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***Research Article***

# A SELF-SIMILAR FLOW OF A MIXTURE OF A NON-IDEAL GAS AND SMALL SOLID PARTICLES BEHIND A SPHERICAL SHOCK WAVE IN THE PRESENCE OF A GRAVITATIONAL FIELD

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## ABSTRACT
Similarity solutions are obtained for one-dimensional unsteady adiabatic flow of a dusty gas behind a spherical shock wave with time dependent energy input. The dusty gas is assumed to be a mixture of small solid particles and a non-ideal gas. It is assumed that the equilibrium flow condition is maintained in the flow-field, and that the viscous stress and heat conduction of the mixture are negligible. The medium is assumed to be under the gravitational field due to heavy nucleus at the origin (Roche Model). The total energy of the flow-field behind the shock is increasing. In order to obtain similarity solutions the density of the undisturbed medium is assumed to be constant. The effects of an increase in the mass concentration of solid particles $k_p$, the ratio of the density of the solid particles to the initial density of the gas $G_1$, the gravitational parameter $G_0$ and the parameter of non-idealness of the gas $\bar{b}$ on the flow-field and on the shock-strength are investigated.

***Key Words:*** *Shock Wave, Similarity Solutions, Mixture of Non-Ideal Gas and Small Solid Particles, Time Dependent Energy Input, Gravitational Field*

## INTRODUCTION
The study of shock waves in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow, lunar ash flow, bomb blasts, coal-mine blasts, underground, volcanic and cosmic explosions, supersonic flights in polluted air, collision of a coma with a planet and many other engineering problems [see Pai et al. (1980), Higashino and Suzuki (1980), Miura and Glass (1983), Gretler and Regenfelder (2005), Popel and Gisko (2006), Vishwakarma and Nath (2006, 2009), Vishwakarma et al. (2008)]. Miura and Glass (1985) obtained an analytical solution for a planar dusty gas flow with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of dust on shock propagation. Pai et al. (1980) generalized the well-known solution of a strong explosion due to an instantaneous release of energy in gas [Sedov (1959), Korobeinikov (1976)] to the case of two-phase flow of a mixture of perfect gas and small solid particles, and brought out the essential effects due to the presence of dusty particles on such a strong shock wave. As they considered the non-zero volume fraction of solid particles in the mixture, their results reflect the influence of both the decrease of mixture's compressibility and the increase of the mixture's inertia on shock propagation [Steiner and Hirschler (2002), Vishwakarma and Nath (2006, 2009)].
The perfect gas law can often be applied to actual gases with sufficient accuracy. This approximation, may, however, be inadequate in a situation such as arises in the case of an explosion. It is then necessary to take into account the deviations of an actual gas from the ideal state which results from the interaction between its component molecules. Anisimov and Spiner (1972) have taken an equation of state for non-ideal gases in a simplified form, and investigated the effect of the parameter for non-idealness on the problem of strong point explosions, which describes the behavior of the medium satisfactorily at low densities. Ranga Rao and Purohit (1976) have studied the self-similar flow of a non-ideal gas driven by an

expanding piston and obtained solutions by taking the equation of state suggested by Anisimov and Spiner (1972). In recent years, several studied have been performed concerning the problem of shock waves in a mixture of a non-ideal gas and small solid particles, in particular Vishwakarma et al. (2007), Vishwakarma and Nath (2009, 2010, 2011) among others.

Carrus et al. (1951) have studied the propagation of shock waves in a gas under the gravitational attraction of a central body of fixed mass (Roche Model) and obtained similarity solutions by numerical method. Rogers (1957) has discussed a method for obtaining analytical solution of the same problem. Singh (1982) has studied the self-similar flow of a non-conducting perfect gas, moving under the gravitational attraction of a central body of fixed mass, behind a spherical shock wave under the assumption that the total energy content between the inner expanding surface and the shock front to be increasing with time.

In the present study, we therefore investigated the self-similar flow behind a spherical shock wave propagating in a dusty gas, which is a mixture of a non-ideal gas and small solid particles. The medium is assumed to be under a gravitational field due to heavy nucleus at the origin (Roche Model). The unsteady model consists of the dusty gas distributed with spherical symmetry around a nucleus having a large mass m. It is assumed that the gravitational effect of the mixture itself can be neglected compared with the attraction of the heavy nucleus. The total energy of the flow-field behind the shock is supposed to be increasing with time [Freeman (1968), Director and Dabora (1977)]. This increase can be obtained by the pressure exerted on the mixture by inner expanding surface [Rogers (1958)]. In order to obtain the similarity solutions of the problem the density of the undisturbed medium is assumed to be constant. It is investigated that how the parameter of non-idealness of the gas in the mixture $\bar{b}$ (which depends on the internal volume of the gas molecules), the mass concentration of solid particles $k_p$, the ratio of the density of solid particles to the initial density of the gas $G_1$ and the gravitational parameter $G_0$ affect the flow-field behind the shock. This work can be treated as extension of the work of Vishwakarma et al. (2007) by taking the medium under the gravitation attraction towards the heavy nucleus at the centre.

