International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2012 Vol. 2 (3) July-September pp.1-8/Kalaimaran Ara

Research Article

GEOMETRICAL ANALYSIS FOR NEW MATHEMATICAL THEOREMS ON QUADRILATERAL

***Kalaimaran Ara**
**Construction and Civil Maintenance Dept., Central Food Technological Research Institute, Mysore-20, Karnataka, India*
**Author for Correspondence*

ABSTRACT

The quadrilateral is a four sided polygon of two dimensional geometrical Figure s. Square, rectangle, parallelogram, rhombus, rhomboid, trapezium or trapezoidal, kite and dart are the members of the quadrilateral family. However the properties are varying from one to another. The author has attempted to develop a new theorem for their properties of each on parallelogram, quadrilateral and kite including necessary equations for derivation with necessary drawings.

Key Words: *Altitude, Diagonals, Kite, Parallelogram, Quadrilateral, Triangle and Vertex.*

INTRODUCTION

Quadrilateral

The *Quadrilateral* (Weisstein Eric, W2003, p.2439) is a four sided polygon of two dimensional geometrical Figure s which is one of the families of the polygon with four straight edges and four vertices. A quadrilateral is also known as a tetragon or quadrangle. However, it is a word often used for an open space where people gather say in building of educational institute or organization the quadrangle is usually a rectangle or square.

- The Regular quadrilateral is a quadrilateral in which all sides are the same length and same interior vertex angle. Square is the regular quadrilateral.
- The Irregular quadrilateral is a quadrilateral in which at least one of its sides is of different length and different interior vertex angle.
- The *Cyclic quadrilateral* (Weisstein Eric W, 2003, p.640) is a quadrilateral for which a circle can be circumscribed so that it touches vertex of each polygon. A quadrilateral that can be both inscribed and circumscribed on some pair of circles is known as a bi-centric quadrilateral.
- The *Tangential quadrilateral* (Weisstein Eric W, 2003, p.2941) is a convex quadrilateral whose sides all lie tangent to a single circle inscribed within the quadrilateral. This circle is called the in-circle of the quadrilateral.
- The *Complete quadrilateral* (Weisstein Eric W, 2003, p.486) is a four sided geometrical Figure that is determined by four coplanar lines, and has four sides and three diagonal axes, each two of which cut the other harmonically in three centers of which one or even two may be at infinity
- The Convex quadrilateral is a quadrilateral with all diagonals in the interior of the quadrilateral. All vertices points are outward concave, at least one vertex point inward towards the center of the quadrilateral. A convex quadrilateral is defined as a quadrilateral with all its interior angles less than 180°.
- The Concave quadrilateral is a non convex. The concave quadrilateral is a simple quadrilateral having at least one interior angle greater than 180°.
- The Complex quadrilateral its path may self-intersect. It looks like combined triangles.

The particular cases of the quadrilaterals are:

A *Square* (Weisstein Eric, 2003) is a regular quadrilateral, in which all four sides are of equal length and the two sets of opposing, parallel sides are perpendicular to each other and all angles fixed at 90°. A *Rectangle* (Weisstein Eric, 2003) is a quadrilateral all of whose angles are equal to right angles, and whose sides may or may not all be the same length. A *Parallelogram* (Weisstein Eric W, 2003) is a closed plane Figure bounded by four line segments with opposite pairs of sides parallel and equal in length. A *Trapezoid* (Weisstein Eric, 2003) or Trapezium is a quadrilateral which has one pair of sides parallel. A *Rhomboid* (Weisstein Eric, 2003) is a parallelogram whose sides are not of same length. A *Rhombus* (Weisstein Eric, 2003, p.2551) is a parallelogram whose sides are all the same length. A *Kite* (Weisstein Eric W, 2003, p.1629) is a quadrilateral in which with two distinct pairs of equal adjacent sides, one pair of opposite angles at the junction of unequal sides are equal, one diagonal bisects the other, diagonals intersect at right angles. A *Dart* (Weisstein Eric, 2003) is an irregular quadrilateral with a reflex-angled trapezium.

Parallelogram

The parallelogram is a family of quadrilateral with opposite sides parallel. The rectangle square and rhombus are special cases of parallelograms. In any parallelogram, the diagonals bisect each other, i.e., they cut each other in half. Any side of a parallelogram is called base, the perpendicular distance from a base to the opposite parallel side is called Altitude, the diagonals of a parallelogram, connecting opposite vertices, bisect one another; each diagonal divides the parallelogram into two congruent triangles.

