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***Research Article***

# MAGNETOGASDYNAMIC SHOCK WAVE IN A NON-IDEAL GAS UNDER A GRAVITATIONAL FIELD WITH HEAT CONDUCTION AND RADIATION HEAT FLUX

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## ABSTRACT
Similarity solutions are obtained for one-dimensional unsteady flow of a non-ideal gas behind a spherical magnetogasdynamic shock wave with heat conduction and radiation heat flux in the presence of a spatially decreasing azimuthal magnetic field. The gas is assumed to be perfectly conducting and under the influence of a gravitational force due to a heavy nucleus at the origin. The heat conduction is expressed in terms of Fourier's law and the radiation is considered to be of the diffusion type for an optically thick grey gas model. The thermal conductivity and the absorption coefficient are assumed to vary with temperature and density. The effects of an increase in the value of the parameter of non-idealness of the gas, the Alfven-Mach number and gravitational parameter on the flow variables are investigated.

***Key Words:*** *Shock wave, Magnetogasdynamics, Non-ideal gas, Self-similar flow, Heat transfer effects, Gravitational field*

## INTRODUCTION
The influence of radiation on the shock wave and on the flow-field behind the shock front has always been of great interest, for instance in the field of nuclear power and space research. Consequently, similarity solutions for shock waves in radiative gas dynamics have been given by Marshak, (1958); Elliot (1960), Wang (1964), Helliwell (1969), Nicastro (1970), Ghoniem et al. (1982), Vishwakarma and Singh (2008) and many others. Marshak (1958) studied the effects of radiation on the shock propagation by introducing the radiation diffusion approximation. Using the same mode of radiation, Elliot (1960) discussed the conditions leading to self-similarity with a specified functional form of the mean-free path of radiation and obtained a solution for self-similar spherical explosions. Wang (1964), Helliwell (1969) and Nicastro (1970) treated the problems of radiating walls, either stationary or moving, generating shocks at the head of self-similar flow-fields. Ghoniem et al., (1982) obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limits of Rosseland radiative diffusion and Plank radiative emission. Vishwakarma and Singh (2008) obtained self-similar solutions for the propagation of a magnetogasdynamic shock wave in a non-uniform gas with heat conduction and radiation heat flux in presence of an azimuthal magnetic field, driven out by a cylindrical or spherical piston moving with time according to power law.

In extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. In recent years, several studies have been performed concerning the problem of shock waves in non-ideal gases, in particular, by Anisimov and Spiner (1972), Ranga Rao and Purohit (1976), Roberts and Wu (1996), Madhumita and Sharma (2004), Arora and Sharma (2006), Vishwakarma and Nath (2007, 2009, 2010, 2011) among others.

In all of the works mentioned above, the influence of gravitational field on the medium is not considered. The gravitational force has considerable effect on many astrophysical problems. Carrus et al., (1951) have studied the propagation of shock waves in a gas under the gravitational attraction of a central body of fixed mass (Roche Model) and obtained the similarity solutions by numerical method. Rogers (1957) has discussed a method for obtaining analytical solution of the same problem. Ojha et al., (1998) have discussed the dynamical behaviour of an unsteady magnetic star by employing the concept of the Roche Model in an electrically conducting atmosphere. Rosenau (1977) has presented self-similar solutions for the equatorial propagation of axisymmetric, piston-driven magnetohydrodynamic shocks into an inhomogeneous ideal gas permeated by an azimuthal magnetic field. The theory is applied to describe shock waves in a gravitational field due to a heavy nucleus at the origin. Singh and Nath (2011) have studied the self-similar flow of a perfectly conducting non-ideal gas, moving under the gravitational attraction of a central body of fixed mass, behind a spherical shock wave in the presence of an azimuthal magnetic field.

The purpose of this study is therefore to obtain self-similar solutions for the propagation of a magnetogasdynamic shock wave in a non-ideal gas with heat conduction and radiation heat flux in presence of an azimuthal magnetic field, driven out by a spherical piston moving with time according to power law. The medium is assumed to be under a gravitational field due to a heavy nucleus at the origin (Roche Model). The unsteady model of Roche consists of a gas distributed with spherical symmetry around a nucleus having a large mass $m$. It is assumed that the gravitational effect of the gas itself can be neglected compared with the attraction of the heavy nucleus. The heat transfer fluxes are expressed in terms of Fourier's law for heat-conduction and a diffusion radiation mode for an optically thick grey gas, which is typical of large-scale explosions. The thermal conductivity and absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoniem *et al.*, 1982). Also, it is assumed that the gas is grey and opaque, and the shock is isothermal. The assumption that the shock is isothermal is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient, this excludes the possibility of temperature jump [Zel'dovich and Raizer (1967), Rosenau and Frankenthal (1976b, 1978), Bhowmick (1981), Singh and Srivastava (1982)]. The counter pressure (the pressure ahead of the shock) is taken into account. The radiation pressure and radiation energy are neglected (Elliott, 1960; Wang 1964; Ghoniem *et al.*, 1982; Abdel-Raouf and Gretler, 1991). The piston velocity is assumed to vary as some power of time and the initial azimuthal magnetic field to vary as some power of distance. In order to obtain the similarity solutions of the problem the density of the undisturbed medium is assumed to be constant. Effects of an increase in the value of the parameter of non-idealness of the gas $\bar{b}$, Alfven-Mach number $M_A$ and the gravitational parameter $G_0$ on the flow-field behind the shock are investigated.

***Equations of Motion and Boundary Conditions***

The fundamental equations for one-dimensional unsteady and spherically symmetric flow of an electrically conducting non-ideal gas with heat conduction and radiation heat flux taken into account under the influence of a gravitational field and an azimuthal magnetic field may, in Eulerian co-ordinates, be expressed as (Christer and Helliwell, 1969; Summers, 1975; Ghoniem *et al.*, 1982; Abdel-Raouf and Gretler, 1991; Gretler and Wehle,1993; Vishwakarma and Nath 2011).

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0, \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] + \frac{G^* m}{r^2} = 0, \quad (2.2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0, \quad (2.3)$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{1}{\rho r^2}\frac{\partial}{\partial r}(r^2 q) = 0, \quad (2.4)$$

Where $r$ and $t$ are independent space and time co-ordinates, $\rho$ is the density, $p$ the pressure, $u$ the flow velocity, $h$ the azimuthal magnetic field, $e$ the internal energy per unit mass, $\mu$ the magnetic permeability, $q$ the heat flux, $G^*$ the gravitational constant and $m$ the mass of the heavy nucleus at the centre.

