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Research Article

AN INNOVATIVE METHOD FOR SOLVING TRANSPORTATION PROBLEM

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ABSTRACT
In this paper 'An Innovative Method named NMD Method is proposed for finding an optimal solution. However a new algorithm in NMD Method which is discussed in this paper gives an initial as well as either optimal solution or near to optimal solution. Numerical examples are provided to illustrate the proposed algorithm. It can be seen that the proposed algorithm gives a better optimal solution to the given transportation Problem.

Key Words: *Transportation Problem, VAM, Optimal Solution, IBFS, Degeneracy*

INTRODUCTION

Transportation problem is a special class of Linear Programming Problem which deals with the distribution of single commodity from various Sources of supply to various destination of demand in such manner that the total transportation cost is minimized..Usually for the initial allocation in the case of a transportation problem methods such as North-West corner Method (NWCM) , Least Cost Method (LCM) , Vogel's Approximation Method (VAM) are used. Finally the purpose for optimality MODI check is carried out.

This Problem was first presented in 1941 by Hitchcock and it was further developed Koopmans (1949) and Dantzig (1951). The Simplex method is not suitable for the Transportation Problem especially for large Scale transportation problem due to its special Structure of the model in 1954 Charnes and Cooper was developed Stepping Stone method for the efficiency reason. An Initial Basic Feasible Solution(IBFS) for the transportation problem can be obtained by using the north-west corner rule row minima,Coiumn minima ,Matrix minima and Vogel's Approximation Method [Reinfeld and Vogel 1958,Goyal"s version of VAM ..Kirca and Stair developed a heuristic method to obtain an efficient initial basic feasible solution.

Sudhakar etal proposed zero suffix method for finding an optimal solution for transportation problem directly in 2012.This paper presents a new simple approach to solve the transportation problem. Even in the above mentioned method needs more iteration to arrive optimal solution . Hence the Proposed method helps to get directly optimal solution with less iteration and its degeneracy .The algorithm of the approach is detailed in the paper and finally numerical examples are given to illustrate the approach and minimized cost comparison table is given.

Paper is organized as follows: Mathematical Representation and degeneracy in section first, VAM is Summarized in the second section .In third section ,proposed algorithm named as NMD Method, Numerical examples of the transportation problem and results are clarified in fourth section . Finally comparison of minimized cost by NMD, NWCM , LCM,VAM & MODI is given in the table. In the last section conclusion is discussed.

1. Mathematical Representation

Let there be m origins o_i having $a_i\,(i = 1,2,\ldots\ldots,m)$ units of Source respectively which are to be transported to n destinations $D_j's$ with $b_j\,(j = 1,2,\ldots\ldots,n)$ units of demand respectively .Let C_{ij} be the cost of source one unit of commodity from origin i to destination j. If x_{ij} represents the units source from origin i to destination j then problem is to determine the transportation Schedule so as to minimize the total transportation Cost satisfying supply and demand condition.

Mathematically the problem can be stated as

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i \; for \; i = 1,2, \ldots, m$ (supply constraints

and $\sum_{i=1}^{m} x_{ij} = b_j \; forj = 1,2, \ldots, n$ (demand constraints)

and $x_{ij} \geq 0 \; for \; all \; i \; and \; j.$

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ otherwise it is called Unbalanced.

Table 1.1: Transportation table

Origins (i)	Destination(j)				SUPPLY(a_i)
	1	2	…………	n	
1	x_{11} C_{11}	x_{12} C_{12}	…………………	x_{1n} C_{1n}	a_1
2	x_{21} C_{21}	x_{22} C_{22}	…………………	x_{2n} C_{2n}	a_2
3	x_{31} C_{31}	x_{32} C_{32}	…………………	x_{3n} C_{3n}	a_3
…………	………….	………….	………..	………….	………….
M	x_{m1} C_{m1}	x_{m2} C_{m2}	…………………	x_{mn} C_{mn}	a_m
Demand (b_j)	b_1	b_2	……………	b_n	$\sum a_i = \sum b_j$

1.2 Basic Definitions

The following terms are to be defined with reference to the transportation problem.

A) Feasible Solution (F.S)

A set of non negative allocation $x_{ij} \geq 0$ which satisfies the row and column restriction is known as Feasible Solution

B) Basic Feasible Solution (BFS)

A feasible solution to a $m -$ origins and $n -$destination problem is said to be Basic Feasible Solution if the number of positive allocation are $(m + n - 1)$. If the number of allocations in basic feasible solution are less than$(m + n - 1)$, it is called Degenerate Basic Feasible Solution (DBFS) otherwise non-degenerate.

C) Optimal Solution

A feasible solution (not necessarily basic) is said to be Optimal if minimizes the total transportation cost.

2 VOGEL'S APPROXIMATION METHOD (VAM)

The Vogel's approximation method is an iterative procedure for computing a basic feasible solution of the transportation problem.

Step I) Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

Step II) Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) along the side of the table against the corresponding column

Step.III) Identify the minimum penalty. If it is along the side of the table make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.

Step IV) If the penalties corresponding to two or more rows or columns are equal . Select the top most row and the extreme left column.

3 NMD METHOD

Algorithm for solving Transportation problem

Step I : Construct the Transportation matrix from given transportation problem

Step II: Select minimum odd cost from all cost in the matrix

Step III: Subtract selected least odd cost only from odd cost in matrix. Now there will be at least one zero and remaining all cost become even

Step IV: Multiply by $\frac{1}{2}$ each column ($i.e\ \frac{1}{2}C_{ij}$) or To get minimum cost 1 in any column ,if possible by dividing cost in that column.

Step V: Again select minimum odd cost in the remaining column except zeros in the column.

