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Research Article

SORET EFFECT ON LINEAR DDC IN A HORIZONTAL SPARSELY PACKED POROUS LAYER

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ABSTRACT
Double diffusive convection (DDC) in a horizontal sparcely packed porous layer in presence of Soret effect is studied analytically using a linear stability analyses. The linear theory is based on the usual normal mode technique. The flow in the porous matrix is modeled by using the Brinkman- extended Darcy model which accounts for friction due to macroscopic shear. The Rayleigh number for both stationary and oscillatory convection is obtained. The effect of Darcy number, Soret parameter, solute Rayleigh number for stationary and oscillatory modes are presented graphically. Also frequency oscillation graphs are drawn.

Key Words: *Double Diffusive Convection (Ddc) · Porous Layer · Soret Parameter· Rayleigh Number*

INTRODUCTION
The problem of double diffusive convection in porous media has attracted considerable interest during the few years because of its wide range of applications, from the solidification of binary mixture to the migration of solutes in water-saturated soils. Other examples include geophysical systems, electro-chemistry and the migration of moisture through air contained in fibrous insulation. Early studies on the phenomena of double diffusive convection in porous media are mainly concerned with the problem of convective instability in a horizontal layer heated and salted from below. During the last two decades there has been a great upsurge of interest in determining the effect of extensions to Darcy's law since many practical applications involve media for which Darcy's law is inadequate. A comprehensive review of the literature concerning double diffusive natural convection in a fluid saturated porous medium may be found in the book by Nield and Bejan (2006). Useful review articles on double diffusive convection in porous media include those by Trevisan and Bejan (1999), Mojtabi and Charrier–Mojtabi [(2000), (2005)] and Mamou (2000).

The problem of convective instability in horizontal porous layer has been a major topic in porous media research in recent years. Starting with the early works of Horton and Rogers (1945) and Lapwood (1948), several studies have been published, in which phenomena related to the onset of convection are investigated. An excellent review of most of these studies has been reported in Cheng (1978).

Of particular interest to the present study are investigations focusing on the phenomenon of double diffusive convection in a porous layer. To this end, Nield (1968), Rubin (1973) and Tauntan and Lightfoot (1972) relied on linear stability analysis to determine the critical values of the problem parameters and the nature (direct mode vs. overstable mode) of the instability. Relative to the large volume of published work focusing on double diffusion in classical fluids (see for e.g. Turner (1973)) double diffusion in porous media has received only limited attention. However, double diffusion in porous media has many engineering applications exemplified by geothermal systems, moisture migration in thermal insulations and stored grain, and the underground spreading of chemical pollutants.

Thermal convection in a binary fluid driven by the Soret and Duffour effect has been investigated by Knobloch (1980). He has shown that equations are identical to the thermo solutal problem except for a relation between the thermal and solute Rayleigh numbers. The double diffusive convection in porous medium in the presence of Soret and Duffour coefficients has been analyzed by Rudraiagh and

Malashetty (1986). This work has been extended to weak nonlinear analysis by Rudraiah and Sidheswar (1998). The effect of temperature dependent viscosity on double diffusive convection in anisotropic porous medium in the presence of Soret coefficient has been studied by Patil and Subramanian (1992). Barten et al. (1995) have observed the non linear traveling wave and stationary onset for the negative values of Soret coefficient in the mixtures of binary fluids. They used the linear stability analysis to find the criteria for the onset of oscillatory convection. A study of convective instability in a fluid mixture heated above with negative separation (Soret coefficient) was performed experimentally by Porta and Surko (1998). Straughan and Hutter (1999) have investigated the double diffusive convection with Soret effect in a porous layer using Darcy –Brinkman model. Bahloul *et al.,* (2003) have carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect. Hill (2005) performed linear and nonlinear stability analysis of double diffusive convection in a fluid saturated porous layer with a concentration based internal heat source using Darcy's law. Double diffusive natural convection within a multilayer anisotropic porous medium is studied numerically and analytically by Bennacer *et al.,* (2005). Recently, Mansour *et al.,* (2006) have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect.