The total heat flux $q$, which appears in the energy equation may be decomposed as

$$q = q_C + q_R, \qquad (2.5)$$

Where $q_C$ the conduction heat is flux, and $q_R$ is the radiation heat flux.

According to the Fourier's law of heat conduction

$$q_C = -k\frac{\partial T}{\partial r}, \qquad (2.6)$$

Where $k$ is the coefficient of thermal conductivity of the gas and $T$ is the absolute temperature of the medium.

Assuming the local thermodynamic equilibrium and using radiative diffusion model for an optically thick grey gas [Pomraning (1973)], the radiative heat flux $q_R$ may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as

$$q_R = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T^4}{\partial r}, \qquad (2.7)$$

Where $\sigma$ is the Stefan-Boltzmann constant and $\alpha_R$ is the Rosseland mean absorption coefficient.

The electrical conductivity of the gas is assumed to be infinite. Therefore the diffusion term from the magnetic field equation is omitted, and the electrical resistivity is ignored. Also, the effect of viscosity on the flow of the gas is assumed to be negligible.

The above system of equations should be supplemented with an equation of state. The equation of state for non-ideal gas is obtained by considering an expansion of the pressure $p$ in powers of the density (Landau and Lifshitz, 1958; Anisimov and Spiner, 1972).

$$p = \Gamma\rho T\left[1 + \rho C_1(T) + \rho^2 C_2(T) + \cdots\right],$$

Where $\Gamma$ is the gas constant and $C_1(T)$, $C_2(T)$, $\cdots$ are virial coefficients. The first term in the expansion corresponds to an ideal gas. The second term is obtained by taking into account the interaction between the pairs of molecules, and subsequent terms must involve the interactions between the groups of three, four, etc. molecules. In the high temperature range, the coefficients $C_1(T)$ and $C_2(T)$ tend to constant values equal to $b$ and $(5/8)b^2$ respectively. For gases $b\rho << 1$, $b$ being the internal volume of the molecules, and therefore it is sufficient to consider the equation of state in the form (Anisimov and Spiner, 1972; Singh and Singh, 1998; Ojha, 2002).

$$p = \Gamma\rho T(1 + b\rho). \qquad (2.8)$$

In this equation the correction to pressure is missing due to neglect of second and higher powers of $b\rho$, i.e. due to neglect of interactions between groups of three, four, etc. molecules of the gas. The internal energy $e$ per unit mass of the non-ideal gas is given by (Singh and Singh, 1998; Ojha, 2002; Vishwakarma and Nath, 2007).

$$e = \frac{p}{(\gamma - 1)\rho(1 + b\rho)}, \qquad (2.9)$$

Where $\gamma$ is the adiabatic index.

For an isentropic change of state of the non-ideal gas, we may calculate the so-called speed of sound in non-ideal gas as follows:

$$a = \left(\frac{dp}{d\rho}\right)_S^{\frac{1}{2}} = \left[\frac{\gamma p(1+2b\rho)}{\rho(1+b\rho)}\right]^{\frac{1}{2}}, \qquad (2.10)$$

Where the subscript's' refers to the process of constant entropy.

The thermal conductivity $k$ and the absorption coefficient $\alpha_R$ are assumed to vary with the temperature and density. These can be written in the form of power laws, namely [Ghoniem et al. (1982), Vishwakarma and Nath (2008), Vishwakarma et al. (2008)]

$$k = k_0\left(\frac{T}{T_0}\right)^{\beta_C}\left(\frac{\rho}{\rho_0}\right)^{\delta_C}, \qquad \alpha_R = \alpha_{R_0}\left(\frac{T}{T_0}\right)^{\beta_R}\left(\frac{\rho}{\rho_0}\right)^{\delta_R}, \qquad (2.11)$$

Where the subscript '0' denotes a reference state. The exponents in the above equations should satisfy the similarity requirements if a self-similar solution is sought.

A spherical shock is supposed to be propagating in the undisturbed non-ideal gas with constant density under the gravitational field. Also, the azimuthal magnetic field in the undisturbed gas is assumed to vary as $h = Ar^{-\omega}$ (Rosenau, 1977). where $A$ and $\omega$ are constants.

The flow variables immediately ahead of the shock front are

$$u = u_1 = 0,$$

$$\rho = \rho_1 = \text{constant},$$

$$h = h_1 = Ar_s^{-\omega},$$

(2.12)

$$p = p_1 = \frac{(1-\omega)\mu A^2}{2\omega r_s^{2\omega}} + \frac{G^* m\rho_1}{r_s} \qquad (2\omega = 1),$$

$$q = q_1 = 0$$ [Laumbach and Probestein (1970)],

Where $r_s$ the shock radius and the subscript '1' is denotes the conditions immediately ahead of the shock.

The shock is assumed to be isothermal (the formation of isothermal shock is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient; this excludes the possibility of a temperature jump; see for example (Zel'dovich and Raizer, 1967; Rosenau and Frankenthal, (1976, 1978) and hence, the conditions across it are

$$\rho_1 V = \rho_2 (V - u_2),$$

$$h_1 V = h_2 (V - u_2),$$

$$p_1 + \rho_1 V^2 + \frac{1}{2}\mu h_1^2 = p_2 + \rho_2 (V - u_2)^2 + \frac{1}{2}\mu h_2^2, \qquad (2.13)$$

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}V^2 + \frac{\mu h_1^2}{\rho_1} + \frac{q_1}{\rho_1 V} = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(V - u_2)^2 + \frac{\mu h_2^2}{\rho_2},$$

$$T_1 = T_2,$$

Where the subscript '2' denotes the conditions immediately behind the shock front, and $V = \frac{dr_s}{dt}$ denotes the velocity of the shock front.

From equations (2.13), we get

$$u_2 = (1-\beta)V,$$

$$\rho_2 = \frac{\rho_1}{\beta},$$

$$h_2 = \frac{h_1}{\beta},$$

(2.14)

$$p_2 = \left[\frac{1}{\gamma M^2} + \frac{1}{2} M_A^{-2}\left(1 - \frac{1}{\beta^2}\right) + (1-\beta)\right]\rho_1 V^2,$$

$$q_2 = (1-\beta)\left[\frac{\bar{b}}{\gamma M^2 (1+\bar{b})} - \frac{1}{2}(1+\beta) + \frac{1}{\beta} M_A^{-2}\right]\rho_1 V^3,$$

Where $M = \left(\frac{\rho_1 V^2}{\gamma p_1}\right)^{\frac{1}{2}}$ is the shock-Mach number referred to the frozen speed of sound $\left(\frac{\gamma p_1}{\rho_1}\right)^{\frac{1}{2}}$, $M_A = \left(\frac{\rho_1 V^2}{\mu h_1^2}\right)^{\frac{1}{2}}$ is the Alfven-Mach number and $\bar{b} = \rho_1 b$ is the parameter of non-idealness of the gas.