## MATERIALS AND METHODS

### *Fundamental Equations and Boundary Conditions*

We consider the medium to be a mixture of small solid particles and a non-ideal gas. The equation of state of the non-ideal gas in the mixture is taken to be [Anisimov and Spiner (1972), Ranga Rao and Purohit (1976)]

$$p_g = R^* \bar{\rho}_g \left(1 + b\bar{\rho}_g\right)T, \tag{2.1}$$

Where $p_g$ and $\bar{\rho}_g$ are the partial pressure and partial density of the gas in the mixture, *T* is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), *R** is the gas constant and *b* is the internal volume of the molecules of the gas. In this equation, the deviations of an actual gas from the ideal state are taken into account, which results from the interaction between its component molecules. It is assumed that the gas is still so rarefied that triple, quadruple etc, collisions between molecules are negligible, and their interaction is assumed to occur only through binary collisions. The quantity '*b*' is in general, a function of temperature *T*, but at high temperature range it tends to a constant value equal to the internal volume of the gas molecules [Anisimov and Spiner (1972), Landau and Lifshitz (1958)]. The effects of dissociation and ionization of gas molecules are assumed to be negligible.

The equation of state of the solid particles in the mixture is, simply,

$$\rho_{sp} = \text{constant}, \tag{2.2}$$

Where $\rho_{sp}$ is the species density of the solid particles. Proceeding on the same lines as in Pai (1977), we obtain the equation of state of the mixture as

$$p = \frac{(1-k_p)}{(1-Z)}\left(1+b\rho(1-k_p)\right)\rho R^* T, \tag{2.3}$$

where $p$ and $\rho$ are the pressure and density of the mixture, $Z = \frac{V_{sp}}{V}$ is the volume fraction and $k_p = \frac{M_{sp}}{M}$ is the mass fraction (concentration) of the solid particles in the mixture, $M_{sp}$ and $V_{sp}$ being, respectively, the mass and volumetric extension of the solid particles in a volume *V* and mass *M* of the mixture.

The relation between $k_p$ and *Z* is given by [Pai (1977)]

$$k_p = \frac{Z\rho_{sp}}{\rho}. \tag{2.4}$$

In equilibrium flow, $k_p$ is constant in the whole flow-field. Therefore, from (2.4)

$$\frac{Z}{\rho} = \text{constant} \tag{2.5}$$

in the whole flow-field.

Also we have the relation [Pai (1977)]

$$Z = \frac{k_p}{G(1-k_p)+k_p}, \tag{2.6}$$

where $G = \frac{\rho_{sp}}{\rho_g}$ is the ratio of the density of solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

$$U_m = \left[k_p C_{sp} + (1-k_p)C_V\right]T = C_{Vm}T, \tag{2.7}$$

where $C_{sp}$ is the specific heat of the solid particles, $C_V$ is the specific heat of the gas at constant volume and $C_{Vm}$ is the specific heat of the mixture at constant volume process.

The specific heat of the mixture at constant pressure process is

$$C_{pm} = k_p C_{sp} + (1-k_p)C_p, \tag{2.8}$$

where $C_p$ is the specific heat of the gas at constant pressure process.

The ratio of the specific heats of the mixture is given by (Pai *et al.*, 1980; Pai, 1977; and Marble, 1970)

$$\Gamma = \frac{C_{pm}}{C_{Vm}} = \gamma \frac{1+\frac{\delta\beta'}{\gamma}}{1+\delta\beta'}, \tag{2.9}$$

where $\gamma = \frac{C_p}{C_V}$, $\delta = \frac{k_p}{1-k_p}$ and $\beta' = \frac{C_{sp}}{C_V}$.

Now,

$$C_{pm} - C_{Vm} = (1-k_p)(C_p - C_V) = (1-k_p)R^*, \tag{2.10}$$

Neglecting the term containing $b^2\rho^2$ (Anisimov and Spiner, 1972; and Singh, 1983). The internal energy per unit mass of the mixture is, therefore, given by

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)[1+b\rho(1-k_p)]}. \tag{2.11}$$

From the first law of thermodynamics and the equation of state (2.3), we may calculate the so called equilibrium speed of sound in the mixture of non-ideal gas and small solid particles, as

$$a = \left(\frac{dp}{d\rho}\right)_S^{1/2} = \left[\frac{\{\Gamma+(2\Gamma-Z)b\rho(1-k_p)\}p}{(1-Z)\{1+b\rho(1-k_p)\}\rho}\right]^{1/2}, \tag{2.12}$$

where $\left(\frac{dp}{d\rho}\right)_S$ denotes the derivative of *p* with respect to $\rho$ at constant entropy S.

The compressibility (adiabatic) of the mixture may be calculated as (Moelwyn-Hughes, 1961)

$$-\frac{1}{v}\left(\frac{dv}{dp}\right)_S = \frac{1}{\rho a^2} = \frac{(1-Z)\{1+b\rho(1-k_p)\}}{\{\Gamma+(2\Gamma-Z)b\rho(1-k_p)\}p}, \tag{2.13}$$

Where $v = \frac{1}{\rho}$.

