International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2012 Vol. 2 (3) July-September, pp.92-101/Prakash and Nithya

Research Article

AN ENTROPIC FUZZY ECONOMIC ORDER QUANTITY FOR ITEMS WITH IMPROPER QUALITY USING FUZZY PARAMETERS

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ABSTRACT
This paper presents an entropic fuzzy economic order quantity with imperfect quality items in a fuzzy situation by employing the type of fuzzy numbers which are triangular. The objective is to determine the optimal order lotsize to maximize the total profit. We first propose a model with fuzzy annual demand is presented. For this case, we employ the signed distance, a ranking method for fuzzy numbers, to find the estimate of total profit per unit time in the fuzzy sense, and then derive the corresponding optimal lot size. Numerical examples are provided to illustrate the results of proposed models.

Key Words: Inventory, Fuzzy set, EOQ, imperfect quality, signed distance, Entropy.

INTRODUCTION

The economic order model is perhaps the most celebrated one in the inventory management literature. It has been widely used and accepted by practitioners and academicians alike. Despite its popularity it has been criticized. Sometimestrongly, because its assumption are too simplistic and subsequently its results are far from reality. Jamal *et al.,* (2009) developed models for the optimal production lotsize problem with provision for rework for defective items. Two policies were considered i) defective items generated in a given cycle are reworked in that cycle, and ii) defective items are accumulated and reworked after several production cycle. Cardenas Barron (2009) corrected solutions to two numerical examples presented by Jamal *et al.,* and later Cardenas Barron (2009) presented an algebraic procedure to determine the optimal solution for two inventory policies that were proposed by Jamal et al. Sarker *et al.,* (2009) extended the single stage model of Jamal *et al.,* (2009) to consider multiple stages, where some of their mathematical expression were corrected by Cardenas Barron (2009) who also presented a closed form expression for the total inventory cost function of Sarker et al. In a later paper Cardenas Barron developed an EPQ inventory model with planned backorders for a single-stage manufacturing process with defective items that are reworked in the same cycle in which they were generated. This paper revisits a corrected version of the work of Salameh and Jaber (2000) i.e.of Maddah and Jaber (2000), and models it using the thermodynamic approach suggested by Jaber (2000).

In recent years, several researchers have applied the fuzzy sets theory and technique to develop and solve the production inventory problems. For example Park (1989) and Vujosevic *et al.,* (1996) extended the classical EOQ model by introducing the fuzziness of ordering cost and holding cost. Chen and Wang (1996) fuzzified the demand, ordering cost, and backorder cost into trapezoidal fuzzy number sin EOQ model with backorder. Roy and Maiti presented a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Chang *et al.,* (1998) presented a fuzzy model for inventory with backorder where the backorder quantity was fuzzified as the Triangular Fuzzy Number. Yao *et al.,* (2000) proposed the EOQ model in the fuzzy sense, where both order quantity and total demand were fuzzified as the triangular fuzzy numbers. Ouyang and Yao (2002) presented a mixture inventory model with variable lead-time, where the annual demand was fuzzified as a triangular fuzzy number and as the statistic fuzzy number.

In section 2, some basic concepts of fuzzy sets, fuzzy numbers and signed distance method are introduced. In section 3, an entropic fuzzy EOQ for items with imperfect quality is discussed. Section 4 provides numerical examples to illustrate the results of the proposed models.

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2. PRELIMINARIES

Before presenting the fuzzy inventory models, we introduce some definitions and properties about fuzzy numbers with relevant operations.

Definition 1. For $0 \le \alpha \le 1$, the fuzzy set $\tilde{a}_z$ defined on $R = (-\infty, \infty)$ is called an α-level fuzzy point if the membership function of $\tilde{a}_z$ is given by

$$\mu_{\tilde{a}_z}(x) = \begin{cases} \alpha, & x = a, \\ 0, & x \neq a. \end{cases} \quad \ldots (2.1)$$

Definition 2. The fuzzy set $\tilde{A} = (a, b, c)$, where $a < b < c$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a)/(b - a), & a \le x \le b, \\ (c - x)/(c - b), & b \le x \le c, \\ 0, & \text{otherwise}. \end{cases} \quad \ldots (2.2)$$

Remark 1.