Kite

The Kite is named for the flying kites, which often have this shape. It has two pairs of sides. Each pair is made up of adjacent sides that are equal in length. The angles are equal where the pairs meet. Diagonals meet at a right angle, the diagonal bisects each other.

New Theorem on Parallelogram

If a point 'P' is anywhere at inside of a parallelogram ABCD and connected with the vertices, 'AC' is the longer diagonal and 'BD' is the shorter diagonal and the distances of point P from vertices A, B, C, D are PA, PB, PC, PD respectively, then the double of difference of sum of square of PA and square of PC and sum of square of PB and square of PD is always equal to difference of square of AC and square of BD. This can be expressed in a mathematical form as $2[(PA^2 + PC^2) - (PB^2 + PD^2)] = AC^2 - BD^2$.

Derivations of Equations and Proof for the Theorem:

Referring Figure 1, let, A, B, C, D are the vertices of a parallelogram, point E is the projection of D on x-axis, F is the projection of C on line x-axis, S is the projection of C and D on y-axis, P is the point anywhere inside of the ellipse, Q is the projection of P on x-axis and R is the projection of P on y-axis. Therefore, DE, PQ, CF are perpendicular to x-axis similarly PR, DS are perpendicular to y-axis.

$AB = CD = m, BC = AD, AB \parallel CD \parallel x - \text{axis}, BC \parallel AD, AE = SD = BF = e \text{ and } AS = ED = FC = n$

$\text{Let, coordinates of } A = (x_1, y_1) = (0,0). \text{ Therefore, } x_1 = 0 \text{ and } y_1 = 0 \quad ----- [1.01]$

$\text{Let, coordinates of } B = (x_2, y_2) = (m, 0). \text{ Therefore, } x_2 = m \text{ and } y_2 = 0 \quad ----- [1.02]$

$\text{Let, coordinates of } C = (x_3, y_3) = (m + e, n). \text{ Therefore, } x_3 = m + e \text{ and } y_3 = n \quad --- [1.03]$

$\text{Let, coordinates of } D = (x_4, y_4) = (e, n). \text{ Therefore, } x_4 = e \text{ and } y_4 = n \quad ----- [1.04]$

$\text{Let, coordinates of } P = (x_p, y_p) = (u, v). \text{ Therefore, } x_p = u \text{ and } y_p = v \quad ----- [1.05]$

Determination of 'PA'

$$PA = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2}$$

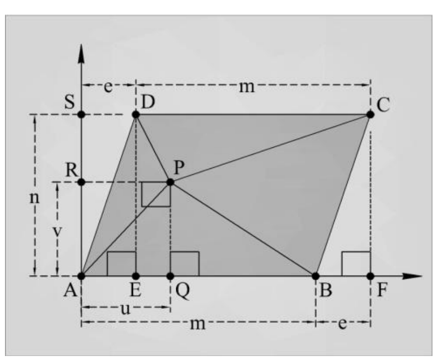

Figure 1: Parallelogram with a point P anywhere inside of the parallelogram

Substituting [1.05] and [1.01]

$\therefore PA = \sqrt{(u-0)^2 + (v-0)^2}$

$\therefore PA = \sqrt{u^2 + v^2}$

$\therefore PA^2 = u^2 + v^2$ ———————————————————————————————[1.06]

Determination of 'PB'

$PB = \sqrt{\left(x_p - x_2\right)^2 + \left(y_p - y_2\right)^2}$

Substituting [1.05] and [1.02]

$\therefore PB = \sqrt{(u-m)^2 + (v-0)^2}$

$\therefore PB = \sqrt{(u-m)^2 + v^2}$

$\therefore PB = \sqrt{u^2 + m^2 - 2um + v^2}$

$\therefore PB^2 = u^2 + m^2 - 2um + v^2$ ————————————————————————— [1.07]

Determination of 'PC'

$PC = \sqrt{\left(x_p - x_3\right)^2 + \left(y_p - y_3\right)^2}$

Substituting [1.05] and [1.03]

