xy,$$
$$\dot{y}(t) = 11x^2 + 3y^2 + z^2 + \beta xz - 3,$$
$$\dot{z}(t) = -\alpha x + 2yz - \beta xy,$$

Where $\alpha = 0.1, \beta = 1$ are positive real parameters and related to the ratios of intensities of the stretch, twist and fold ingredients of the flow. If we apply an external force acos (30^0) then the system will be define as:

$$\dot{x}(t) = \alpha z - 8xy + a\cos(30),$$
$$\dot{y}(t) = 11x^2 + 3y^2 + z^2 + \beta xz - 3,$$
$$\dot{z}(t) = -\alpha x + 2yz - \beta xy,$$

First, we will discuss equilibria of this nonlinear system.
Let

$$\alpha z - 8xy + a\cos(30) = 0$$
$$11x^2 + 3y^2 + z^2 + \beta xz - 3 = 0$$
$$-\alpha x + 2yz - \beta xy = 0$$

The system has three equlibria, which are respectively described as follows:

0(0, 0, 0), $E_1(x_1, y_1, z_1)$, $E_2(x_2, y_2, z_2)$

We operate above these nonlinear algebraic equations and obtain

0(0, 0, 0), E_1 (0.087, 1, 0), E_2 (-0.087,-1, 0)

For equilibrium 0(0, 0, 0), system (2) are linearized, Jacobian matrix is defined as

$$J_0 = \begin{bmatrix} -8y & -8x & \alpha \\ -22x - \beta z & 6y & -2z - \beta x \\ \alpha + \beta y & \beta x - 2z & 2y \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{bmatrix}$$

To gain its eigenvalues, we let

$$|\lambda I - J_0| = 0$$

These eigenvalues that corresponding to equilibrium 0(0, 0, 0) are respectively obtained as follows:

$$\lambda_1 = -0.1, \lambda_2 = 0, \lambda_3 = 0.1$$

Here λ_3 is a positive real number, λ_1 is a negative real number. So, this equilibria point 0(0, 0, 0) is stable.

Next, linearizing the system (2) about the other equilibria such as E_1, E_2 yields the following characteristic operation.

For equilibrium points E_1, has a Jacobian matrix equal to

$$J_1 = \begin{bmatrix} -8 & -0.696 & 0.1 \\ -1.914 & 6 & -0.087 \\ 1.1 & 0.087 & 2 \end{bmatrix}$$

We let

$$|\lambda I - J_1| = 0$$

These eigenvalues corresponding to the equilibrium point $E_1(x_1, y_1, z_1)$ are

$$\lambda_1 = -8.1050, \lambda_2 = 6.0937, \lambda_3 = 2.0113$$

Here λ_1 is a negative real number, λ_2 and λ_3 are two positive real number.

The equilibria point E_1 is a saddle-focus point; this equilibrium point is unstable.

For equilibria point E_2:

$$J_2 = \begin{bmatrix} 8 & 0.696 & 0.1 \\ 1.914 & -6 & 0.087 \\ -0.9 & -0.087 & -2 \end{bmatrix}$$