(i) When $\alpha = 1$, the membership function of the 1-level fuzzy point $\tilde{a}_1$ becomes the characteristic function, ie., $\mu_{\tilde{a}_1}(x) = 1$ if $x = a$ and $\mu_{\tilde{a}_1}(x) = 0$ if $x \neq a$. In this case, the real number $a \in R$ is the same as the fuzzy point $\tilde{a}_1$ except for their representations.

(ii) If $c = b = a$, then the triangular fuzzy number $\tilde{A} = (a, b, c)$ is identical to the 1-level fuzzy point $\tilde{a}_1$.

Definition 3. For $0 \le \alpha \le 1$, the fuzzy set (a_α, b_α) defined on R is called an α-level fuzzy interval if the membership function of (a_α, b_α) is given by

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0, & \text{otherwise}. \end{cases} \quad \ldots (2.3)$$

Definition 4. Let $\tilde{B}$ be a fuzzy set on R, and $0 \le \alpha \le 1$. The α-cut B(a) $\tilde{B}$ consists of points x such that $\mu_{\tilde{B}}(x) \ge \alpha$, that is, $B(\alpha) = \{x | \mu_{\tilde{A}}(x) \ge \alpha\}$.

Decomposition Principle. Let $\tilde{B}$ be a fuzzy set on R, and $0 \le \alpha \le 1$. Suppose the α-cut of $\tilde{B}$ to be closed interval $(B_L(\alpha), B_U(\alpha))$, that is, $B(\alpha) = (B_L(\alpha), B_U(\alpha))$. Then, we have (see, (16))

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \alpha B(\alpha) \quad \ldots (2.4)$$

or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \le \alpha \le 1} \alpha C_{B(\alpha)}(x) \quad \ldots (2.5)$$

where

(i) $\alpha B(\alpha)$ is a fuzzy set with membership function

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0, & \text{otherwise}. \end{cases}$$

(ii) $C_{B(\alpha)}(x)$ is a characteristic function $B(\alpha)$, that is

$$C_{B(\alpha)}(x) = \begin{cases} 1, & x \in B(\alpha) \\ 0, & x \notin B(\alpha). \end{cases}$$

Remark 2. From the Decomposition Principle and (5), we obtain

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} \alpha B(\alpha) = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \qquad \text{. . . (2.6)}$$

or

$$\mu_{\tilde{B}}(x) = \underset{0 \le \alpha \le 1}{V} \alpha C_{B(\alpha)}(x) = \mu_{\tilde{B}}(x) = \underset{0 \le \alpha \le 1}{V} \mu_{[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]}(x) \qquad \text{. . . (2.7)}$$

For any a, b, c, d, k $\in$ R, a < b and c < d the interval operations are as follows (16) :

(i) (a, b) (+) (c, d) = (a + c, b + d)

(ii) (a, b) (-) (c, d) = (a - c, b - d) . . . (2.8)

(iii) $k\ (\cdot)\ (a, b) = \begin{cases} [ka, kb], & k > 0, \\ [kb, ka], & k < 0, \end{cases}$

further, for a > 0 anc d > 0.

(iv) (a, b) (·) (c, d) = (ac, bd)

(v) $(a, b)\ (\div)\ (c, d) = \left[\frac{a}{d}, \frac{b}{c}\right]$

Next, as in Yao and Wu (15), we introduce the concept of the signed distance of fuzzy set. We first consider the signed distance on R.