The equations of motion for one-dimensional adiabatic unsteady spherically symmetric flow of a mixture of non-ideal gas and small solid particles under the influence of a gravitational field are [Vishwakarma (2000), Rogers (1957)]

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0, \tag{2.14}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{G^* m}{r^2} = 0, \tag{2.15}$$

$$\frac{\partial U_m}{\partial t} + u\frac{\partial U_m}{\partial r} - \frac{p}{\rho^2}\left[\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right] = 0, \tag{2.16}$$

where *u* is the flow velocity, *r* the radial distance, *t* the time, *m* the mass of heavy nucleus at the centre and *G** the gravitational constant. Here, it is assumed that the gravitating effect of the medium itself is negligible in comparison with the attraction of the heavy nucleus.

We consider that a spherical shock wave is propagating into a medium (mixture of small solid particles and non-ideal gas) of constant density $(\rho_1 = \text{constant})$ at rest $(u_1 = 0)$.

The pressure immediately ahead of the shock front is given by, from equation (2.15),

$$p_1 = \frac{\rho_1 m G^*}{R}, \tag{2.17}$$

where *R* is the shock radius.

The jump conditions across the moving shock are as follows:

$$u_2 = (1-\beta)\dot{R}, \quad \rho_2 = \frac{\rho_1}{\beta}, \quad p_2 = p_1 + (1-\beta)\rho_1\dot{R}^2, \quad Z_2 = \frac{Z_1}{\beta}, \tag{2.18}$$

where $\dot{R} = \left(\frac{dR}{dt}\right)$ denotes the shock velocity, and subscripts '1' and '2' refer to the values just ahead and just behind the shock front. The quantity $\beta$ is given by the equation

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$$\gamma(\Gamma+1)\{1+\bar{b}(1-k_p)\}\beta^2-[\{2\Gamma M^{-2}+\gamma(2Z_1+\Gamma-1)\}+2\bar{b}(1-k_p)(\Gamma M^{-2}+\gamma Z_1)-\gamma(\Gamma-1)\bar{b}^2(1-k_p)^2]\beta$$
$$-[\bar{b}(1-k_p)\{2(\Gamma-Z_1)M^{-2}+\gamma(\Gamma-1)\}+(\Gamma-1)\bar{b}^2(1-k_p)^2(\gamma+2M^{-2})]=0 \tag{2.19}$$

where $\bar{b}=b\rho_1$, $Z_1=\dfrac{k_p}{G_1(1-k_p)+k_p}$, (2.20)

$\bar{b}=b\rho_1$ is the parameter of non-idealness of the gas in the mixture, $M$ is the shock-Mack number referred to the speed of sound in the dust-free ideal gas $\left(\dfrac{\gamma p_1}{\rho_1}\right)^{1/2}$ and $G_1$ the ratio of the density of the solid particles to the initial density of gas.

The total energy $E$ of the flow behind the shock is assumed to be increasing with time as [Rogers (1958), Freeman (1968), Director and Dabora (1977)]

$$E=E_0t^q, \tag{2.21}$$

where $q$ and $E_0$ are constants. This increase of energy may be achieved by the pressure exerted on the fluid by a piston. The piston may be physically, the surface of the stellar corona or the condensed explosives or the diaphragm containing a very high-pressure driven gas. By sudden explosion of the stellar corona or the detonation products or the driver gas into the ambient gas, a shock wave is produced in the ambient gas. The shocked gas is separated from this expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave.

***Similarity Solutions***

Let us take the solution of the equations (2.14) to (2.16) in the form

$$u=\frac{r}{t}U(\eta),\quad \rho=r^kt^\lambda D(\eta),\quad p=r^{k+2}t^{\lambda-2}P(\eta), \tag{3.1}$$

where $\eta=r^at^b$, (3.2)

while $k$, $\lambda$, $a$ and $b$ are constants. We choose the shock front to be given by $\eta_0=\text{constant}$. This choice fixes the velocity of the shock surface as

$$\dot{R}=-\frac{b}{a}\frac{R}{t} \tag{3.3}$$

which represents an outgoing shock if $a<0$ ($b>0$).

The total energy $E$ of the disturbance consists of three parts, namely, the heat energy, the kinetic energy and the gravitational energy. Hence the total energy

$$E=4\pi\int_{r_p}^{R}\rho\left(U_m+\frac{1}{2}u^2-\frac{G*m}{r}\right)r^2dr \tag{3.4}$$

where $r_p$ is the radius of the inner contact surface or piston.

Now using the transformations

$$g=\frac{\rho}{\rho_2}=\frac{Z}{Z_2},\ Y=\frac{p}{p_2},\ W=\frac{u}{\dot{R}},\ x=\frac{r}{R}, \tag{3.5}$$

equation (3.4) becomes

$$\frac{E}{4\pi\rho_1 J} = R^3 \dot{R}^2, \tag{3.6}$$

where

$$J = \int_{x_p}^{1} \left[ \frac{gW^2}{2\beta} - \frac{G_0 g}{\beta x} + \frac{\beta Y\left(1 - \frac{Z_1}{\beta} g\right)\{1 + (1-\beta)\gamma M^2\}}{(\Gamma - 1)\gamma M^2 \{\beta + g\bar{b}(1-k_p)\}} \right] x^2 dx, \tag{3.7}$$

$G_0 \left( = \frac{mG^*}{R\dot{R}^2} = \frac{1}{\gamma M^2} \right)$ being the parameter of gravitation and $x_p$ being the value of $x$ at the inner expanding surface.