A linear stability analysis was carried out by Poulikakos (1986) to study double diffusive convection in horizontal sparsely packed porous layer. To model the flow in the porous medium, the Brinkman-extended momentum equation Cheng (1978), Brinkman (1949) is used. This equation accounts for friction caused by macroscopic shear which is omitted in the Darcy flow model. The Results for pure viscous fluid and Darcy (densely packed) porous medium were obtained from his analysis as limiting cases.

Although some work on double diffusive convection in porous medium is available, attention has not been given to the study of double diffusive convection in a horizontal sparsely packed porous layer with Soret effect. The objective of this study is therefore to investigate the effect of Soret parameter on double diffusive convection in a horizontal sparsely packed porous layer using linear stability analysis. The effect of Darcy number, Soret parameter, solute Rayleigh number and frequency oscillation for stationary and oscillatory mode are presented graphically. However, our article provides quantitative results on the effect of Soret parameter on double diffusive convection (DDC) in a horizontal sparsely packed porous layer.

Mathematical Formulation

The system of interest consists of a horizontal porous layer bounded from above and below by two impermeable boundaries. These boundaries are kept at constant temperature and solute concentration. The convection adopted in this study is that if $\Delta T = T_L - T_U > 0$ or $\Delta S = S_L - S_U < 0$, the respective temperature or concentration gradient is destabilizing. The subscript L is lower surface and U is upper surface. Regarding the boundary conditions on the velocity field, both the horizontal walls are assumed to be impermeable and free (i.e. the shear stress vanishes on these walls). For sparcely packed porous media the Brinkman–extended Darcy flow model is appropriate to describe the fluid phenomenon in the porous matrix. The equations for the conservation of mass, momentum, energy and species are:

$$\nabla.\vec{\boldsymbol{q}} = 0, \tag{1}$$

$$\nabla.p = -\rho_0 g[1-\alpha_T(T-T_0)+\alpha_S(S-S_0)]\vec{k} - \frac{\mu}{K}q + \mu_e \nabla^2 \vec{q}, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t} + \vec{q}.\nabla T = D_1\nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + \vec{q}.\nabla S = [D_2 S + D_3 T]\nabla^2, \tag{4}$$

In the above equation $\vec{q} = u\vec{i} + w\vec{i}$ is the velocity vector, T the temperature, p the pressure, S the solute concentration, t the time, g the gravitational acceleration, K the porous medium permeability, ρ the fluid density, μ the fluid viscosity, ε the porosity and D_1 and D_2 the thermal and mass(solute) diffusivity of the porous medium respectively. S_0, T_0, ρ_0 are reference solute concentration, temperature and density respectively. Here D_3 quantifies the contribution to the mass flux due to the temperature gradient. The effective viscosity of the porous medium μ_e, may not, in general, be equal to the fluid viscosity. The parameter γ is the heat capacity ratio Cheng (1978) is defined as

$$\gamma = \frac{\varepsilon(\rho C_p)_f + (1-\varepsilon)(\rho C_p)_s}{(\rho C_p)_f}, \tag{5}$$

Where ε is the porosity

It is worth noting that in writing governing Eqs. (1)– (4), the Boussinesq approximation was taken into account , whereby the density was assumed to be constant everywhere except in the buoyancy term in the momentum Eq.(2) where it was approximated by the following linear function of temperature and species concentration

$$\rho = \rho_0[1-\alpha_T(T-T_0)+\alpha_S(S-S_0)], \tag{6}$$

The constant α_T and α_S are the coefficients of thermal and solute expansion, respectively. Subscript 0 stands for a reference state.