Definition 5. For any a and 0 $\in$ R, define the signed distance from a to 0 as $d_0(a, 0) = a$. If a > 0, the distance from a to 0 is $a = d_0(a, 0)$; if a < 0, the distance from a to 0 is $-a = -d_0(a, 0)$. Hence, $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R with which the α-cut $B(\alpha) = [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ exists for every $\alpha \in (0, 1)$, and both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous function on $\alpha \in (0, 1)$. Then, for any $\tilde{B} \in \Omega$, from (8) we have

$$\tilde{B} = \bigcup_{0 \le \alpha \le 1} [B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \qquad \text{. . . (2.9)}$$

From Definition 5, the signed distance of two end points, $B_L(\alpha)$ and $B_U(\alpha)$, of the α-cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ of $\tilde{B}$ to the origin 0 us $d_0(B_L(\alpha), 0) = B_L(\alpha)$ and $d_0(B_U(\alpha), 0) = B_U(\alpha)$, respectively. The average, $\frac{[B_L(\alpha) + B_U(\alpha)]}{2}$.

In addition, for every $\alpha \in (0, 1)$, there is a one-to-one mapping between the α-level fuzzy interval $[B_L(\alpha), B_U(\alpha)]$ and the real interval $[B_L(\alpha), B_U(\alpha)]$, that is, the following correspondence one-to-one mapping.

$$[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha] \leftrightarrow [B_L(\alpha), B_U(\alpha)] \qquad \text{. . . (2.10)}$$

Also, the l-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0. Hence, the signed distance of $[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$ to $\tilde{0}_1$ can be defined as

$$d\left([B_L(\alpha)_\alpha, B_U(\alpha)_\alpha], \tilde{0}_1\right) = d\left([B_L(\alpha), B_U(\alpha)], 0\right) = \frac{(B_L(\alpha), B_U(\alpha))}{2}.$$

Moreover, $\tilde{B} \in \Omega$, since the above function is continuous on $0 \le \alpha \le 1$, we can use the integration to obtain the mean value of the signed distance as follows :

$$\int_0^1 d\left(\left[B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha\right],\ \tilde{0}_1\right) dx \ = \ \frac{1}{2}\int_0^1 \left(B_L(\alpha) + B_U(\alpha)\right) dx \qquad \text{. . . (2.11)}$$

Then, from (11) and (13) we have the following definition.

Definition 6. For $\tilde{B} \in \Omega$, define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ (ie. y-axis) as

$$d\left(\tilde{B}, \tilde{0}_1\right) = \int_0^1 d\left(\left[B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha\right],\ \tilde{0}_1\right) dx \ = \ \frac{1}{2}\int_0^1 \left(B_L(\alpha) + B_U(\alpha)\right) dx \qquad \text{. . . (2.12)}$$

According to Definition 6, we obtain the following property.

Property 1. For the triangular fuzzy number $\tilde{A}$ = (a, b, c), the α-cut of $\tilde{A}$ is $\left[A_L(\alpha),\ A_U(\alpha)\right], \alpha \in (0, 1)$, where $A_L(\alpha) = a + (b - a)\alpha$ and $A_U(\alpha) = c - (c - b)\alpha$. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is

$$d\left(\tilde{A}, \tilde{0}_1\right) \ = \ \frac{1}{4}(a + 2b + c) \qquad \text{. . . (2.13)}$$

Furthermore, for two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$, where $\tilde{B} = \bigcup_{0 \le \alpha \le 1} \left[B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha\right]$ and $\tilde{G} = \bigcup_{0 \le \alpha \le 1} \left[G_L(\alpha)_\alpha,\ G_U(\alpha)_\alpha\right]$, and $k \in R$, using (10) and (12), we have

(i) $\quad \tilde{B}\,(+)\,\tilde{G} = \bigcup_{0 \le \alpha \le 1} \left[B_L(\alpha)_\alpha + G_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha + G_U(\alpha)_\alpha\right]$

(ii) $\quad \tilde{B}\,(-)\,\tilde{G} = \bigcup_{0 \le \alpha \le 1} \left[B_L(\alpha)_\alpha - G_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha - G_U(\alpha)_\alpha\right]$ . . . (2.14)

(iii) $$\tilde{k}_1\ (-)\ \tilde{B} = \begin{cases} \bigcup_{0 \le \alpha \le 1} \left[kB_L(\alpha)_\alpha,\ kB_U(\alpha)_\alpha\right], & k > 0, \\ \bigcup_{0 \le \alpha \le 1} \left[kB_U(\alpha)_\alpha,\ kB_L(\alpha)_\alpha\right], & k > 0, \\ \tilde{0}_1, & k = 0. \end{cases}$$

From the above and Definition 6, we obtain the following property.