$\therefore PC = \sqrt{(u - [m+e])^2 + (v-n)^2}$

$\therefore PC = \sqrt{u^2 + [m+e]^2 - 2u[m+e] + (v-n)^2}$

$\therefore PC = \sqrt{u^2 + m^2 + e^2 + 2me - 2um - 2ue + v^2 + n^2 - 2vn}$

$\therefore PC^2 = u^2 + m^2 + e^2 + 2me - 2um - 2ue + v^2 + n^2 - 2vn$ ——————————— [1.08]

Determination of 'PD'

$PD = \sqrt{\left(x_p - x_4\right)^2 + \left(y_p - y_4\right)^2}$

Substituting [1.05] and [1.04]

$\therefore PD = \sqrt{(u-e)^2 + (v-n)^2}$

$\therefore PD = \sqrt{u^2 + e^2 - 2ue + v^2 + n^2 - 2vn}$

$\therefore PD^2 = u^2 + e^2 - 2ue + v^2 + n^2 - 2vn$ ———————————————————— [1.09]

Determination of $PB^2 - PC^2$

Subtracting equation [1.08] from equation [1.07]

$PB^2 - PC^2 = u^2 + m^2 - 2um + v^2 - (u^2 + m^2 + e^2 + 2me - 2um - 2ue + v^2 + n^2 - 2vn)$

$\therefore PB^2 - PC^2 = u^2 + m^2 - 2um + v^2 - u^2 - m^2 - e^2 - 2me + 2um + 2ue - v^2 - n^2 + 2vn$

$\therefore PB^2 - PC^2 = -e^2 - 2me + 2ue - n^2 + 2vn$ ———————————————————[1.10]

Determination of $PD^2 - PA^2$

Subtracting equation [1.06] from equation [1.09]

$PD^2 - PA^2 = u^2 + e^2 - 2ue + v^2 + n^2 - 2vn - (u^2 + v^2)$

$PD^2 - PA^2 = +e^2 - 2ue + n^2 - 2vn$ ——————————————————————— [1.11]

Determination of $(PD^2 - PA^2) + (PB^2 - PC^2)$

Adding equation [1.11] and equation [1.10]

$PD^2 - PA^2 + PB^2 - PC^2 = -2me$

$PA^2 - PB^2 + PC^2 - PD^2 = 2me$ ——————————————————————— [1.12]

Determination of 'AC' and 'BD'

In Figure1, $AF = AB + BF$

Substituting AB = m and BF = e

$\therefore AF = m + e$ ———————————————————————————— [1.13]

In right-angled triangle AFC,

$AC^2 = AF^2 + FC^2$

Substituting, equation [1.13] in above equation ,
$AC^2 = (m + e)^2 + n^2$ ———————————————————————————————[1.14]
In right-angled triangle BED,
$BD^2 = BE^2 + ED^2$
$BD^2 = (AB - AE)^2 + ED^2$
Substituting AB = m, AE =BF = e and ED = n in above equation ,
$\therefore BD^2 = (m - e)^2 + n^2$ ——————————————————————————————[1.15]
Subtracting, equation [1.15] from equation [1.14]
$AC^2 - BD^2 = (m + e)^2 + n^2 - [(m - e)^2 + n^2]$
$\therefore AC^2 - BD^2 = m^2 + e^2 + 2me + n^2 - m^2 - e^2 + 2me - n^2$
$\therefore AC^2 - BD^2 = 4me$
$AC^2 - BD^2 = 4me$ ————————————————————————————————[1.16]
Final equation of the theorem
Equating, equation [1.16] and equation [1.12]

$$\therefore PA^2 - PB^2 + PC^2 - PD^2 = \frac{AC^2 - BD^2}{2}$$

$$\therefore PA^2 + PC^2 - PB^2 - PD^2 = \frac{AC^2 - BD^2}{2}$$

$$\therefore PA^2 + PC^2 - (PB^2 + PD^2) = \frac{1}{2}(AC^2 - BD^2)$$

This can be rewritten as
$2[(PA^2 + PC^2) - (PB^2 + PD^2)] = AC^2 - BD^2$ —————————————————— [1.17]
The equation [1.17] is the mathematical form of the theorem.