Using the equations (2.21) and (3.3) in the equation (3.6), we obtain,

$$R = \left( \frac{E_0}{4\pi\rho_1 J} \right)^{\frac{1}{5}} \left( \frac{a}{b} \right)^{\frac{2}{5}} t^{\frac{q+2}{5}}. \tag{3.8}$$

Also from equation (3.2),

$$R = \eta_0^{1/a} t^{-b/a} \tag{3.9}$$

Comparing the equations (3.8) and (3.9), we get

$$\frac{q+2}{5} = -\frac{b}{a}. \tag{3.10}$$

By direct substitution of (3.1) in the shock conditions and equations of motion, we find that our solutions are consistent with the similarity conditions if

$$k=0, \quad \lambda = 0. \tag{3.11}$$

Using the transformations (3.5), the equations of motion (2.14) to (2.16) take the form

$$(W - x)\frac{dg}{dx} + g\frac{dW}{dx} + \frac{2gW}{x} = 0, \tag{3.12}$$

$$(W - x)\frac{dW}{dx} + \frac{\beta}{g}\left( \frac{1}{\gamma M^2} + 1 - \beta \right)\frac{dY}{dx} + \left( \frac{a}{b} + 1 \right)W + \frac{G_o}{x^2} = 0, \tag{3.13}$$

$$(W - x)\left(1 - \frac{Z_1}{\beta} g\right)\frac{dY}{dx} - (W - x)\left[ \frac{\Gamma\beta + \left(2\Gamma - \frac{Z_1}{\beta} g\right)\bar{b}(1-k_p)g}{\beta + \bar{b}(1-k_p)g} \right]\frac{Y}{g}\frac{dg}{dx} + 2\left( \frac{a}{b} + 1 \right)Y\left(1 - \frac{Z_1}{\beta} g\right) = 0 \tag{3.14}$$

The equations (3.12) to (3.14) which give the solution of our problem, are consistent with similarity conditions only if the parameter $M^2$ occurring in the equations (3.13) and (3.14) is independent of *t*. This requires that

$$\frac{b}{a} = -\frac{2}{3}. \tag{3.15}$$

Comparing (3.10) and (3.15), we obtain

$$q = \frac{4}{3} \tag{3.16}$$

which corresponds to a decreasing velocity shock. This shows that the similarity solution of the present problem exists only when the total energy of the flow-field behind the shock increases as $t^{4/3}$.
Without any loss of generality we may choose

$$a = -3 \text{ and } b = \frac{3}{5}(q+2). \tag{3.17}$$

In terms of dimensionless variables *x, W, Y* and *g* the shock conditions take the form

$$x = 1, \quad W = 1-\beta, \quad g = 1, \quad Y = 1. \tag{3.18}$$

From equations (3.12), (3.13) and (3.14) we have,

$$\frac{dg}{dx} = A\left[\frac{\beta Y\left(\frac{1}{\gamma M^2} + 1 - \beta\right)}{W-x} - \frac{gW}{2} + \frac{gG_0}{x^2} - \frac{2gW(W-x)}{x}\right], \tag{3.19}$$

$$\frac{dY}{dx} = \frac{AY}{g\left(1-\frac{Z_1}{\beta}g\right)}\left[\frac{\Gamma\beta + \left(2\Gamma - \frac{Z_1}{\beta}g\right)\bar{b}g(1-k_p)}{\beta + \bar{b}g(1-k_p)}\right]\left[\frac{\beta Y\left(\frac{1}{\gamma M^2} + 1 - \beta\right)}{W-x} - \frac{gW}{2} + \frac{gG_0}{x^2} - \frac{2gW(W-x)}{x}\right] + \frac{Y}{W-x}, \tag{3.20}$$

$$\frac{dW}{dx} = -\frac{(W-x)A}{g}\left[\frac{\beta Y\left(\frac{1}{\gamma M^2} + 1 - \beta\right)}{W-x} - \frac{gW}{2} + \frac{gG_0}{x^2} - \frac{2gW(W-x)}{x}\right] - \frac{2W}{x}, \tag{3.21}$$

where

$$A = \frac{\left(1-\frac{Z_1}{\beta}g\right)\{\beta + \bar{b}g(1-k_p)\}g}{(W-x)^2\left(1-\frac{Z_1}{\beta}g\right)\{\beta + \bar{b}g(1-k_p)\}g - \beta Y\left(\frac{1}{\gamma M^2} + 1 - \beta\right)\left\{\Gamma\beta + \left(2\Gamma - \frac{Z_1}{\beta}g\right)\bar{b}g(1-k_p)\right\}}.$$

Now, the equations (3.19) to (3.21) may be integrated numerically with boundary conditions (3.18) to obtain the values of *W, Y* and *g*.