Property 2. For two fuzzy sets $\tilde{B}, \tilde{G} \in \Omega$ and $k \in R$,

$$d\left(\tilde{B}, \tilde{0}_1\right) = \int_0^1 d\left(\left[B_L(\alpha)_\alpha,\ B_U(\alpha)_\alpha\right],\ \tilde{0}_1\right) dx \ = \ \frac{1}{2}\int_0^1 \left(B_L(\alpha) + B_U(\alpha)\right) dx.$$

(i) $\quad d\left(\tilde{B}\,(+)\,\tilde{G}, \tilde{0}_1\right) = d\left(\tilde{B}, \tilde{0}_1\right) + d\left(\tilde{G}, \tilde{0}_1\right)$

(ii) $\quad d\left(\tilde{B}\,(-)\,\tilde{G}, \tilde{0}_1\right) = d\left(\tilde{B}, \tilde{0}_1\right) - d\left(\tilde{G}, \tilde{0}_1\right)$ . . . (2.15)

(iii) $\quad d\left(\tilde{k}_1\ (\square)\ \tilde{B}, \tilde{0}_1\right) = kd\left(\tilde{B}, \tilde{0}_1\right)$

Notations

P is selling price of a good quality item ($/unit).

C_s is the salvage price of an imperfect quality ($/unit).
h is the holding cost per unit per unit of time (($/unit/year).
y is the lotsize quantity (units/lot).
d demand rate (unit/year).
x is the screening rate (unit/year) where $x > d$.
C is the unit purchase price ($/unit) $C > C_s$.
γ is the unit inspection / screening cost ($/unit).
A is the order / setup cost (($/lot).
ρ is a random variable, which follows uniform distribution and represents the percentage of imperfect quality items in a lot of size y and it is bound as $0 < a < \rho < b < 1$ and
T cycle time, where $T = (1 - \rho)y/d$ and its expected value of is $E(T) = (1 - E(P))y/d$.
The additional notations that will be used in developing the entropic version of the model of Salameh and Jaber (2000) and subsequently Maddah and Jaber (2000), are
P_0 is selling price of a good quality item of a competitor brand in the market ($/unit) where $P_0 > P$
g_0 is quality index of an item of a competitor brand (1/unit) where $0 < g_0 < 1$.
g is quality index of an item (1/unit) where $0 < g < 1$.
d(p,g) demand rate (unit/year) as a function of p and g, where $p/g > P_0 / g_0$ and
K elasticity (unit/year/$) if a demand function d(p,g) and C_{inv} is the investment required to increase quality g by % (p/%/year).

The model of Salameh and Jaber (2000)
Salameh and Jaber (2000) modified the EOQ/EPQ model by assuming that each shipment contains a random percentage of imperfect quality items. Upon replenishing the inventory instantaneously the received lot is subject to a 100% screening process. Items that do not confirm to quality are withdrawn from the inventory and sold at a discounted price as a single batch, this perhaps why Salameh and Jaber (2000) used the term "imperfect quality" rather that "defective" items. The cycle is then duplicated indefinitely. The other assumptions remain to be the same as for the EOQ/EPQ model which are constant demand rate, no shortages, zero-lead time, etc.