The quantity $\beta(0 < \beta < 1)$ is obtained by

$$\beta^3 - \beta^2\left(\frac{1}{\gamma M^2} + 1 + \frac{1}{2} M_A^{-2}\right) + \beta\left(\frac{1}{\gamma M^2(1+\bar{b})}\right) + \frac{\bar{b}}{\gamma M^2(1+\bar{b})} + \frac{1}{2} M_A^{-2} = 0. \qquad (2.15)$$

***Self-similarity transformations***

*T*he inner boundary of the flow-field behind the shock is assumed to be an expanding piston. In the framework of self-similarity (Sedov, 1959). the velocity $U_p = \frac{dr_p}{dt}$ of the piston is assumed to follow a power law which results in (Steiner and Hirschler, 2002; Vishwakarma and Nath, 2009, 2011; Nath, (2007, 2011).

$$U_p = \frac{dr_p}{dt} = U_0\left(\frac{t}{t_0}\right)^n, \qquad (3.1)$$

Where $r_p$ is the radius of the piston and $t_0$ denotes the time at a reference state, $U_0$ is the piston velocity at $t = t_0$ and $n$ is a constant. The consideration of ambient pressure $p_1$ and the ambient magnetic field $h_1$ sets a value of $n$ as $n = -\frac{1}{3}$ (see equation (3.5)). Thus, the piston velocity jumps almost instantaneously, from zero to infinity leading to the formation of a shock of high strength in the initial phase. The piston is then decelerated. Concerning the shock boundary conditions, self-similarity requires that the velocity of the shock $V = \frac{dr_s}{dt}$ is proportional to the velocity of the piston, that is

$$V = \frac{dr_s}{dt} = CU_0\left(\frac{t}{t_0}\right)^n, \qquad (3.2)$$

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***Research Article***

Where '*C*' is a dimensionless constant. The time and space co-ordinate can be transformed into a dimensionless self-similarity variable as follows:

$$\eta = \frac{r}{r_s} = \left[\frac{(n+1)t_0^{\ n}}{CU_0}\right]\left(\frac{r}{t^{n+1}}\right). \qquad (3.3)$$

Evidently, the variable $\eta$ assumes the value '1' at the shock front and $\eta = \eta_p = \frac{r_p}{r_s}$ at the piston. To obtain the similarity solutions, we write the flow variables in the form (Ghoniem *et al.*, 1982; Abdel-Raouf and Gretler, 1991; Vishwakarma and Yadav, 2003; Vishwakarma and Singh, 2008).

$u = VU(\eta)$, $\rho = \rho_1 D(\eta)$, $p = \rho_1 V^2 P(\eta)$, $\sqrt{\mu}h = \sqrt{\rho_1}VH(\eta)$, $q = \rho_1 V^3 Q(\eta)$, (3.4) where *U*, *D*, *P*, *H* and *Q* are functions of $\eta$ only.

For the existence of similarity solutions *M* and $M_A$ should be constant, therefore

$$n = -\frac{1}{3}. \qquad (3.5)$$

Thus,

$$M^2 = \frac{2M_A^{\ 2}}{\gamma\left[1 + 2G_0(n+1)M_A^{\ 2}\right]}, \qquad (3.6)$$

Where $G_0 = G^* m\left(\frac{t_0^{\ n}}{CU_0}\right)^3$ is the gravitational parameter .

With the help of equations (3.4), the conservation equations (2.1) to (2.4) can be transformed into a system of ordinary differential equations with respect to $\eta$

$$(U-\eta)\frac{dD}{d\eta} + D\frac{dU}{d\eta} + \frac{2UD}{\eta} = 0, \qquad (3.7)$$

$$(U-\eta)\frac{dU}{d\eta} + \left(\frac{n}{n+1}\right)U + \frac{1}{D}\frac{dP}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + \frac{H^2}{D\eta} + \frac{(n+1)G_0}{\eta^2} = 0, \qquad (3.8)$$

$$(U-\eta)\frac{dH}{d\eta} + \left(\frac{n}{n+1}\right)H + H\frac{dU}{d\eta} + \frac{HU}{\eta} = 0, \qquad (3.9)$$

$$(U-\eta)\frac{dP}{d\eta} + \left(\frac{2n}{n+1}\right)P - \frac{\gamma P(1+\bar{b}D)(U-\eta)}{D}\frac{dD}{d\eta} + \frac{2(\gamma-1)(1+\bar{b}D)Q}{\eta} + (\gamma-1)(1+\bar{b}D)\frac{dQ}{d\eta} = 0. \qquad (3.10)$$

By using equations (2.6), (2.7) and (2.11) in (2.5), we get

$$q = -\left[\frac{k_0}{T_0^{\ \beta_C}\rho_0^{\ \delta_C}}T^{\beta_C}\rho^{\delta_C} + \frac{16\sigma T_0^{\ \beta_R}\rho_0^{\ \delta_R}}{3\alpha_{R_0}}T^{-\beta_R+3}\rho^{-\delta_R}\right]\frac{\partial T}{\partial r}. \qquad (3.11)$$

Using equations (2.8) and (3.4) in (3.11), we get

$$Q=-(n+1)\left[\begin{array}{l}\dfrac{k_0}{T_0^{\beta_C}\rho_0^{\delta_C}\Gamma^{\beta_C+1}}\left\{\dfrac{P}{(1+\bar{b}D)}\right\}^{\beta_C}V^{2\beta_C-2+\frac{1}{n}}\rho_1^{\delta_C-1}D^{\delta_C-\beta_C}\dfrac{(CU_0)^{\frac{1}{n}}}{t_0}\\ +\dfrac{16\sigma T_0^{\beta_R}\rho_0^{\delta_R}}{3\alpha_{R_0}\Gamma^{4-\beta_R}}\left\{\dfrac{P}{(1+\bar{b}D)}\right\}^{3-\beta_R}V^{4-2\beta_R+\frac{1}{n}}\rho_1^{-\delta_R-1}D^{-\delta_R+\beta_R-3}\dfrac{(CU_0)^{\frac{1}{n}}}{t_0}\end{array}\right]\frac{\partial}{\partial\eta}\left[\frac{P}{(1+\bar{b}D)D}\right]. \tag{3.12}$$