Basic State

The basic state of a fluid assumed to be a quiescent and is given by

$$\vec{q}_b = (u,v,w) = (0,0,0),\ p = p_b(z), T = T_b(z), \rho = \rho_b(z), S = S_b(z). \tag{7}$$

Using (7), Eqs. (1)- (4) One can obtain

$$\frac{dp}{dz} = -\rho g, \frac{d^2T_b}{dz^2} = 0, \frac{d^2S_b}{dz^2} = 0,\ \rho_b = \rho_0[1-\beta_T(T_b-T_0)+\beta_S(S_b-S_0)]. \tag{8}$$

Perturbed State

On the basic state, we superpose small perturbation in the form

$$\mathbf{q} = \mathbf{q}_b(z) + \mathbf{q}', \rho = \rho_b(z) + \rho', T = T_b(z) + T', S = S_b(z) + S', p = p_b(z) + p' \tag{9}$$

Where the primes indicate perturbations. We consider any two dimensional distribution and define stream function ψ by

$$u' = \frac{\partial \psi}{\partial z},\ w' = -\frac{\partial \psi}{\partial x}\quad . \tag{10}$$

Introducing (9) in Eqs. (1)- (4) And using basic Eqs. (8) and the transformations

$$(x,z) = (x^*, z^*)/d,\ \psi = \frac{\psi'}{D_1}, t = t^*/\frac{d^2\gamma}{D_1}, T = \frac{T'}{\Delta T}, S = \frac{S'}{\Delta S}\quad , \tag{11}$$

To render the resulting equations dimensionless we (after dropping the asterisks for simplicity),

$$(Da\nabla^2 - 1)\nabla^2\psi = R_T\frac{\partial T}{\partial x} - \tau R_S\frac{\partial S}{\partial x}, \tag{12}$$

$$(\frac{\partial}{\partial t} - \nabla^2)T = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, T)}{\partial(x,z)}, \tag{13}$$

$$(\frac{\varepsilon}{\gamma}\frac{\partial}{\partial t} - \tau\nabla^2)S - Sr\frac{R_T}{R_S}\nabla^2 T = -\frac{\partial \psi}{\partial x} + \frac{\partial(\psi, S)}{\partial(x,z)}, \tag{14}$$

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Where,

$Da = \frac{\mu_e}{\mu}\frac{K}{d^2}$, Darcy number; $\tau = \frac{D_2}{D_1}$, Diffusivity ratio; $Sr = \frac{D_2}{D_1}\frac{\alpha_S}{\alpha_T}$, Soret parameter; $R_T = \frac{\alpha_T g \Delta T d K}{\nu D_1}$,Thermal Rayleigh number; $R_S = \frac{\alpha_S g \Delta S d K}{\nu D_2}$, Solute Rayleigh number.

The boundary conditions at the two horizontal walls are

$$\psi = 0, \frac{\partial^2 \psi}{\partial z^2} = 0, T = S = 0$$

at z = 0, 1. (15)

These boundary conditions state that all perturbation quantities vanish on the walls of the system and both the walls are shear stress free.

Linear Stability Theory

In this section, we discuss the linear stability analysis. To make this study, we neglect the Jacobians in eqs. (12) - (14) and assume the solutions to be periodic waves of the form

$$\begin{bmatrix} \psi \\ T \\ S \end{bmatrix} = e^{\sigma t}\begin{bmatrix} \psi_0 sin(\alpha x) \\ \theta_0 cos(\alpha x) \\ \phi_0 cos(\alpha x) \end{bmatrix} Sin(n\pi z), \quad \text{(n= 1, 2, 3…)}, \tag{16}$$

Where σ is the growth rate and in general a complex quantity $(\sigma = \sigma_r + i\sigma_i)$, and α is horizontal wave number and $n\pi$ is vertical wavenumber. Substituting Eqs. (21) In linearized version of Eqs. (17)- (19), we get

$$k^2(k^2 Da + 1)\psi_0 = R_T(\alpha\theta_0) + \tau R_S(\alpha\phi_0), \tag{17}$$

$$(P + k^2)\theta_0 = -\alpha\psi_0, \tag{18}$$

$$\frac{\gamma}{\varepsilon}\sigma + \tau k^2)\phi_0 + Sr\frac{R_T}{R_S}k^2\theta_0 = -\alpha\psi_0, \tag{19}$$

Where $k^2 = \alpha^2 + (n\ \pi)^2$ is total wave number?