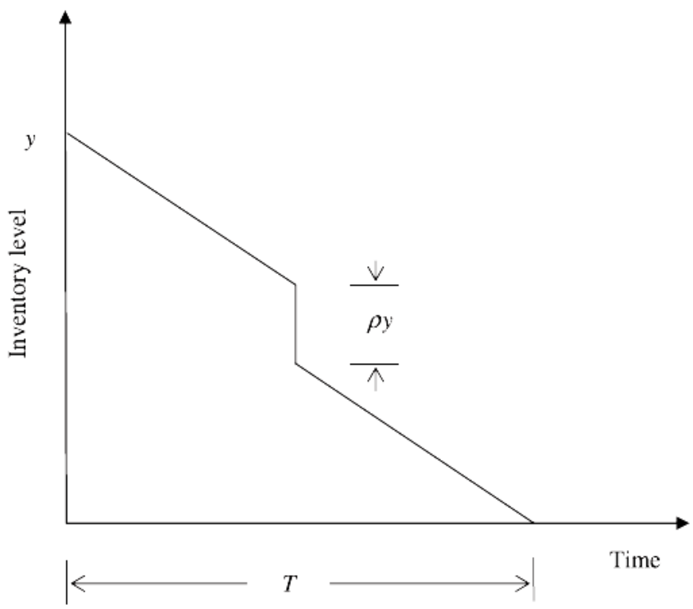

Fig. 1. The behaviour of inventory over time [15].

The per unit time profit function of the model of Salameh and Jaber (2000) is given as :

$$TPU(y) = \frac{d}{y(1-\rho)} \times py(1-\rho) + C_s y\rho - A - Cy - \gamma y - h\left(\frac{y(1-\rho)}{2} + \frac{\rho y^2}{x}\right)$$ which Salameh and Jaber (2000) chose to rewrite after rearrange its term as

$$TPU(y) = d\left(p - C_s + \frac{hy}{x}\right) + d\left(C_s - \frac{hy}{x} - C - \gamma - \frac{A}{y}\right)\frac{1}{1-\rho} - \frac{hy(1-\rho)}{2} \quad \text{. . . (1)}$$

The optimal solution for equation (1) is

$$y^* = \sqrt{\frac{2Ad[1/(1-\rho)]}{h(1-\rho-2(d/x)(1-(1/(1-\rho))}} \quad \text{. . . (2)}$$

Equation (2) is the corrected version of that of Salameh and Jaber (2000), where the '2' in the denominator was missing from the term, which was later added by Cardenas Barron who also noted that this minor error did not affect the main idea and the remainder of the paper of Salameh and Jaber (2000). Goyal and Cardenas Barron provided a simplified and a very reasonable approximation of the model of Salameh and Jaber (2000). Finally Maddah and Jaber (2000) corrected a flaw in the work of Salameh and Jaber (2000) by applying the renewal theory, that is by dividing the profit per cycle by the cycle length. The per unit time profit function and the optimal lotsize are given respectively from Maddah and Jaber (2000) as

$$TPU(y) = d\,\frac{\left(p(1-\rho) + C_s p - C - \gamma\right) - \left(\frac{Ad}{\gamma} + hy\frac{(1-\rho)^2}{2} + hy\frac{d}{x}\rho\right)}{1-\rho}$$

$$y^* = \sqrt{\frac{2Ad}{h(1-\rho)^2 + 2h\rho(d/x)}} \quad \text{. . . (3)}$$

This approach of Maddah and Jaber (2000) will be used in this paper where a thermodynamic version of the model will be developed.

The Thermodynamic version of the model of Salameh and Jaber (2000)

In this section we modify the model in Equation (3) by replacing the constant d with d(p, g) and subtracting the entropy cost per unit of time expressions and the investment cost from Equation (3) which becomes

$$TPU_E(y) = \frac{d(p,g)\left\{p(1-\rho) + C_s\rho - \frac{A}{y} - C - \gamma\right\} - hy\left(\frac{(1-\rho)^2}{2} + \frac{\rho d(p,g)}{x}\right)}{1-\rho}$$

$$+ \frac{d(p,g)}{(1-\rho)y}\left[\frac{PP_0}{Pg_0 - P_0 g} + \frac{C_s C}{C_s g_0 - Cg_s}\right] - C_{inv}\frac{(g - g_0)}{g_0} \quad \text{. . . (4)}$$

$$y_E^* = \sqrt{\frac{2d(p,g)\left(A - \frac{PP_0}{Pg_0 - P_0 g} + \frac{C_s C}{C_s g_0 - Cg_s}\right)}{h\left[(1-\rho)^2\right] + 2h\frac{d(p,g)}{x}\rho}} \quad \text{. . . (5)}$$