**RESULTS AND DISCUSSION**

To obtain the solutions, we start numerical integration of the equations (3.19) to (3.21) from the shock front $(x = 1)$ and proceed inwards until the inner expanding surface $(x = x_p)$ is reached. Values of the flow variables *W, Y, g* are obtained for $\gamma = 1.4$; $\bar{b} = 0, 0.05, 0.1$; $k_p = 0, 0.2, 0.4$;

$G_1 = 1, 100$; $M^{-2} = 0.014, 0.14$ [see Pai et al. (1980), Miura and Glass (1985) and Vishwakarma et al. (2007)], and solutions are shown in Figures 1 to 3 and 5 to 7. The case $\bar{b} = 0$, $k_p \neq 0$ corresponds to a mixture of a perfect gas and small solid particles; and the case $\bar{b} \neq 0$, $k_p \neq 0$ to a mixture of a non-ideal gas and small solid particles.

The quantity $G_0$ which is a parameter of gravitation depends on $\gamma$ and $M$ and is tabulated in Table 1 for $\gamma = 1.4$ and $M^{-2} = 0.014, 0.14$. The density ratio $\beta$ across the shock front and position of the inner expanding surface $x_p$ are tabulated in Table 2 for various values of $k_p$, $G_1$ and $G_0$ with $\bar{b} = 0.05$. These quantities are also shown in Table 3 for various values of $\bar{b}$ and $G_0$ with $k_p = 0.2$ and $G_1 = 1$.

**Table 1: Values of $G_0$ (a parameter of gravitation) for different values of $M^{-2}$ and $\gamma = 1.4$**

| $M^{-2}$ | $G_0$ |
|---|---|
| 0.014 | 0.01 |
| 0.014 | 0.10 |

**Table 2: Density ratio $\beta$ across the shock front and position of the inner expanding surface $x_p$ for different values of $k_p$, $G_1$ and $G_0$ with $\bar{b} = 0.05$**

| $k_p$ | $G_1$ | $G_0$ | $\beta$ | $x_p$ |
|---|---|---|---|---|
| **0** | - | 0.01 | 0.211972 | 0.896933 |
| | | 0.10 | 0.318813 | 0.838660 |
| **0.2** | 1 | 0.01 | 0.333858 | 0.853050 |
| | | 0.10 | 0.439710 | 0.789144 |
| | 100 | 0.01 | 0.179194 | 0.912301 |
| | | 0.10 | 0.283390 | 0.855514 |
| **0.4** | 1 | 0.01 | 0.479309 | 0.791082 |
| | | 0.10 | 0.581108 | 0.719498 |
| | 100 | 0.01 | 0.145261 | 0.928566 |
| | | 0.10 | 0.246666 | 0.873143 |

**Table 3: Density ratio $\beta$ across the shock front and position of the inner expanding surface $x_p$ for different values of $\bar{b}$ and $G_0$ with $k_p = 0.2$ and $G_1 = 1$**

| $\bar{b}$ | $G_0$ | $\beta$ | $x_p$ |
|---|---|---|---|
| **0** | 0.01 | 0.321724 | 0.858818 |
| | 0.10 | 0.424138 | 0.796848 |
| **0.05** | 0.01 | 0.333858 | 0.853050 |
| | 0.10 | 0.439710 | 0.789144 |
| **0.1** | 0.01 | 0.344813 | 0.847711 |
| | 0.10 | 0.453561 | 0.782080 |

Figures 1 and 5 show that the velocity $W$ is higher at the inner expanding surface than that at the shock front. In fact, the velocity of the inner expanding surface is higher than the fluid velocity just behind the shock due to increasing energy input given by equation (2.21).

Figures 2, 3, 6 and 7 show that the density $g$ and the pressure $Y$ decrease as we move inwards from the shock front to the inner expanding surface, in general.

The behavior of the curves 7 and 8 in figures 1 to 3 is somewhat different from others. For curves 7 and 8, $G_1 = 1$ and $k_p = 0.4$; and therefore 40 percent of the volume is occupied by small solid particles causing a heavy loss in compressibility of the dusty gas. This fact is responsible for the different behavior of curves 7 and 8. The loss of compressibility of the medium also results in the increase of the distance of the inner expanding surface from the shock front, i.e. in the decrease of the value of $x_p$ (see Table 2). The behavior of compressibility of the medium for different values of $k_p$, $G_1$.

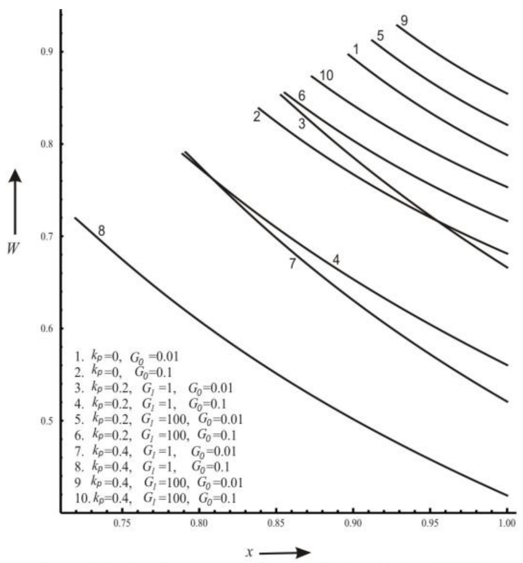


Figure 1. Variation of dimensionless flow velocity $W$ in the flow-field behind the shock front with $\bar{b}$=0.05