For a non-trivial solution ψ_0, θ_0, ϕ_0 we require

$$\begin{aligned} &k^2(k^2 Da + 1)\frac{\varepsilon}{\gamma}\sigma^2 + [k^4(k^2 Da + 1)(\frac{\varepsilon}{\gamma} + \tau) - \alpha^2(R_T\frac{\varepsilon}{\gamma} - R_S\tau)]\sigma \\ &+ \tau k^6(k^2 Da + 1) - ((1 + Sr)R_T - R_S)\tau\alpha^2 k^2 = 0 \end{aligned} . \tag{20}$$

The onset of double diffusive convection is described by the solution of Eq. (20) for various combinations of parameters.

$$R_T = \frac{(\sigma + k^2)(\frac{\varepsilon}{\gamma}\sigma + \tau k^2)(k^2(k^2 Da + 1))}{\alpha^2[(\frac{\varepsilon}{\gamma}\sigma + \tau k^2) + \tau k^2 Sr]} + \frac{R_S\tau\alpha^2(\sigma + k^2)}{\alpha^2[(\frac{\varepsilon}{\gamma}\sigma + \tau k^2) + \tau k^2 Sr]}. \tag{21}$$

Stationary State

If σ is real, then marginal stability occurs when $\sigma = 0$. Then eq. (21) gives the stationary Rayleigh number R_T^{st} at the marginal stability in the form

$$R_T^{\ st} = \frac{(\alpha^2+\pi^2)^2}{\alpha^2(1+Sr)}[(\alpha^2+\pi^2)Da+1] + \frac{R_S}{(1+Sr)}. \qquad (22)$$

The minimum value of Rayleigh number $R_T^{\ st}$ occurs at the critical wave number $\alpha = \alpha_c$,
Where $\alpha^2 = x$ and satisfies the equation

$$2Dax^2 + (1+Da\pi^2)x - \pi^2(1+Da\pi^2) = 0, \qquad (23)$$

Where the critical wave number is given by

$$\alpha^2_{\ cr} = \frac{-(1+Da\pi^2) + \{(1+Da\pi^2)^2 + 8Da\pi^2((1+Da\pi^2)\}^{1/2}}{4Da}. \qquad (24)$$

Oscillatory State

The boundary defining the region in which over stable modes take place is obtained from (21) for $\sigma = i\sigma_i(\sigma_r = 0, \sigma_i \neq 0)$ and is given by

$$R_T^{\ osc} = \frac{(\alpha^2+\pi^2)^2}{\alpha^2}(1+\lambda)[(\alpha^2+\pi^2)Da+1] + \lambda R_S \qquad (25)$$

With the non-dimensional frequency $\sigma_i^{\ 2}$ in the form

$$\sigma_i^{\ 2} = \lambda\left[-\lambda(\alpha^2+\pi^2)^2(1+Sr) + \alpha^2\frac{(1-\lambda)}{((\alpha^2+\pi^2)Da+1)}(1+Sr)R_S\right] \qquad (26)$$

RESULT AND DISCUSSION

The onset of double diffusive convection in horizontal sparsely packed porous layer with Soret effect is investigated analytically using the linear theory. In this linear stability theory the expressions for stationary and oscillatory Rayleigh numbers are obtained analytically along with expression for frequency of oscillation.

The effect of Darcy number Da on the critical thermal Rayleigh number and frequency of oscillation values of governing parameters are depicted in Figs. 1-5. Fig. 1 shows that the effect of Darcy number on critical Rayleigh number for different values of soret parameter Sr and for a fixed value R_S =150. We observed from this figure that as Sr increases the critical thermal Rayleigh number $R_T^{\ st}$ decreases implying that the increase of soret effect is to destabilize the system. The effect of Darcy number Da on the critical thermal Rayleigh number for stationary mode instability $R_T^{\ st}$ is illustrated in fig.2. We observe from this figure that increase in the value of R_S is to destabilize the system while increase in negative values of R_S decreases the Rayleigh number implying the effect of R_S is to destabilize the system.