Equation (3.12) shows that similarity solution of the present problem exists only when

$$\beta_C=1+\frac{1}{2n} \quad \text{And} \quad \beta_R=2-\frac{1}{2n}. \tag{3.13}$$

Therefore, equation (3.12) becomes

$$Q=-X\left(\frac{1}{(1+\bar{b}D)D}\frac{dP}{d\eta}-\frac{P}{D^2}\frac{dD}{d\eta}\right), \tag{3.14}$$

Where

$$X=(n+1)\left(\frac{P}{(1+\bar{b}D)D}\right)^{1+\frac{1}{2n}}\left[\Gamma_C D^{\delta_C}+\Gamma_R D^{-\delta_R}\right],$$

and $\Gamma_C$ and $\Gamma_R$ are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters $\Gamma_C$ and $\Gamma_R$ depend on the thermal conductivity $k$ and the mean-free path of radiation $\frac{1}{\alpha_R}$, respectively and also on the exponent $n$ and they are given by

$$\Gamma_C=\frac{k_0\rho_1^{\delta_C}}{T_0\Gamma^2\rho_0^{\delta_C}t_0}\left[\frac{CU_0}{\sqrt{T_0\Gamma}}\right]^{\frac{1}{n}} \quad \text{and} \quad \Gamma_R=\frac{16T_0^{\,2}\rho_0^{\delta_R}\rho_1^{-\delta_R-1}}{3\alpha_{R_0}\Gamma^2 t_0}\left[\frac{CU_0}{\sqrt{T_0\Gamma}}\right]^{\frac{1}{n}}.$$

Using the similarity transformations (3.4) and the equation (3.2), equations (2.14) can be re-written as

$$\begin{aligned}&U(1)=(1-\beta),\\ &D(1)=\frac{1}{\beta},\\ &H(1)=\frac{M_A^{-1}}{\beta},\\ &P(1)=\left[\frac{1}{\gamma M^2}+\frac{1}{2}M_A^{-2}\left(1-\frac{1}{\beta^2}\right)+(1-\beta)\right],\\ &Q(1)=(1-\beta)\left[\frac{\bar{b}}{\gamma M^2(1+\bar{b})}-\frac{1}{2}(1+\beta)+\frac{1}{\beta}M_A^{-2}\right].\end{aligned} \tag{3.15}$$

By solving equations (3.7) to (3.10) and equation (3.14) for $\frac{dU}{d\eta}$, $\frac{dH}{d\eta}$, $\frac{dP}{d\eta}$, $\frac{dQ}{d\eta}$ and $\frac{dD}{d\eta}$, we have

$$\frac{dU}{d\eta}=-\left(\frac{U-\eta}{D}\right)\frac{dD}{d\eta}-\frac{2U}{\eta}, \tag{3.16}$$

$$\frac{dH}{d\eta}=\frac{H}{D}\frac{dD}{d\eta}-\left(\frac{n}{n+1}\right)\frac{H}{U-\eta}-\frac{HU}{\eta(U-\eta)}, \qquad (3.17)$$

$$\frac{dP}{d\eta}=\left[(U-\eta)^2-\frac{H^2}{D}\right]\frac{dD}{d\eta}-\left(\frac{n}{n+1}\right)UD+\frac{2UD(U-\eta)}{\eta}+\frac{H^2[(2n+1)\eta-2(n+1)U]}{(n+1)\eta(U-\eta)}-\frac{(n+1)G_0D}{\eta^2}, \qquad (3.18)$$

$$\frac{dQ}{d\eta}=\frac{(U-\eta)}{(\gamma-1)D}\left[\gamma P+\frac{H^2}{(1+\bar{b}D)}-\frac{(U-\eta)^2D}{(1+\bar{b}D)}\right]\frac{dD}{d\eta}-\frac{(U-\eta)}{(\gamma-1)(1+\bar{b}D)}\times$$
$$\left[\frac{2UD(U-\eta)}{\eta}-\left(\frac{n}{n+1}\right)UD+\frac{H^2[(2n+1)\eta-2(n+1)U]}{(n+1)\eta(U-\eta)}-\frac{(n+1)G_0D}{\eta^2}\right] \qquad (3.19)$$
$$-\left(\frac{2n}{n+1}\right)\frac{P}{(\gamma-1)(1+\bar{b}D)}-\frac{2Q}{\eta},$$

$$\frac{dD}{d\eta}=\frac{D^2}{\left[P(1+\bar{b}D)-(U-\eta)^2D+H^2\right]}\left[\begin{matrix}\frac{Q(1+\bar{b}D)}{X}-\left(\frac{n}{n+1}\right)U+\frac{2U(U-\eta)}{\eta}\\ +\frac{H^2[(2n+1)\eta-2(n+1)U]}{(n+1)\eta(U-\eta)D}-\frac{(n+1)G_0}{\eta^2}\end{matrix}\right]. \qquad (3.20)$$

Because of the dependence of the equations (3.15) to (3.20) on $\bar{b}$, similarity solution exists only when $\bar{b}$ is constant, i.e. only when the initial density $\rho_1$ is constant. The problem with the flow of a non-ideal gas is different from that of the perfect gas problem. In the latter case, similarity solution exists for initial density varying as some power of distance (Rogers, 1957; Rosenau, 1977; Singh, 1982; Ojha *et al.*, 1998). but it is not true for the problem with the flow of a non-ideal gas.
The total energy of the disturbance is given by

$$E=4\pi\int_{r_p}^{r_s}\rho\left(e+\frac{u^2}{2}+\frac{\mu h^2}{2\rho}-\frac{G^*m}{r}\right)r^2dr. \qquad (3.21)$$

Using equations (3.4) and (2.9), equation (3.21) becomes

$$E=4\pi\rho_1(CU_0)^{\frac{2}{(n+1)}}(n+1)^{\frac{2n}{(n+1)}}t_0^{\frac{-2n}{(n+1)}}r_s^{3+\frac{2n}{(n+1)}}\int_{\eta_p}^{1}\left[\frac{P}{(\gamma-1)(1+\bar{b}D)}+\frac{U^2D}{2}+\frac{H^2}{2}-\frac{G_0D(n+1)}{\eta}\right]\eta^2d\eta. \qquad (3.22)$$

Hence the total energy of the shock wave is non-constant and varies as $r_s^{3+\frac{2n}{(n+1)}}$.