New Theorem on Quadrilateral

For any quadrilateral, if a, b, c and d are the sides of a quadrilateral and p and q are the diagonals such that the p is the common largest side for triangles of other sides of c, d and a, b similarly q is the common largest side for triangles of other sides of a, d and b, c then the following condition is always satisfied.

$$\frac{\sqrt{[(a+b)^2 - p^2][p^2 - (a-b)^2]} + \sqrt{[(c+d)^2 - p^2][p^2 - (c-d)^2]}}{\sqrt{[(a+d)^2 - q^2][q^2 - (a-d)^2]} + \sqrt{[(b+c)^2 - q^2][q^2 - (b-c)^2]}} = 1 \text{ (Ref: Figure 2)}$$

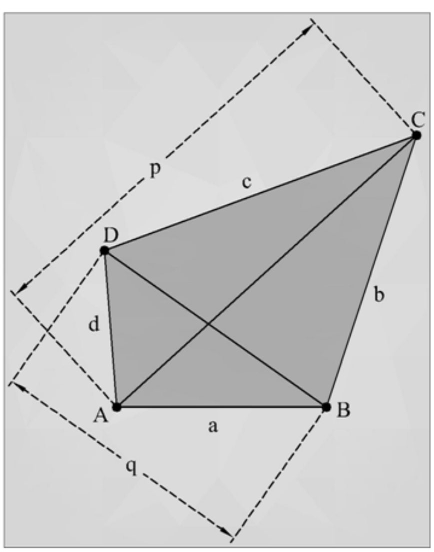

Figure 2: A Quadrilateral with its two diagonals intersecting each other

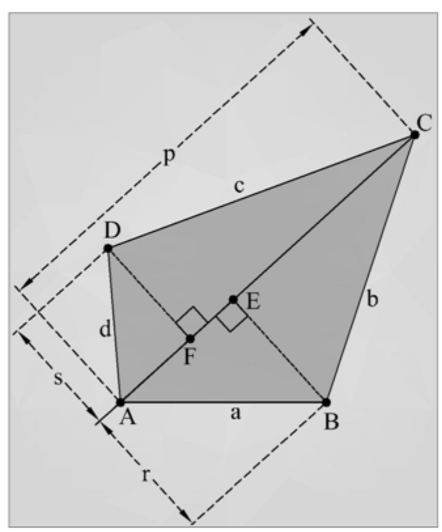

Figure 3: A Quadrilateral with its two altitudes

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Derivations of Equations and Proof for the Theorem:

Referring Figure 3, let points 'A', 'B', 'C' and 'D' are the vertices of a quadrilateral. 'AC' is one of the diagonals. Point 'E' is the projection of vertex D of triangle ABC on line AC. Point 'F' is the projection of vertex D of triangle ADC on line AC.

BE and DF with Respect to Diagonal AC:

Let, AB = a, BC = b, CD = c, DA = d, AC = p, BE = r and DF = s. Therefore, BE ⊥ AC and DF ⊥ AC.
According to the *Maran's formula on scalene triangle* (Kalaimaran Ara, 2006, p.10),

$$4 \times \mathrm{BE}^2 \times \mathrm{AC}^2 = [(\mathrm{AB}+\mathrm{BC})^2 - \mathrm{AC}^2] \times [\mathrm{AC}^2 - (\mathrm{AB}-\mathrm{BC})^2]$$

$$\therefore \mathrm{BE}^2 = \frac{1}{4 \times \mathrm{AC}^2}[(\mathrm{AB}+\mathrm{BC})^2 - \mathrm{AC}^2] \times [\mathrm{AC}^2 - (\mathrm{AB}-\mathrm{BC})^2]$$

$$\therefore r^2 = \frac{1}{4p^2}[(a+b)^2 - p^2] \times [p^2 - (a-b)^2]$$

$$\therefore r = \frac{1}{2p}\sqrt{[(a+b)^2 - p^2] \times [p^2 - (a-b)^2]} \text{ ------------------ [2.01]}$$

According to Maran's theorem on scalene triangle,

$$4 \times \mathrm{DF}^2 \times \mathrm{AC}^2 = [(\mathrm{AD}+\mathrm{CD})^2 - \mathrm{AC}^2] \times [\mathrm{AC}^2 - (\mathrm{AD}-\mathrm{CD})^2]$$

$$\therefore \mathrm{DF}^2 = \frac{1}{4 \times \mathrm{AC}^2}[(\mathrm{AD}+\mathrm{CD})^2 - \mathrm{AC}^2] \times [\mathrm{AC}^2 - (\mathrm{AD}-\mathrm{CD})^2]$$

$$\therefore s^2 = \frac{1}{4c^2}[(d+c)^2 - p^2] \times [p^2 - (d-c)^2]$$

$$\therefore s = \frac{1}{2c}\sqrt{[(d+c)^2 - p^2] \times [p^2 - (d-c)^2]} \text{ ------------------ [2.02]}$$