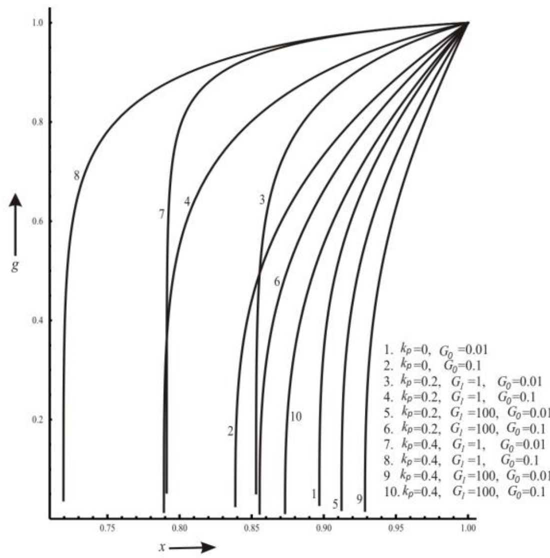


Figure 2. Variation of dimensionless density $g$ in the flow-field behind the shock front with $\bar{b}$=0.05

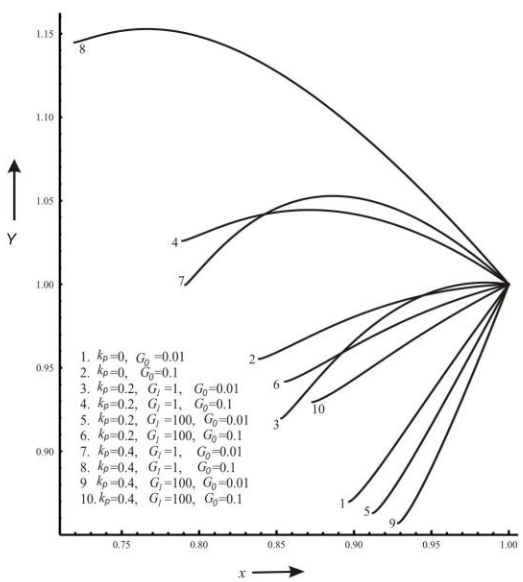


Figure 3. Variation of dimensionless pressure $Y$ in the flow-field behind the shock front with $\bar{b}$=0.05

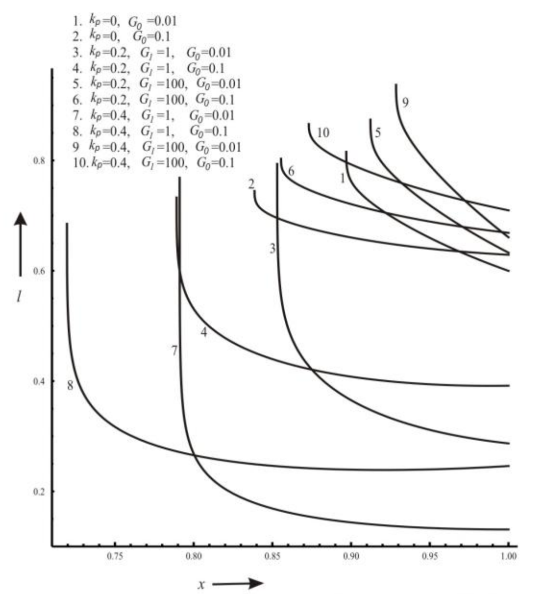


Figure 4. Variation of dimensionless compressibility $l$ in the flow-field behind the shock front with $\bar{b}$=0.05

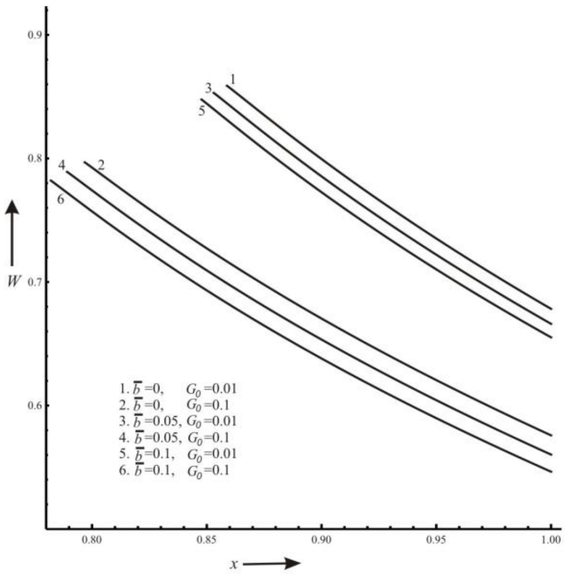


Figure 5. Variation of dimensionless flow-velocity $W$ in the flow-field behind the shock front with $k_p$=0.2 and $G_l$=1