We let

$$|\lambda I - J_2| = 0$$

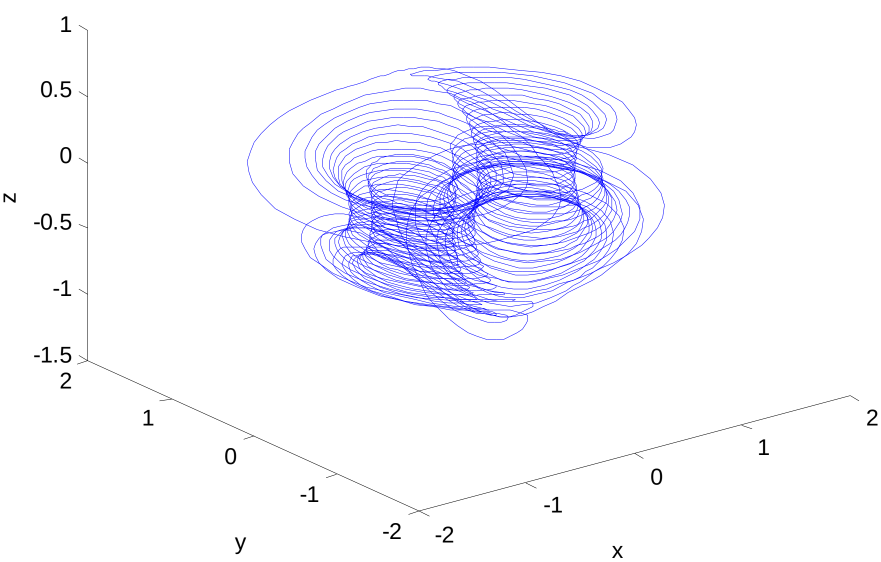
These eigenvalues of E_2 are

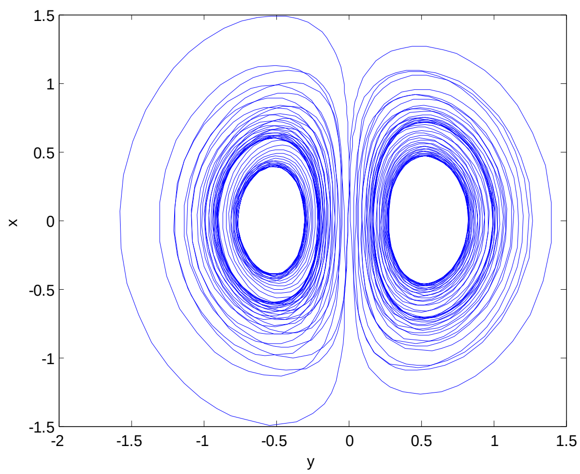
$$\lambda_1 = 8.0851, \lambda_2 = -6.0938, \lambda_3 = -1.9914$$

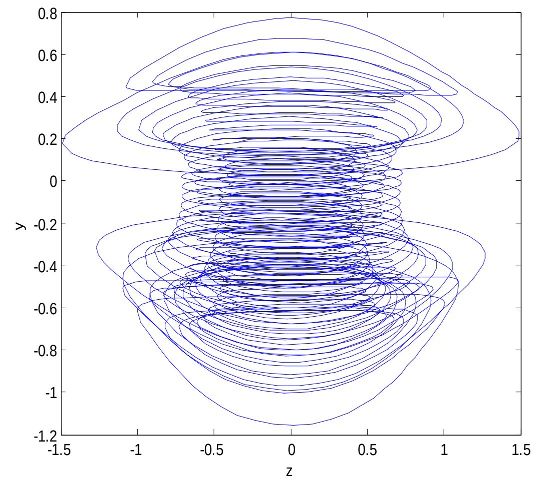
Result show that λ_1 is a positive real number, λ_2 and λ_3 are two negative numbers.

The equilibria point E_2 is also a saddle-focus point, this equilibria point is unstable.

The above brief analyses show that the three equilibrium points of the nonlinear system are two saddle focus-nodes. Using MATLAB program, the numerical simulation have been completed. The nonlinear system is shown in figure 1, 2 and 3.







In dynamical system (2), a volume element V_0 is apparently contracted by the flow into a volume element $V_0 e^{pt}$ in time t. It means that each volume containing the trajectory of this dynamical system shrinks to zero as $t \to \infty$. So, all this dynamical system orbits are eventually confined to a specific subset that has zero volume (Lu *et al.*, 2002).

. As is well known, the Lyapunov exponents measure the exponential rates of divergence or convergence of nearby trajectories in phase space, according to the detailed numerical as well as theoretical analysis, the largest value of positive Lyapunov exponents of this chaotic system is obtained as

$$\lambda_{L1} = 8.09$$

It is related to the expanding nature of different direction in phase space

Another one Lyapunov exponent is

$$\lambda_{L2} = 0$$

It is related to the critical nature between the expanding and the contracting nature of different direction in phase space while negative Lyapunov exponent is

$$\lambda_{L3} = -8.11$$

It is related to the contracting nature of different direction in phase space.

The Lyapunov dimension of this nonlinear system, it is described as

$$D_L = j + \frac{\sum_{i=1}^{j} \lambda_{Li}}{|\lambda_{Lj+1}|} = 2 + \frac{(\lambda_{L1} + \lambda_{L2})}{|\lambda_{L3}|} = 2 + \frac{0.4412 + 0}{|-3.9418|} = 2.112$$

The fractal nature of an attractor does not merely imply non-periodic orbits, it also causes nearby trajectories to diverge. They soon diverge and follow totally different paths in this system (Wolf *et al.*, 1985). Therefore, there is really chaos in this nonlinear system.

CONTROLLER PARAMETER

In this section we will add controller parameter in z-dimension. The autonomous differential equations of its controlled system are expressed as

$$\dot{x}(t) = \alpha z - 8xy,$$
$$\dot{y}(t) = 11x^2 + 3y^2 + z^2 + \beta xz - 3,$$
$$\dot{z}(t) = -\alpha x + 2yz - \beta xy + e,$$

In this system, e is the parameter of control, the value of it can be changed within a certain range.
When the parameter e is changed, the chaos behavior of this system can effectively be controlled. So it is a controller.

Let $e = 0.01,$

The system evolves into partial but is still bounded in this time.

Let $e = 0.02,$

The system evolves into doubling bifurcations.

Let $e = 0.03,$

The system evolves into partial but still bounded in this time.

Let $e = 0.04,$

The system evolves into a period-doubling bifurcation.

In the controller, one can see when $|e|$ large enough, chaos is going to disappear; when $|e|$ is small enough, a complete chaos going to appears. So $|e|$ is an important parameter to control chaos in the nonlinear-system (Chong *et al.*, 2004; and Jinhu *et al.*, 2002).