The piston path coincides at $\eta_p=\frac{r_p}{r_s}$ a particle path. Using equations (3.1) and (3.4) the relation

$$U(\eta_p)=\eta_p=\frac{1}{C}=\frac{U_p}{V} \qquad (3.23)$$

Can be derived. In addition to the shock conditions (3.15), the kinematic condition (3.23) at the piston surface must be satisfied.
For exhibiting the numerical solutions, it is convenient to write the flow variables in the non-dimensional form as

$$\frac{u}{u_2}=\frac{U(\eta)}{U(1)},\ \frac{\rho}{\rho_2}=\frac{D(\eta)}{D(1)},\ \frac{p}{p_2}=\frac{P(\eta)}{P(1)},\ \frac{h}{h_2}=\frac{H(\eta)}{H(1)},\ \frac{q}{q_2}=\frac{Q(\eta)}{Q(1)}. \tag{3.24}$$

**RESULTS AND DISCUSSION**

Distribution of the flow variables in the flow-field behind the shock front are obtained by numerical integration of the equations (3.16) to (3.20) with the boundary conditions (3.15) by the Runge-Kutta mathod of fourth order. For the purpose of numerical integration, values of the constant parameters are taken to be ( Rosenau and Frankenthal, 1976; Rosenau, 1977; Ghoniem *et al.*, 1982; Vishwakarma *et al.*, 2007). $\gamma=\frac{5}{3}$; $M_A^{-2}=0,\ 0.01,\ 0.05,\ 0.1$; $\delta_C=1$; $\delta_R=2$; $\Gamma_C=0.1,\ 1,\ 10$; $\Gamma_R=10,\ 100,\ 1000$; $n=-\frac{1}{3}$; $G_0=0,\ 0.01,\ 0.05,\ 0.1$ and $\bar{b}=0,\ 0.05,\ 0.1$. The values $\bar{b}=0$ corresponds to the perfect gas case and $M_A^{-2}=0$ corresponds to the non-magnetic case. Also, $n=-\frac{1}{3}$ corresponds to the decelerated piston. The set of values $\delta_C=1,\ \delta_R=2$ is representative of the case of a high-temperature, low density medium (Ghoniem *et al.*, 1982). For a fully ionized gas $\gamma=\frac{5}{3}$ and therefore it is applicable to stellar medium. (Rosenau and Frankenthal, 1976), have shown that the effects of magnetic field on the flow-field behind the shock are significant when $M_A^{-2}\geq 0.01$; therefore the above values of $M_A^{-2}$ are taken for calculations in the present problem.

Table 1 shows the variation of density ratio $\beta$ across the shock front and the position of the piston surface $\eta_p$ for different values of $\bar{b}$ and $M_A^{-2}$ with $\gamma=\frac{5}{3}$, $n=-\frac{1}{3}$, $\delta_C=1,\ \delta_R=2$, $\Gamma_C=0.1$, $\Gamma_R=10$ and $G_0=0.05$. Table 2 shows the variation of density ratio $\beta$ across the shock front and the position of the piston surface $\eta_p$ for different values of $\bar{b}$ and $G_0$ with $\gamma=\frac{5}{3}$, $n=-\frac{1}{3}$, $\delta_C=1,\ \delta_R=2$, $\Gamma_C=0.1$, $\Gamma_R=10$ and $\mathrm{M}_\mathrm{A}^{-2}=0.01$.

Figures 1 to 5 and 6 to 10 show the variation of the flow variables $\frac{u}{u_2}$, $\frac{\rho}{\rho_2}$, $\frac{p}{p_2}$, $\frac{h}{h_2}$ and $\frac{q}{q_2}$ with $\eta$ for various values of the parameters $\bar{b}$ and $M_A^{-2}$, and $\bar{b}$ and $G_0$ respectively. It is shown that as we move inward from the shock front towards the inner contact surface (piston), the reduced fluid velocity $\frac{u}{u_2}$, the reduced magnetic field $\frac{h}{h_2}$ and the reduced total heat flux $\frac{q}{q_2}$ increase. These flow variables have higher values at the piston than that at the shock front. In the figures, it is also seen that the reduced density $\frac{\rho}{\rho_2}$ and the reduced pressure $\frac{p}{p_2}$ decrease, as we move inward from the shock front.

can be seen from the equation (3.20) for non-dimensional density *D*, there is a singularity at the piston where $U=\eta$ because this equation becomes singular there.

**Table 1:** Density ratio $\beta\left(=\frac{\rho_1}{\rho_2}\right)$ across the shock front and the position of the piston surface $\eta_p$ for different values of $\bar{b}$ and $M_A^{-2}$ with $\gamma=\frac{5}{3}, n=-\frac{1}{3}$, $G_0=0.05$, $\delta_C=1$, $\delta_R=2$, $\Gamma_C=0.1$ and $\Gamma_R=10$

| $\bar{b}$ | $M_A^{-2}$ | $\beta$ | **Position of the piston** $\eta_p$ |
|---|---|---|---|
| | 0 | 0.0333330 | 0.988012 |
| | 0.01 | 0.0956224 | 0.941219 |
| 0 | 0.05 | 0.2051780 | 0.869634 |
| | 0.1 | 0.3000000 | 0.809904 |
| | 0 | 0.0598532 | 0979275 |
| | 0.01 | 0.1070770 | 0.938922 |
| 0..05 | 0.05 | 0.2134630 | 0.867968 |
| | 0.1 | 0.3083540 | 0.808019 |
| | 0 | 0.0741826 | 0.974477 |
| | 0.01 | 0.1162940 | 0.937384 |
| 0.1 | 0.05 | 0.2206610 | 0.866618 |
| | 0.1 | 0.3157050 | 0.806598 |

**Table 2:** Density ratio $\beta\left(=\frac{\rho_1}{\rho_2}\right)$ across the shock front and the position of the piston surface $\eta_p$ for different values of $\bar{b}$ and $G_0$ with $\gamma=\frac{5}{3}$, $n=-\frac{1}{3}, M_A^{-2}=0.01$, $\delta_C=1$, $\delta_R=2$ , $\Gamma_C=0.1$ and $\Gamma_R=10$ .