We already know that the area of quadrilateral ABCD is given by the any one of the following formulae

$$\text{(i) Area} = \frac{1}{2} \times \mathrm{AC} \times (\mathrm{BE}+\mathrm{DF}) \text{ (or) (ii) Area} = \frac{1}{2} \times \mathrm{BD} \times (\mathrm{AG}+\mathrm{CH})$$

(i) Determination of area ABCD by using the first formula

$$\text{Area} = \frac{1}{2} \times \mathrm{AC} \times (\mathrm{BE}+\mathrm{DF})$$

$$\text{Therefore, Area ABCD} = \frac{1}{2} \times p \times (r+s)$$

Substituting equation [2.01] and [2.02] in above equation ,

$$\text{Area} = \frac{1}{2} \times p \times \left(\frac{1}{2p}\sqrt{[(a+b)^2 - p^2] \times [p^2 - (a-b)^2]} + \frac{1}{2c}\sqrt{[(d+c)^2 - p^2] \times [p^2 - (d-c)^2]}\right)$$

$$\therefore \text{Area} = \frac{p}{4p} \times \left(\sqrt{[(a+b)^2 - p^2] \times [p^2 - (a-b)^2]} + \sqrt{[(d+c)^2 - p^2] \times [p^2 - (d-c)^2]}\right)$$

$$\therefore \text{Area} = \frac{1}{4}\left(\sqrt{[(a+b)^2 - p^2][p^2 - (a-b)^2]} + \sqrt{[(d+c)^2 - p^2][p^2 - (d-c)^2]}\right) \text{ ----- [2.03]}$$

Referring Figure 4, let, points 'A', 'B', 'C' and 'D' are the vertices of a quadrilateral. 'BD' is another diagonal. Point 'G' is the projection of vertex A of triangle DAB on line BD. Point 'H' is the projection of vertex C of triangle CBD on line BD.

Let, AB = a, BC = b, CD = c, DA = d, BD = q, AG =u and CH = s. Therefore, AG is ⊥ to BD and CH is ⊥ to BD.

According to the Maran's theorem on scalene triangle,

$$4 \times \mathrm{AG}^2 \times \mathrm{BD}^2 = [(\mathrm{AB}+\mathrm{AD})^2 - \mathrm{BD}^2] \times [\mathrm{BD}^2 - (\mathrm{AB}-\mathrm{AD})^2]$$

$$\therefore \mathrm{AG}^2 = \frac{1}{4 \times \mathrm{BD}^2}[(\mathrm{AB}+\mathrm{AD})^2 - \mathrm{BD}^2] \times [\mathrm{BD}^2 - (\mathrm{AB}-\mathrm{AD})^2]$$

$$\therefore u^2 = \frac{1}{4q^2}[(a+d)^2 - q^2] \times [q^2 - (a-d)^2]$$

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An Online International Journal Available at http://www.cibtech.org/jpms.htm
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$$\therefore u = \frac{1}{2q}\sqrt{[(a+d)^2 - q^2] \times [q^2 - (a-d)^2]} \quad ------------------ [2.04]$$

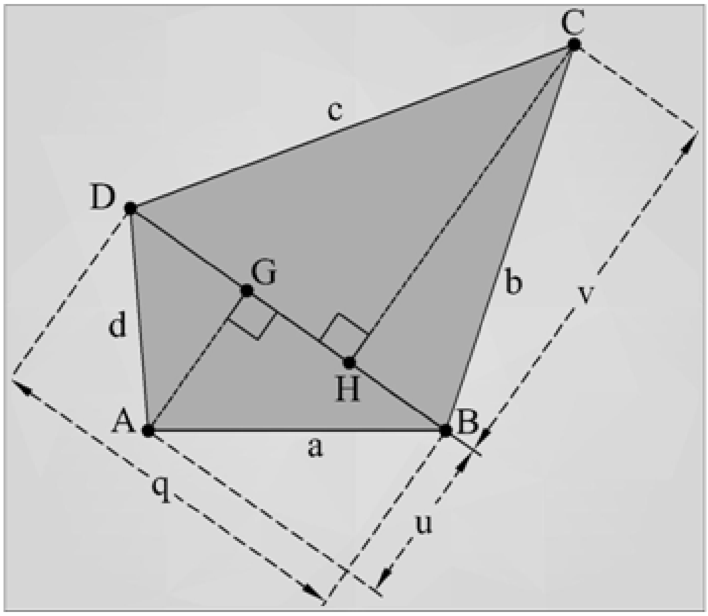

Figure 4: A Quadrilateral with its two altitudes AG and CH with respect to diagonal BD