Fig.3 depicts the effect of Darcy number Da on critical thermal Rayleigh number for dimensionless group λ in oscillatory mode. We find from this figure that oscillatory critical Rayleigh number $R_T^{\ osc}$ increases with increase in the value of λ when the concentration gradient R_S =150 is fixed. We also observed from the same figure that when R_S =-150 is fixed, decreases oscillatory Rayleigh number $R_T^{\ osc}$ with increase in the value of λ implying that the effect of λ is to destabilize the system.

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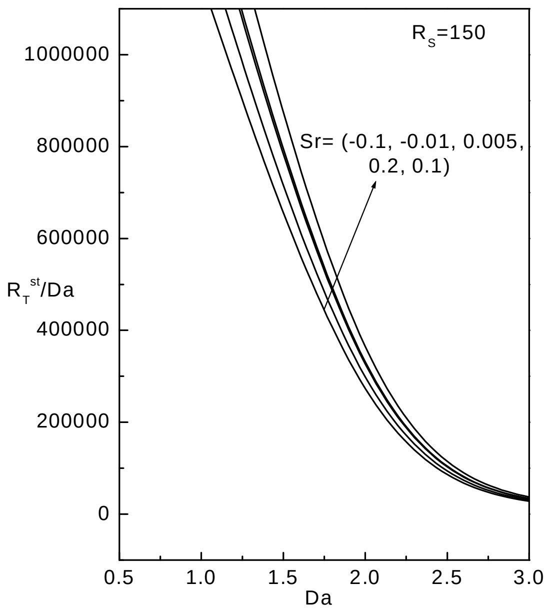

Figure 1: The effect of Darcy number Da on the stationary Rayleigh number for different values of Soret parameter Sr

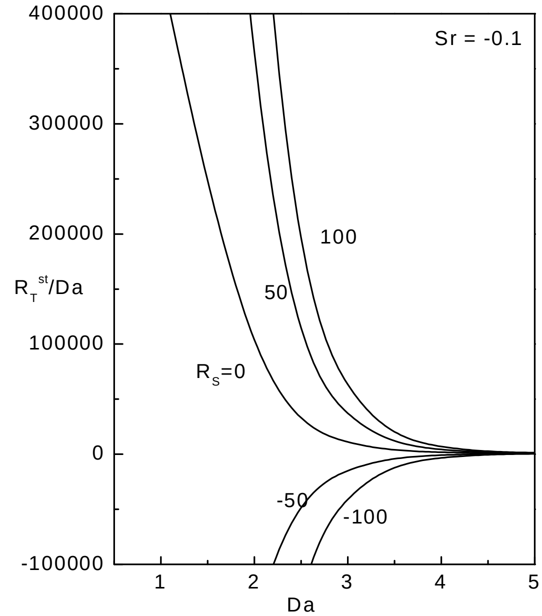

Figure 2: The effect of Darcy number Da on the stationary Rayleigh number for different values of solute Rayleigh number R_S

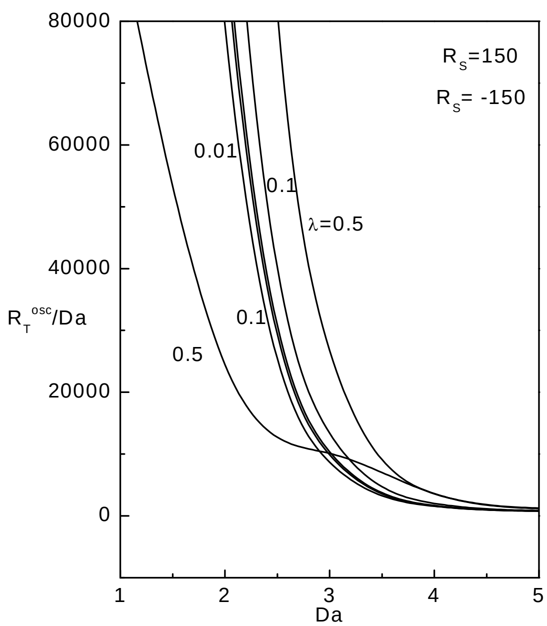

Figure 3: The effect of Darcy number Da on the oscillatory Rayleigh number for different values of dimensionless group λ