| $\bar{b}$ | $G_0$ | $\beta$ | **Position of the piston** $\eta_p$ |
|---|---|---|---|
| | 0 | 0.0758872 | 0.947443 |
| | 0.01 | 0.0795334 | 0.946780 |
| 0 | 0.05 | 0.0956224 | 0.941219 |
| | 0.1 | 0.1187660 | 0.934383 |
| | 0 | 0.0775472 | 0.947230 |
| | 0.01 | 0.0833333 | 0.945605 |
| 0..05 | 0.05 | 0.1070770 | 0.938922 |
| | 0.1 | 0.1377420 | 0.930246 |

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***Research Article***

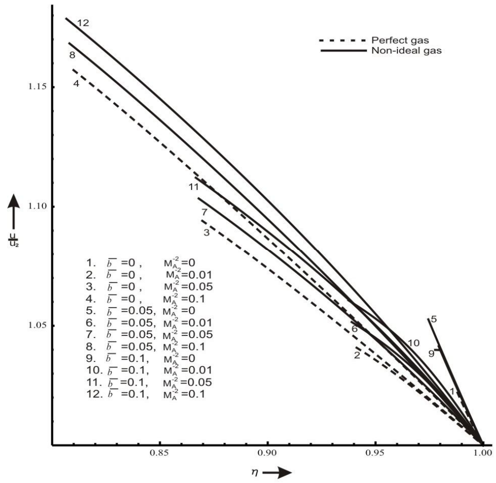


Figure 1: Variation of reduced velocity in the flow-field behind the shock with $G_0$=0.05

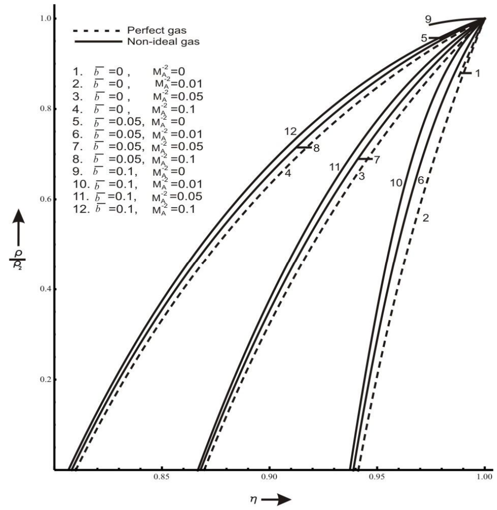


Figure 2: Variation of reduced density in the flow-field behind the shock with $G_0$=0.05

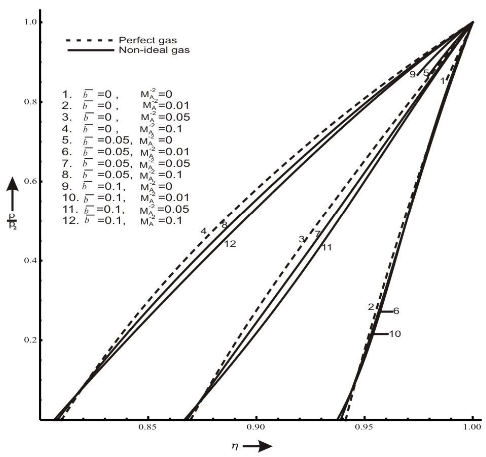


Figure 3: Variation of reduced pressure in the flow-field behind the shock with $G_0$=0.05

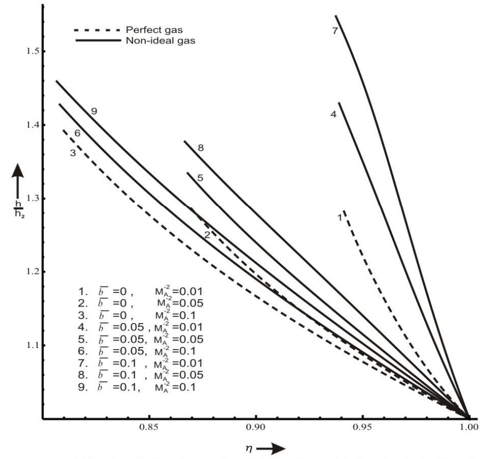


Figure 4: Variation of reduced magnetic field in the flow-field behind the shock with $G_0$=0.05

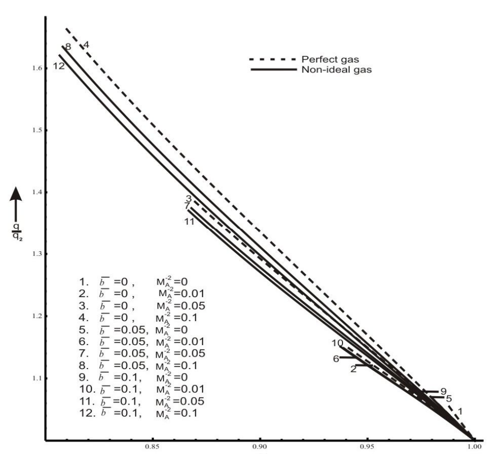


Figure 5: Variation of reduced total heat flux in the flow-field behind the shock with $G_0$=0.05

***Research Article***

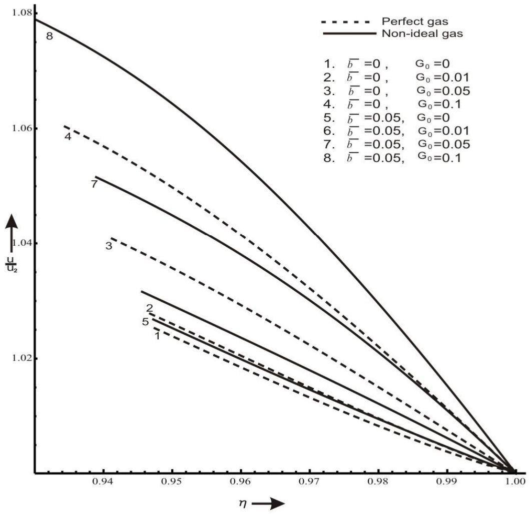


Figure 6: Variation of reduced velocity in the flow-field behind the shock with $M_A^{-2}$=0.01

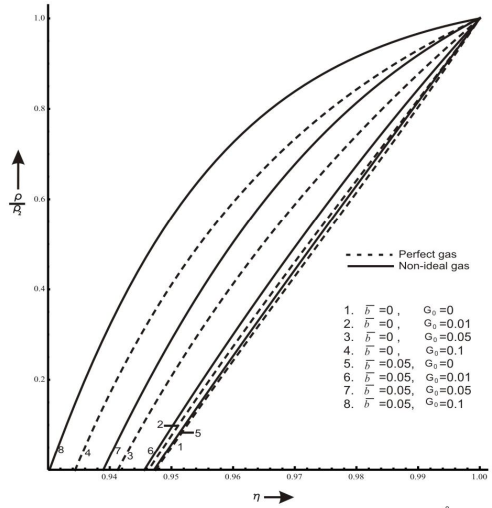


Figure 7: Variation of reduced density in the flow-field behind the shock with $M_A^{-2}$=0.01