According to the Maran's formula on scalene triangle,

$$4 \times CH^2 \times BD^2 = [(BC + CD)^2 - BD^2] \times [BD^2 - (BC - CD)^2]$$

$$\therefore CH^2 = \frac{1}{4 \times BD^2}[(BC + CD)^2 - BD^2] \times [BD^2 - (BC - CD)^2]$$

$$\therefore v^2 = \frac{1}{4q^2}[(b+c)^2 - q^2] \times [q^2 - (b-c)^2]$$

$$\therefore v = \frac{1}{2q}\sqrt{[(b+c)^2 - q^2] \times [q^2 - (b-c)^2]} \quad ------------------ [2.05]$$

(ii) Determination of area ABCD by using the second formula

$$\text{Area} = \frac{1}{2} \times BD \times (AG + CH)$$

$$\text{Area} = \frac{1}{2} \times q \times (u + v)$$

Substituting equation [2.04] and [2.05] in above equation ,

$$\text{Area} = \frac{1}{2} \times q \times \left(\frac{1}{2q}[(a+d)^2 - q^2] \times [q^2 - (a-d)^2] + \frac{1}{2q}\sqrt{[(b+c)^2 - q^2] \times [q^2 - (b-c)^2]}\right)$$

$$\therefore \text{Area} = \frac{q}{4q} \times \left(\sqrt{[(a+d)^2 - q^2] \times [q^2 - (a-d)^2]} + \sqrt{[(b+c)^2 - q^2] \times [q^2 - (b-c)^2]}\right)$$

$$\therefore \text{Area} = \frac{1}{4}\left(\sqrt{[(a+d)^2 - q^2][q^2 - (a-d)^2]} + \sqrt{[(b+c)^2 - q^2][q^2 - (b-c)^2]}\right) \quad ---- [2.06]$$

We know that the area calculated by these two formulae must be same.
Therefore, equating equation [2.03] and [2.06],

$$\sqrt{[(a+b)^2 - p^2][p^2 - (a-b)^2]} + \sqrt{[(d+c)^2 - p^2][p^2 - (d-c)^2]}$$
$$= \sqrt{[(a+d)^2 - q^2][q^2 - (a-d)^2]} + \sqrt{[(b+c)^2 - q^2][q^2 - (b-c)^2]}$$

Therefore,

$$\frac{\sqrt{[(a+b)^2-p^2][p^2-(a-b)^2]}+\sqrt{[(c+d)^2-p^2][p^2-(c-d)^2]}}{\sqrt{[(a+d)^2-q^2][q^2-(a-d)^2]}+\sqrt{[(b+c)^2-q^2][q^2-(b-c)^2]}}=1 \quad -------- [2.07]$$

The equation [2.07] is the mathematical expression of the relation between the sides and diagonals of any quadrilateral.

New Theorem on Kite

Suppose A, B, C and D are the vertices of a kite, and point P is the intersection of diagonals of a kite and connected by these vertices. If the kite is symmetrically divided by a diagonal AC, PA and PC are the cut length of the diagonal AC, sides AB and CD and AD and BC are opposite to each other, then the difference of the squares of any pair of opposite sides of a kite is equal to the difference of squares of the cut length PA and PC. This can be expressed in a mathematical form as

$AB^2 - CD^2 = AD^2 - BC^2 = PA^2 - PC^2$. (Ref: Figure 5)

Derivations of Equations and Proof for the Theorem:

Referring Figure 5, let points 'A', 'B', 'C' and 'D' are the vertices of a kite, PD = PB . Point 'P' is the intersection of these two diagonals. According to the properties of kite, AB = AD, BC = CD, PB = PD = BD/2. AC and BD are the Ortho-diagonal to each other.