3. AN ENTROPIC FUZZY ECONOMIC ORDER QUANTITY FOR ITEMS WITH IMPERFECT QUALITY

Model with fuzzy demand rate :

$$TPU_E(y) = \frac{d(p,g)\left\{p(1-\rho) + C_s\rho - \frac{A}{y} - C - \gamma\right\} - hy\left(\frac{(1-\rho)^2}{2} + \frac{\rho d(p,g)}{x}\right)}{1-\rho}$$

$$+\frac{d(p,g)}{(1-\rho)y}\left[\frac{PP_0}{Pg_0 - P_0g} + \frac{C_sC}{C_sg_0 - Cg_s}\right] - C_{inv}\frac{(g-g_0)}{g_0}$$

In this section, we modify the crisp demand rate d(p, g) in (4) by incorporating the fuzziness of the demand rate. Then, we defuzzify demand rate to be a triangular fuzzy number $\tilde{d}(p,g) = \left(d(p,g) - \Delta_1, d(p,g), d(p,g) + \Delta_2\right)$ where Δ_1 and Δ_2 are determined by decision makers and should satisfy the conditions $0 < \Delta_1 < d(p,g)$ and $0 < \Delta_2$.

$$T\tilde{P}U_E(y) = \frac{\tilde{d}(p,g)\left\{p(1-\rho) + C_s\rho - \frac{A}{y} - C - \gamma\right\} - hy\left(\frac{(1-\rho)^2}{2} + \frac{\rho\tilde{d}(p,g)}{x}\right)}{1-\rho}$$

$$+\frac{\tilde{d}(p,g)}{(1-\rho)y}\left[\frac{PP_0}{Pg_0 - P_0g} + \frac{C_sC}{C_sg_0 - Cg_s}\right] - C_{inv}\frac{(g-g_0)}{g_0}$$

Now we defuzzify $T\tilde{P}U_E(y)$ using the signed distance method. The signed distance of $TPU_E(y)$ to $\tilde{0}_1$ is given by

$$d\left(T\tilde{P}U_E(y), \tilde{0}_1\right) = \frac{d\left(\tilde{d}(p,g), \tilde{0}_1\right)\left\{p(1-\rho) + C_s\rho - \frac{A}{y} - C - \gamma\right\} - hy\left(\frac{(1-\rho)^2}{2} + \frac{\rho\tilde{d}(p,g)}{x}, \tilde{0}_1\right)}{1-\rho}$$

$$+\frac{d\left(\tilde{d}(p,g), \tilde{0}_1\right)}{(1-\rho)y}\left[\frac{PP_0}{Pg_0 - P_0g} + \frac{C_sC}{C_sg_0 - Cg_s}\right] - C_{inv}\frac{(g-g_0)}{g_0}$$

where $d\left(\tilde{d}(p,g), \tilde{0}_1\right)$ is measured as follows.

The signed distance of fuzzy number $\tilde{d}(p,g)$ to $\tilde{0}_1$ is

$$d\left(\tilde{d}(p,g), \tilde{0}_1\right) = \frac{1}{4}\left(d(p,g) - \Delta_1 + 2d(p,g) + d(p,g) + \Delta_2\right)$$

$$= d(p,g) + \frac{1}{4}\left(\Delta_2 - \Delta_1\right)$$

We have

$$TPU_E^*(y) = d\left(T\tilde{P}U_E(y), \tilde{0}_1\right)$$

$$= \frac{d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)\left\{p(1-\rho)+C_s\rho-\frac{A}{y}-C-\gamma\right\}-hy\frac{(1-\rho)^2}{2}+\frac{\rho(d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1))}{x}}{1-\rho}$$

$$+\frac{d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}{(1-\rho)y}\left[\frac{PP_0}{Pg_0-P_0g}+\frac{C_sC}{C_sg_0-Cg_s}\right]-C_{inv}\frac{(g-g_0)}{g_0} \qquad \ldots (6)$$