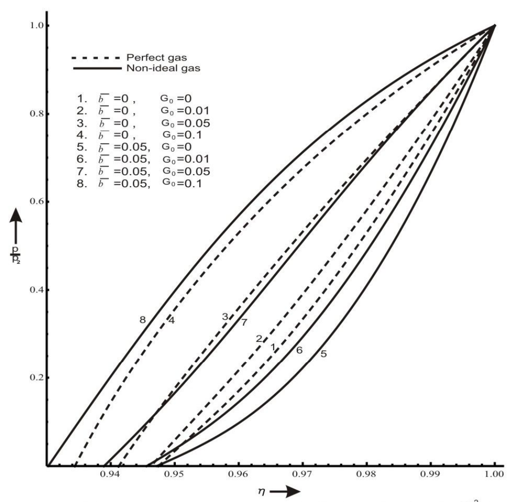


Figure 8: Variation of reduced pressure in the flow-field behind the shock with $M_A^{-2}$=0.01

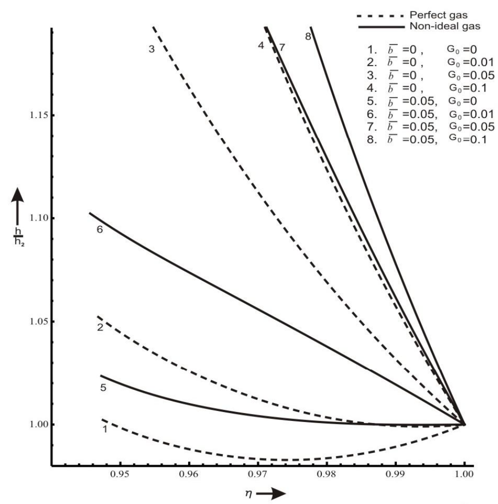


Figure 9: Variation of reduced magnetic field in the flow-field behind the shock with $M_A^{-2}$=0.01

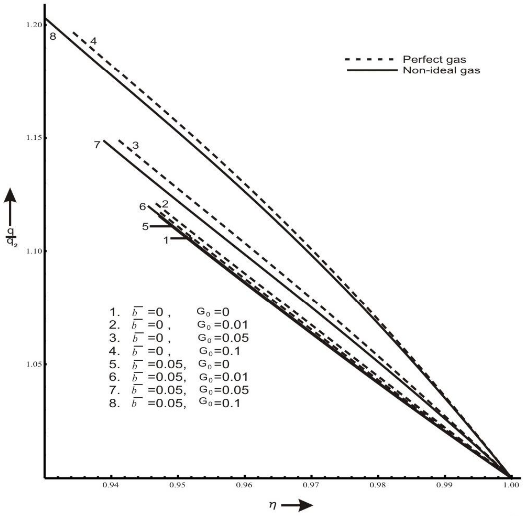


Figure 10: Variation of reduced total heat flux in the flow-field behind the shock with $M_A^{-2}$=0.01

The singularity is non-removable and the derivative of the density tends to negative infinity in magnetic case, as shown in figures 2 and 7. This singularity can be physically interpreted as follows (Steiner and Hirschler, 2002; Vishwakarma and Nath, 2009). the path of the decelerated piston diverges from the path of the particle immediately ahead rarefying the gas.

It is found that the effects of an increase in the value of the parameter of non-idealness $\bar{b}$ of the gas are:

To increase the value of $\beta$ (i.e. to decrease the shock strength, see Tables 1 and 2);

To increase the distance of the piston $(1-\eta_p)$ from the shock front (see Tables 1 and 2), i.e. the flow-field behind the shock becomes somewhat rarefied. This shows the same result as in (i), i.e. there is a decrease in the shock strength;

As To increase the reduced fluid velocity $\frac{u}{u_2}$, the reduced density $\frac{\rho}{\rho_2}$ and the reduced magnetic field $\frac{h}{h_2}$ at any point in the flow-field behind the shock front (see Figs 1, 2, 4, 6, 7 and 9);

To decrease the reduced total heat flux $\frac{q}{q_2}$ at any point in the flow-field behind the shock front (see Figs 5 and 10);

To increase the reduced pressure $\frac{p}{p_2}$ for $M_A^{-2}=0$ (non-magnetic case) and to decrease it when $M_A^{-2} \neq 0$ (see Figs 3 and 8).

The effects of an increase in the value of $M_A^{-2}$ (i.e. effects of an increase in the strength of ambient magnetic field) are:

To increase the value of $\beta$ (i.e. to decrease the shock strength, see Table 1);

To increase the distance of the piston from the shock front (see Table 1). Physically it means that the gas behind the shock is less compressed, i.e. the shock strength is reduced, which is same as given in (i) above;

To decrease the reduced velocity $\frac{u}{u_2}$, the reduced density $\frac{\rho}{\rho_2}$, the reduced pressure $\frac{p}{p_2}$ and the reduced total heat flux $\frac{q}{q_2}$ when $M_A^{-2}$ is increased from zero to 0.01, but to increase them when $M_A^{-2}$ is further increased from 0.01 to 0.1 (see Figs. 1, 2, 3 and 5);

To decrease the reduced magnetic field $\frac{h}{h_2}$ at any point in the flow-field behind the shock front (see Fig. 4).

The effects of an increase in the value of the gravitational parameter $G_0$ are:

To increase the value of $\beta$ (i.e. to decrease the shock strength, see Table 2);

To increase the distance of the piston from the shock front (see Table 2);

To increase the flow variables at any point in the flow-field behind the shock front (see Figs 6 - 10).

**REFERENCES**

**Marshak RE (1958)**. Effects of radiation on shock wave behaviour, Physics Fluids **1** 24-29. Elliot ss**LA (1960).** Similarity methods in radiation and hydrodynamics, Proceeding Royal Society London A **258**(3) 287-301.

**Wang KC (1964)**. The 'piston problem' with thermal radiation, Journal Fluid Mechanics **20**(3) 447-455.

**Helliwell JB (1969).** Self-similar piston problem with radiative heat transfer, *Journal Fluid Mechanics* **37**(3) 497-512.