$\text{In right}-\text{angled triangle APB}, AB^2 = PA^2 + PB^2 \quad ---------[3.01]$

$\text{In right}-\text{angled triangle BPC}, BC^2 = PB^2 + PC^2 \quad ---------- [3.02]$

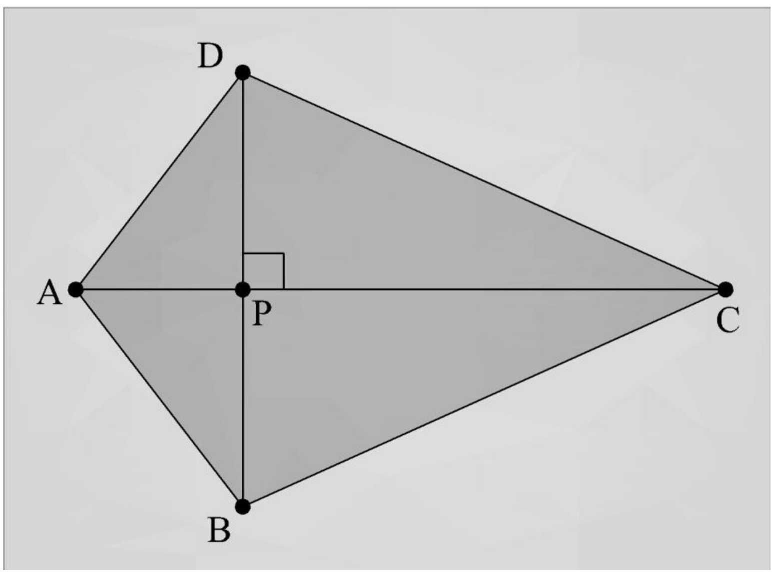

Figure5: A Kite with a point P is the intersection of diagonals

$\text{In right}-\text{angled triangle CPD}, CD^2 = PC^2 + PD^2 \quad ---------- [3.03]$

$\text{In right}-\text{angled triangle APD}, AD^2 = PA^2 + PD^2 \quad ---------- [3.04]$

Subtracting equation [3.03] from equation [3.01]

$AB^2 - CD^2 = (PA^2 + PB^2) - (PC^2 + PD^2)$

$AB^2 - CD^2 = PA^2 + PB^2 - PC^2 - PD^2$

$AB^2 - CD^2 = PA^2 - PC^2 \ (\because PB = PD) \quad ----------------- [3.05]$

Subtracting equation [3.04] from equation [3.02]

$AD^2 - BC^2 = (PA^2 + PD^2) - (PB^2 + PC^2)$

$AD^2 - BC^2 = PA^2 + PD^2 - PB^2 - PC^2$

$AD^2 - BC^2 = PA^2 - PC^2 \ (\because PB = PD) \quad ----------------- [3.06]$

From equation [3.05]and [3.06], $AB^2 - CD^2 = AD^2 - BC^2 = PA^2 - PC^2 \quad ------[3.07]$

The equation [3.07] is the mathematical form of the property.

CONCLUSION

In this article, new theorems each on Parallelogram, Quadrilateral and Kite have been developed including necessary equations derived for new properties including illustrations where ever necessary. These theorems, which have been defined in this article, may be useful for that work is related to geometry, research or further study in the quadrilateral, parallelogram and kite. This may also be very useful for students, research scholars, etc.

ACKNOWLEDGEMENTS

It gives immense pleasure to thank all those who extended their coordination, moral support, guidance and encouraged to reach the publication of the article. My special thanks to Prof. Ram Rajasekharan and Rao, Director, CSIR-CFTRI, Mysore. My special thanks to Raghava, KSMS, Head-FE Dept. and Murthy, Principal Scientist, FE Dept. CSIR-CFTRI for his valuable suggestions and cooperation. I would also like to express my gratitude to Krishna, Head-PMC Dept. and all Scientific, Technical and Administrative staff and colleague in CSIR- CFTRI, Mysore for their full support and co-ordination. I would also like to express my heartiest thanks to my college batch mates Muthuselvam, Rajendiran and Rajarajan, Taluk, Balasubramaniyan, EE, Madras Port Trust, K. Alagar, Kallal, Ilayangudi Taluk, Ramnad Dist, T.N, Iyyasamy, A.E, TWAD Board, Aranthangi, T.N, Nehrukumar, Thanjavur, T.N, Mohanraj, Ladapuram, Taluk, Trichirapalli Dist, T.N, Vellaisamy, Srivilliputhur, Dindigul Dist, T.N and Porchezhiyan, Perunthottam, Taluk, Nagapattinam Dist, T.N.

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