Step VI: Go to Step III. Now there will be at least one zero and remaining all cost will become even.

Step VII: Repeat step IV and V , for remaining sources and destinations till $(m+n-1)$ cells are allocated.

Step VIII: Start the allocation from minimum of supply and demand. Allocate this minimum of supply/demand at the place of 0 first and then 1.

Step IX: Finally total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand

i.e. Total Cost $= \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij}$

3 NUMERICAL EXAMPLES

We illustrate the proposed method by the following transportation problem having three origins and five destinations, which is in table 1.1

Example:1 **T**ransportation problem having three origins and five destinations

Table 1.1

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	4	1	2	4	4	60
O_2	2	3	2	2	3	35
O_3	3	5	2	4	4	40
Demand	12	45	20	18	30	135

Step 1:The minimum odd cost value is 1.Subtract 1 from all odd cost ,which shown in Table 1.2

Table 1.2

	D_1	D_2	D_3	D_4	D_5	**Supply**
O_1	4	0	2	4	4	60
O_2	2	2	2	2	2	35
O_3	2	4	2	4	4	40
Demand	22	45	20	18	30	

Allocate the cell (O_1, D_2) min (45 ,60) =45, we get x_{12}=45 and delete column D_2 , as for demand exhausted supply as (60-45) =15, which shown in Table 4.3

Table1.3

	D_1	D_3	D_4	D_5	**Supply**
O_1	4	2	4	4	15
O_2	2	2	2	2	35
O_3	2	2	4	4	40
Demand	22	20	18	30	

Step 2: Multiply $\frac{1}{2}D_1$, $\frac{1}{2}D_3$, $\frac{1}{2}D_4$ $\frac{1}{2}D_5$ which shown in Table 4.4

Table 1.4

	D_1	D_3	D_4	D_5	**Supply**
O_1	2	1	2	2	15
O_2	1	1	1	1	35
O_3	1	1	2	2	40
Demand	22	20	18	30	

Step 3:The minimum odd cost value is 1.Subtract 1 from all odd cost ,which shown in Table 4..5

Table 1.5

	D_1	D_3	D_4	D_5	**Supply**
O_1	1	0	1	1	15
O_2	0	0	0	0	35
O_3	0	0	1	1	40
Demand	22	20	18	30	

Allocate the cell (O_1, D_3) min (20 ,15) =15, we get x_{13}=15 and delete row O_1.

Table 1.6

	D_1	D_3	D_4	D_5	**Supply**
O_2	0	0	0	0	35
O_3	0	0	1	1	40
Demand	22	5	18	30	

Allocate the cell (O_3, D_3) min (5 ,40) =5, we get x_{33}=5 and delete column D_3.And allocate the cell (O_2,D_4) min (18 ,35) =18, cell (O_2, D_1) min (22 ,17)=17, cell (O_3, D_1) min (5,35)=5,cell (O_3, D_5) min (30 ,30) =30, we get x_{24} =18 , x_{21} =17 , x_{31} = 5, x_{35} =30 respectively.

Therefore x_{12}= 45 , x_{13}=15, x_{21}=17, x_{24} =18 , x_{31} =5 , x_{33} = 5, x_{35} =30

Total Cost = (1*45) + (2*15)+ (2*17)+ (2*18)+(3*5)+(2*5)+(4*30) = 290

Example: 2 **T**ransportation problem having three origins and four destinations

Table 2.1

	D_1	D_2	D_3	D_4	**Supply**
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	34

Solution by NMD Method $x_{11} = 5$, $x_{14} = 2$, $x_{22} = 2$,$x_{23}=7$,$x_{32} = 6$, $x_{14}= 12$.
Total Cost = (19*5) + (10*2)+ (30*2)+(40*7)+(8*6)+(20*12)= 743

Example: 3 **T**ransportation problem having three origins and three destinations

Table 3.1

	D_1	D_2	D_3	**Supply**
O_1	6	4	1	50
O_2	3	8	7	40
O_3	4	4	2	60
Demand	20	95	35	150

Solution by NMD Method $x_{12}= 15$, $x_{13} = 35$, $x_{21} = 20$,$x_{22} = 20$,$x_{32} = 60$.
Total Cost = (4*15)+ (1*35)+ (3*20)+(8*20)+(4*60)= 555

Example: 4 **T**ransportation problem having four origins and six destinations

Table 4.1

	D_1	D_2	D_3	D_4	D_5	D_6	**Supply**
O_1	9	12	9	6	9	10	5
O_2	7	3	7	7	5	5	6
O_3	6	5	9	11	3	11	2
O_4	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	22

Solution by NMD Method $x_{13}= 5$,$x_{22} = 4$, $x_{26} =2$, $x_{31} = 1$,$x_{33} = 1$,$x_{41} = 3$ $x_{44} = 2$. $X_{45} = 4$
Total Cost = (9*5)+ (3*4)+ (5*2)+(6*1)+(9*1)+ (6*3)+(2*2)+(2*4) = 112

5. COMPARISON OF TOTAL COST OF TRANSPORTATION PROBLEM FROM VARIOUS METHODS

Table 5.1

TABLE No.	PROBLEM SIZE	NMD	NWCM	LCM	VAM	MODI
1.1	3×5	**290**	363	305	**290**	**290**
2.1	3×4	**743**	1015	814	779	**743**
3.1	3×3	**555**	730	555	**555**	**555**
4.1	4×6	**112**	139	114	**112**	**105**

CONCLUSION

In this paper, simple algorithm for solving transportation problem has been developed. The proposed algorithm is easy to understand and apply. The optimal solution obtained in this investigation is same as that of MODI method or Vogel's Approximation method . However the NMD method presented in this paper gives an optimal solution which is shown in table 5.1

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