**Nicastro JR (1970).** Similarity analysis of radiative gasdynamics with spherical symmetry, *Physics Fluids* **13(8)** 2000-2006.

**Ghoniem AF, Kamel MM, Berger SA and Oppenheim AK (1982).** Effects of internal heat transfer on the structure of self-similar blast waves, *Journal Fluid Mechanics* **117**(1) 473-491.

**Vishwakarma JP and Singh AK (2008).** Magnetogasdynamic shock waves in a non-uniform gas with heat conduction and radiation heat-flux, *International Journal of Applied Mechanics and Engineering* **13(3)** 797-815.

**Anisimov SI and Spiner OM (1972).** Motion of an almost ideal gas in the presence of a strong point explosion, *Journal of Applied Mathematics Mechanics* **36**(5) 883-887.

**Ranga Rao MP and Purohit NK (1976).** Self-similar problem in non-ideal gas, International Roberts **PH and Wu CC (1996).** Structure and stability of a spherical implosion, Physics Letters a **213**(1) 59-64.

**Madhumita G and Sharma VD (2004).** Imploding cylindrical and spherical shock waves in a non-ideal medium, *Journal of Hyperbolic Differential Equations* **1**(3) 521-530.

**Arora R and Sharma VD (2006).** Convergence of strong shock in a van der Waals gas, *SIAM Journal on Applied Mathematics* **66**(5) 1825-1837.

**Vishwakarma JP and Nath G (2007).** Similarity solutions for the flow behind an exponential shock in a non-ideal gas, Meccanica **42**(4) 331-339.

**Vishwakarma JP and Nath G (2009).** A self-similar solution of a shock propagation in a mixture of a non-ideal gas and small solid particles, Meccanica **44**(3) 239-254.

**Vishwakarma JP and Nath G (2010).** Propagation of a cylindrical shock wave in a rotating dusty gas with heat conduction and radiation heat flux, Physica Scripta, **81** 045401 (9pp).

**Vishwakarma JP and Nath G (2011).** Similarity solution for a cylindrical shock wave in a rotational axisymmetric dusty gas with heat conduction and radiation heat-flux, Communications in Non-linear Science and Numerical Simulation **17**(1) 154-169.

**Carrus P, Fox P, Hass F and Kopal Z (1951).** The propagation of shock waves in a steller model with continuous density distribution, *Astrophysical Journal* **113** 496-518.

**Rogers MH (1957).** Analytic solutions for blast wave problem with an atmosphere of varying density, *Astrophysical Journal* **152** 478-493.

**Ojha SN, Takhar HS and Nath O (1998).** Dynamical behaviours of an unstable magnetic star, *Journal MHD Plasma Research* **8** 1-14.

**Rosenau P (1977).** Equatorial propagation of axisymmetric magnetohydrodynamic shocks II, Physics Fluids **20** 1097-1103.

**Singh KK and Nath B (2011).** Self-similar flow of a non-ideal gas with increasing energy behind a magnetogasdynamic shock wave under a gravitational field, *Journal Theoretical Applied Mechanics* **49** (2) 501-513.

**Zel'dovich YaB and Raizer YuP (1967).** Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Vol II, Academic Press, New York.

**Rosenau P and Frankenthal S (1976).** Shock disturbances in a thermally conducting solar wind, *Astrophysical Journal* **208** 633-637.

**Rosenau P and Frankenthal S (1978).** Propagation of magnetohydrodynamic shocks in a thermally conducting medium, Physics Fluids **21** 559-566.

**Bhowmick JB (9181).** An exact analytical solution in radiation gas dynamics, *Astrophysics Space Science* **74** (2) 481-485.

**Singh JB and Srivastava SK (1982).** Propagation of spherical shock waves in an exponential medium with radiation heat flux, *Astrophysics Space Science* **88**(2) 277-282.

**Christer AH and Helliwell JB (1969).** Cylindrical shock and detonation waves in magnetogasdynamics, *Journal of Fluid Mechanics* **39**(4) 705-725.

**Summers D (1975).** An idealised model of a magnetohydrodynamic spherical blast wave applied to a flare produced shock in the solar wind, *Astronomy and Astrophysics* **45**(1) 151-158.

**Abdel-Raouf AM and Gretler W (1991).** Quasi-similar solutions for blast wave with internal heat transfer effects, *Fluid Dynamics Research* **8** 273-285.

**Gretler W and Wehle P (1993).** Propagation of blast waves with exponential heat release and internal heat conduction and thermal radiation, Shock Waves **3** 95-104.

**Pomraning GC (1973).** The Equations of Radiation Hydrodynamics, International Series of Monographs in Natural Philosophy, Pergamann Press, Oxford 54.

**Landau LD and Lifshitz EM (1958).** Course of Theoretical Physics, Statistical Physics, Vol 5, Pergaman Press, Oxford.

**Singh RA and Singh JB (1998).** Analysis of diverging shock waves in non-ideal gas, *Indian Journal of Theoretical Physics* **46**(2) 133-138.

**Ojha SN (2002).** Shock waves in non-ideal fluids, *International Journal of Applied Mechanics and Engineering* **7**(2) 445-464.

**Vishwakarma JP, Nath G and Singh KK (2008).** Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density, Physica Scripta **78** 035402 (11pp).

**Laumbach DD and Probestein RF (1970).** Self-similar strong shocks with radiation in a decreasing exponential atmospheres, Physics Fluids **13** 1178-1183.

**Sedov LI (1959).** Similarity and Dimensional Methods in Mechanics, Chapter IV, Academic Press, New York.

**Steiner H and Hirschler T (2002).** A self-similar solution of a shock propagation in a dusty gas, *European Journal of Mechanics- B/Fluids* **21** 371-380.

**Nath G (2007).** Shock waves generated by piston moving in a non-ideal gas in the presence of a magnetic field: Isothermal flow, *South East Asian Journal Mathematics and Mathemetical Science* **5**(2) 69-83.

**Nath G (2011).** Magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of a perfect gas with variable density, *Advances in Space Research* **47** 1463-1471.

**Vishwakarma JP and Yadav AK (2003).** Self-similar analytical solutions for blast waves in inhomogeneous atmosphere with frozen-in-magnetic field,*European Physical Journal-B* **34**(2) 247-253.

**Vishwakarma JP, Choube Vinay and Patel Arvind (2009).** Self-similar solution of a shock propagation in a non-ideal gas, *International Journal of Applied Mechanics and Engineering* 12(3) 813-830.*Journal of Engineering Science* **14**(1) 91-97.