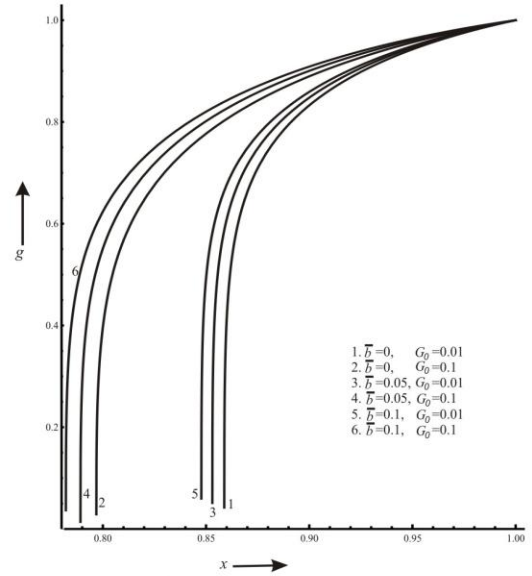


Figure 6. Variation of dimensionless density $g$ in the flow-field behind the shock front with $k_p$=0.2 and $G_l$=1

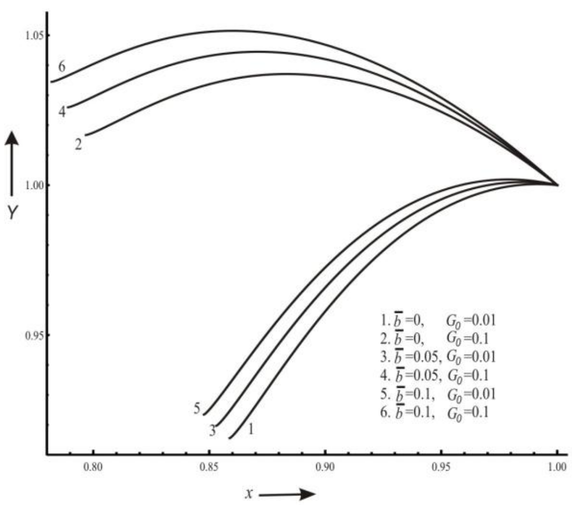


Figure 7. Variation of dimensionless pressure $p$ in the flow-field behind the shock front with $k_p$=0.2 and $G_l$=1

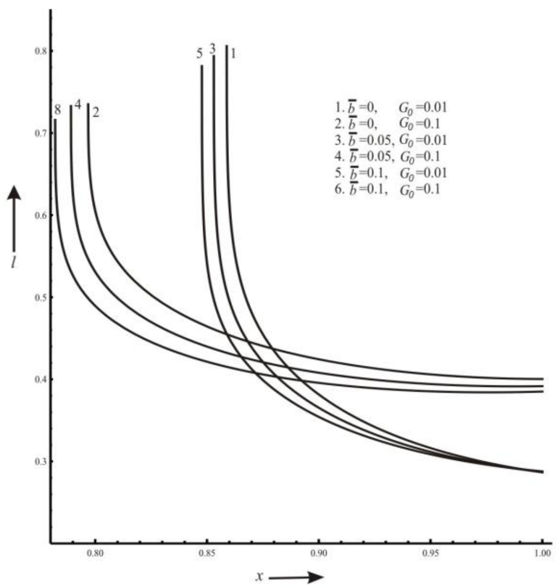


Figure 8. Variation of dimensionless compressibility $l$ in the flow-field behind the shock front with $k_p$=0.2 and $G_l$=1

$G_0$ and $\bar{b}$ may be seen in Figures 4 and 8 in which we have drawn profiles of a dimensionless compressibility *l(x)*, defined by

$$l(x)=\frac{p_2}{\rho a^2}=\frac{\left(1-\frac{Z_1}{\beta}g\right)\{\beta+\bar{b}g91-k_p)\}}{\left\{\Gamma\beta+\left(2\Gamma-\frac{Z_1}{\beta}g\right)\bar{b}g(1-k_p)\right\}Y}.$$

***Research Article***

It is found that the effects of an increase in the mass concentration of solid particles $k_p$ are:

- To decrease the shock strength (i.e. to increase the value of $\beta$) when $G_1 = 1$; and to increase it when $G_1 = 100$ (see Table 2);
- To increase the distance of the inner expanding surface from the shock front when $G_1 = 1$ and to decrease that when $G_1 = 100$ (see table 2);
- To decrease the velocity $W$ and the compressibility $l$ for $G_1 = 1$ and to increase those for $G_1 = 100$ (see Figures 1 and 4);
- To increase the density $g$ and the pressure $Y$ for $G_1 = 1$ and to decrease those for $G_1 = 100$ (see Figures 2 and 3).

An increase in $G_1$ greatly influences the flow variables for higher values of $k_p$ (see Figures 1, 2 and 3). Also, an increase in $G_1$ decreases the distance of the inner expanding surface from the shock front and hence increases the shock strength. Further, an increase in $G_1$ increases the dimensionless compressibility $l$ (see Figure 4). Physically, this increase in $l$ causes more compression of the medium behind the shock, which results in the increase of the shock strength.