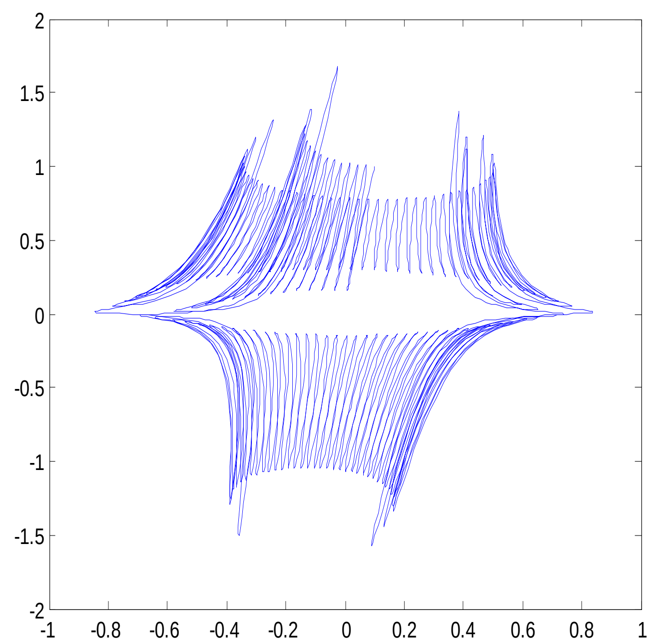


Figure 1: x,z-direction ($e = 0.01,$)

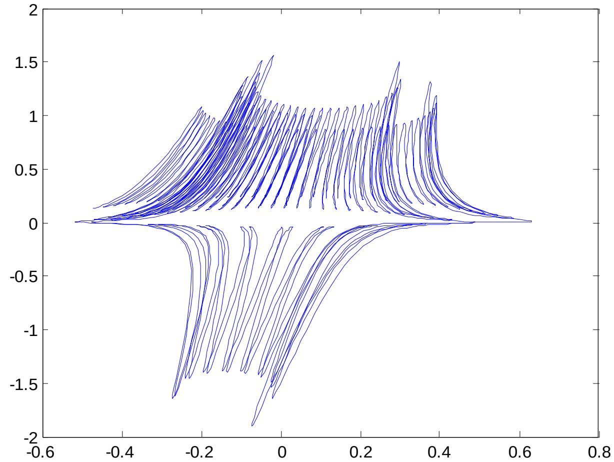


Figure 2: x,z-direction ($e = 0.02,$)

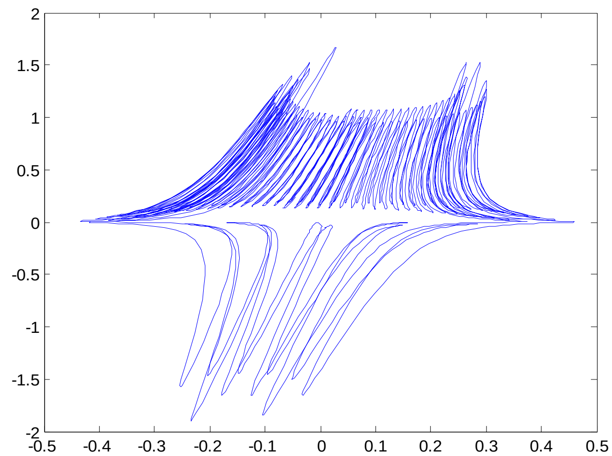


Figure 3: x, z-direction ($e = 0.03,$)

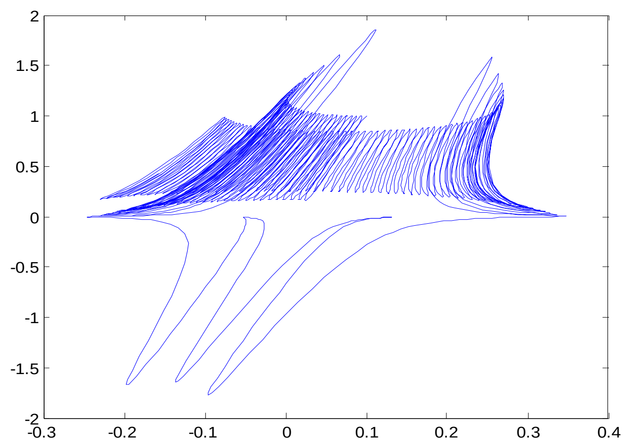


Figure 4: x, z-direction ($e = 0.04,$)

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CONCLUSION

In this system, there are abundant and complex dynamical behaviors. We successfully applied an external force on this dynamical system that shown the stability and unstability by using Jacobian matrix. Control parameter is able to show the chaos in this nonlinear system. This dynamical system and their forming mechanism need further to study and explore. Their topological structure should be completely and thoroughly investigated, it is expecting that more detailed theory analysis and simulation investigation will be provided elsewhere a great deal of achievements will be obtained in the near future.

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