$TPU_E^*(y)$ is regarded as the estimate of the total profit per unit time in the fuzzy sense. The objective of the problem is to find he optimal lot size quantity y_E^*. Such that $TPU_E^*(y)$ has a maximum value. We take the first partial derivative of $TPU_E^*(y)$ with respect by

$$\frac{\partial\left(TPU_E^*(y)\right)}{\partial y} = \frac{d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)\frac{A}{y^2}-h\frac{(1-\rho)^2}{2}+\frac{\rho h}{x}d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}{1-\rho}$$

$$-\frac{d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}{(1-\rho)y^2}\left[\frac{PP_0}{Pg_0-P_0g}+\frac{C_sC}{C_sg_0-Cg_s}\right]$$

$\frac{\partial^2\left(TPU_E^*(y)\right)}{\partial y^2} < 0$ is $TPU_E^*(y)$ is concave in y and hence the maximum value of $TPU_E^*(y)$ will occur at the point satisfies $\frac{d\left(TPU^*(y)\right)}{dy} = 0$

$$\Rightarrow \quad \frac{1}{y^2}\frac{d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}{1-\rho}\left[A-\frac{PP_0}{Pg_0-P_0g}+\frac{C_sC}{C_sg_0-Cg_s}\right]$$

$$\frac{-h\frac{(1-\rho)^2}{2}+\frac{\rho h}{x}d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}{1-\rho} = 0$$

$$\frac{1}{y^2}2\left(d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)\right)\left[A-\frac{PP_0}{Pg_0-P_0g}+\frac{C_sC}{C_sg_0-Cg_s}\right]$$

$$= h(1-\rho)^2+\frac{2\rho h}{x}d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)$$

$$y^2 = \frac{2\left(d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)\right)\left[A-\frac{PP_0}{Pg_0-P_0g}+\frac{C_sC}{C_sg_0-Cg_s}\right]}{h(1-\rho)^2+\frac{2\rho h}{x}d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)}$$

$$y_E^* = \sqrt{\frac{2\left(d(p,g)+\frac{1}{4}(\Delta_2-\Delta_1)\right)\left[A - \frac{PP_0}{Pg_0 - P_0 g} + \frac{C_s C}{C_s g_0 - Cg_s}\right]}{h(1-\rho)^2 + \frac{2\rho h}{x}d(p,g) + \frac{1}{4}(\Delta_2-\Delta_1)}} \quad \ldots (7)$$

Remark :

If $\Delta_1 = \Delta_2 = \Delta$. Then $d\left(\tilde{d}(p,g), \tilde{0}_1\right) = d(p,g)$ when $\Delta_1 = \Delta_2 = \Delta \to 0$. The estimate of the total profit per unit time in the fuzzy sense (6) is identical to the crisp case (4). Hence the crisp demand rate model is a special case of the fuzzy model present here. By taking $\Delta \to 0$. We then have

$$y_E^* = \sqrt{\frac{2d(p,g)\left[A - \frac{PP_0}{Pg_0 - P_0 g} + \frac{C_s C}{C_s g_0 - Cg_s}\right]}{h(1-\rho)^2 + \frac{2\rho h d(p,g)}{x}}}$$

which is classical EOQ formula.

4. NUMERICAL EXAMPLE

To illustrate the results of the proposed model we consider on inventory system with the data : P = 50, C = 25, A = 100, h = 5, x = 175200, C_s = 20, c = 50000, γ = 0.5, ρ = 0.02, g = 0.7, K = 7000, g_s = 0.6, g_0 = 0.7, C_{inv} = \$10000. Δ_1 = 500, $\Delta 2$ = 350.

y_E^* = 5340.21 TPU(y_E^*) = 1192005.58

CONCLUSION

This paper proposed an entropic fuzzy EOQ model for item with imperfect quality using fuzzy parameters. In this model with demand rate is represented by a fuzzy number while the lotsize quantity is treated as a fixed constant. For this fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of total profit per unit time in the fuzzy sense, and then the corresponding optimal lotsize is derived to maximize the total profit. Numerical examples are carried out to investigate the behavior of our proposed models and the results are compared with those obtained from the crisp model.

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