The effects of an increase in the value of the gravitational parameter $G_0$ are:

- To increase the value of $\beta$, i.e. to decrease the shock strength (see Tables 2 and 3);
- To increase the distance of inner expanding surface from the shock front (see Tables 2 and 3). This means that an increase in the value of gravitational parameter has an effect of decreasing the shock strength, which is the same as indicated in (i) above;
- To decrease the velocity $W$ and to increase the density $g$ and the pressure $Y$ at any point in the flow field behind the shock (see Figures 1 to 3 and 5 to 7).
- The effects of on increase in the value of the parameter of the non-idealness of the gas $\bar{b}$ are:
- To increase the value of $\beta$, i.e. to decrease the shock strength (see Table 3);
- To increase the distance of inner expanding surface from the shock front (see Table 3);
- To decrease the velocity $W$ but to increase the density $g$ and the pressure $Y$ at any point in the flow-field behind the shock (see Figures 5 to 7).

Actually, an increase in $\bar{b}$, decreases the compressibility of the medium (see Figure 8) and this decrease of compressibility results in the decrease of shock strength.

## REFERENCES

**Pai SI, Menon S and Fan ZQ (1980).** Similarity solution of a strong shock wave propagating in a mixture of a gas and dusty particles. *International Journal of Engineering Science* **18(12)** 1365-1373.

**Higashino F and Suzuki T (1980).** The effect of particles on blast wave in a dusty gas. *Zeitschrift Fuer Naturforschung Teila* **35a** 1330-1336.

**Miura H and Glass II (1983).** On the passage of a shock wave through a dusty-gas layer. *Proceedings of the Royal Society of London* **385**(1788) 85-105.

**Gretler W and Regenfelder R (2005).** Strong shock waves generated by a piston moving in a dust-laden gas under isothermal condition. *European Journal of Mechanics B* **24** 205-218.

**Popel S and Gisko AA (2006).** Charge dust and shock phenomena in the solar system. *Nonlinear Processes in Geophysics* **13** 223-229.

**Vishwakarma JP and Nath G (2006).** Similarity solutions for unsteady flow behind an exponential shock in a dusty gas. *Physica Scripta* **74**(4) 493-498.

**Vishwakarma JP and Nath (2009).** Self-similar solution of a shock propagation in a mixture of a non-ideal gas and small solid particles. *Meccanica* **44** 239-254.
**Vishwakarma JP, Nath G and Singh KK (2008).** Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density. *Physica Scripta* **78** 035402.
**Miura H and Glass II (1985).** Development of the flow induced by a piston moving impulsively in a dusty gas, *Proceedings of the Royal Society of London A* **397** 295-309.
**Sedov LI (1959).** Similarity and Dimensional Methods in Mechanics, Academic Press, New York.
**Korobeinikov VP (1976).** Problems in the theory of point-explosion in gases. Proceeding of the Steklov Institute of Mathematics. *American Mathematical society, Providence* **119**.
**Steiner H and Hirschler T (2002).** A self-similar solution of shock propagation in a dusty gas. *European Journal of Mechanics-B/Fluids* **21(3)** 371-380.
**Anisimov SI and Spiner OM (1972).** Motion of an almost ideal gas in the presence of a strong point explosion, *Journal of Applied Mathematics and Mechanics* **36(5)** 883-887.
**Ranga Rao MP and Purohit NK (1976).** Self-similar problem in non-ideal gas, *International Journal of Engineering Science* **14** 91-97.
**Vishwakarma JP, Vishwakarma S and Sharma AK (2007).** Self-similar flow of a mixture of a non-ideal gas and small solid particles with increasing energy behind a shock wave. *Modelling Measurement and Control B* **76**(2) 62-75.
**Vishwakarma JP and Nath G (2010).** Propagation of a cylindrical shock wave in a rotating dusty gas with heat conduction and radiation heat flux. *Physica Scripta* **81** 045401.
**Carrus PA, Fox PA, Hass F and Kopal Z (1951).** Propagation of shock waves in the generalized Roche Model. *Astrophysical Journal* **113** 193-209.
**Rogers MH (1957).** Analytic solutions for blast wave problem with an atmosphere of varying density. *Astrophysical Journal* **152** 478-493.
**Singh JB (1982).** A self-similar flow in generalized Roche Model with increasing energy, *Astrophysics Space Science* **88**(2) 269-275.
**Freeman RA (1968).** Variable energy blast wave. *Journal of Physics D: Applied Physics* **1** 1697-1710.
**Director MN and Dabora EK (1977).** An experimental investigation of variable energy blast waves. *Acta Astronautica* **4**(3-4) 391-407.
**Rogers MH (1958).** Similarity flows behind strong shock waves. *Quarterly Journal of Mechanical and Applied Mathematics* **11**(4) 411-422.
**Landau LD and Lifshitz EM (1958).** Course of theoretical Physics. *Statistical Physics* (Pergamon Press, Oxford) **5**.
**Pai SI (1977).** Two phase flow (chap V) View eg. *Tracts in Pure and Applied Physics* (Vieweg, Braunschweig) **3**.
**Marble FE (1970).** Dynamics of dusty gases. *Annual Reviews of Fluid Mechanics* **2** 397-446.
**Singh JB (1983).** An explosion in non-ideal gas with magnetic field II. *Astrophysics Space Science* **96**(2) 239-244.
**Moelwyn-Hughes EA (1961).** *Physical Chemistry* (Pergamon Press, London).
**Vishwakarma JP (2000).** Propagation of shock waves in a dusty gas with exponentially varying density. *European Physical Journal B* **16**(2) 369-372.