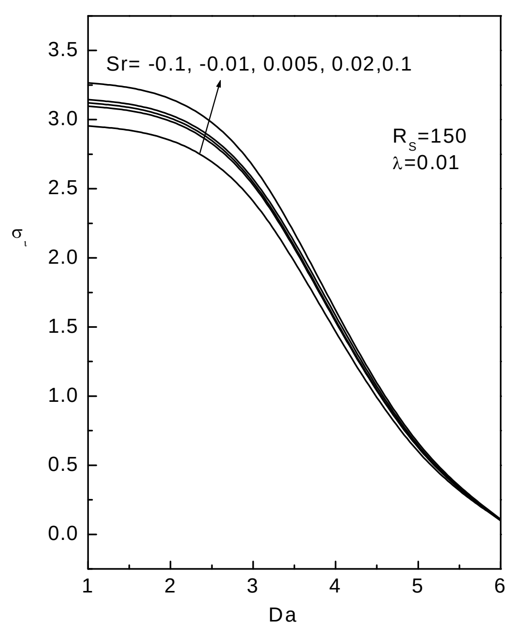

Figure 4: The effect of Darcy number Da on the frequency oscillation for different values of Soret parameter Sr

Fig.4 shows that the effect of Darcy number on frequency of oscillation for fixed values of R_S=150, λ=0.001. It is observed from this figure that as increasing the soret parameter Sr increases the frequency of oscillation.

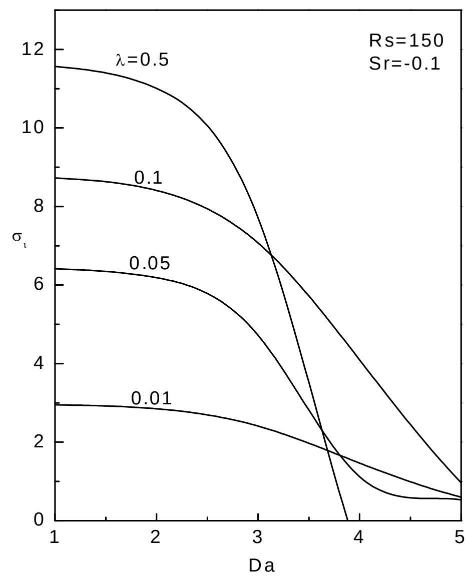

Figure 5: The effect of Darcy number Da on the frequency oscillation for different values of dimensionless group λ

The effect of Darcy number Da on the frequency of oscillation for a different values of λ and for a fixed values of Sr = -0.1 and R_S=150 is shown in fig.5. From this figure we observed that increasing dimensional group λ increases the frequency of oscillation.

CONCLUSIONS

The double diffusive convection in horizontal sparsely packed porous layer in the presence of Soret effect is studied analytically. The linear theory depends on normal mode technique. The flow in the porous matrix is modeled by using the Brinkman-extended Darcy model which accounts for a friction due to macroscopic shear. The effect of Darcy number, Soret parameter, solute Rayleigh number and frequency oscillation for stationary and oscillation mode are presented graphically. From the present analytical investigation the following conclusions are drawn:

When R_S is positive, the solute gradient is stabilizing, while negative values of R_S correspond to destabilizing solute gradients.

It is reasonable to state that for Da>0, the system R_T^{st} approaches that of classical fluids.

For a fixed values of Da and for stabilizing solute gradients (R_S=150) increasing λ increases R_T^{osc}.

When the solute gradient is destabilizing (R_S= -150) increasing λ decreases R_T^{osc}. As the value of λ decreases the effect of R_S on R_T^{osc} weakens considerably.

Increases Da decreases the frequency oscillation. For a fixed value of Da, increasing λ increases the frequency of oscillation. In the Darcy flow limit (small values of Da) the dependence of σ_i on Da is marginal.

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