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Research Article

LRS BIANCHI TYPE-I COSMOLOGICAL MODELS WITH PERFECT FLUID IN GENERAL RELATIVITY

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ABSTRACT
Einstein's field equations with variable gravitational constant and cosmological constants are considered in the presence of perfect fluid for LRS Bianchi type -I universe by assuming the cosmological term proportional to R^{-m} (R is scale factor and m is a constant). A variety of solutions are presented. The physical significance of the cosmological models have also been discussed.

Key Words: *Cosmology. Variable cosmological Term Perfect Fluid Models.* PACS 98.80, Es, 98.80. –k

INTRODUCTION

The Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form of the matter content or suppose that space-time admits killing vector symmetry (Kramer, 1980).

Solutions to the field equations may also be generated by law of variation of scale factor which was proposed by Pavon, (1991). The behavior of the cosmological scale factor R (t) in solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies. In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Holye *et al.*, (1997); Olson *et al.*, (1987); Pavon, (1991); Maia *et al*., (1994); Silveria *et al.*, (1994,1997); Torres *et al.*, (1996). Chen and Wu, (1990) considered Λ varying R^{-2} (R is the scale factor), Carvalho and Lima, (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background.

The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac, (1937); Lau (1985) working in the framework of general relativity, proposed modification linking the variation of G with that of Λ. This modification allows us to use Einstein's field equations formally unchanged since variation in Λ is accompanied by a variation of G. A number of authors investigated FRW models and Bianchies models, using this approach (Abdel-Rahman, 1990; Berman, 1991; Sisterio, 1991; Kalligas,*et al.*,1992; Singh *et al*.,2007; Vishwakarma,1991,2000,2005; Pradhan, 2006; Singh *et al*., 2007). Borges and Carneiro, (2005); Singh and Tiwari, (2008) have considered as cosmological term is proportional to the Hubble parameter in FRW model and LRS Bianchi type-I model respectively. Recently, Tiwari, (2008, 2009) considered as cosmological term is proportional to Hubble parameter in Bianchi type-I and R–W models with varying G andΛ.

In this paper we study homogeneous LRS Bianchi type -I space time with variables G and Λ containing matter in the form of perfect fluid. We obtain solutions of the field equations assuming that cosmological term is proportional to R^{-m} (where R is scale factor and m is constant). The paper is organized as follows.Basic equations of the models in sec. 2. and solution in sec. 3. We discuss the models and conclude our results in sec.4.

MODEL AND FILED EQUATIONS

We consider the space-time admitting LRS Bianchi type-I group of motion in the form

$$ds^2 = -dt^2 + A^2 (t)\, dx^2 + B^2 (t)\, (dy^2 + dz^2) \quad .....(1)$$

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \quad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

We take the equation of state

$$p = \omega\rho \quad , \quad 0 \le \omega \le 1 \quad .....(3)$$

The Einstein's field equations with time dependent G and Λ given by [28].

$$R_{ij} - ½ R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda(t) g_{ij} \quad .....(4)$$

for the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (4) yields.

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = -8\pi G p + \Lambda \quad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \quad ....(6)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 = 8\pi G \rho + \Lambda \quad ....(7)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \quad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j} = 0$, yields

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \quad ....(9)$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \quad ....(10)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t. From (10) implying that Λ is a constant whenever G is constant.

Let R be the average scale factor of LRS Bianchi type -I universe i.e.

$$R^3 = AB^2 \quad .....(11)$$

Using equation (3) in equation (9) and then integrating, we get,

$$\rho = \frac{k}{R^{3(\omega+1)}} \quad .....(12)$$

where k > 0 is constant of integration.

From (5) and (6), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \quad .....(13)$$

where k_1 is constant of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H = \frac{\theta}{3} = \frac{\dot{R}}{R}$$

$$\sigma = \frac{k}{\sqrt{3}R^3},$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(7) and (9) can be written in terms of H, σ and q as

$$H^2(2q-1) - \sigma^2 = 8\pi G p - \Lambda \quad .....(14)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \quad .....(15)$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \quad .....(16)$$

Overduin and Cooperstock, (1998) define

$$\rho_c = \frac{3H^2}{8\pi G} \quad .....(17)$$

$$\rho_v = \frac{\Lambda}{8\pi G} \quad .....(18)$$

and $\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}$ .....(19)

are respectively critical density, vacuum density and density parameter
From (15), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{8\pi G\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.
From (14), and (15), we have

$$\frac{d\theta}{dt} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3\sigma^2}{2} = -12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ, and for positive Λ, universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of R.

SOLUTION OF THE FIELD EQUATIONS

The system of equations (3), (5)-(7), and (10), supply only five equations in six unknowns (A, B,ρ p, G andΛ). One extra equation is needed to solve the system completely. The phenomenological Λ decay scenario have been considered by number of authors (Chen and Wu, 1990) considered $\Lambda \propto a^{-2}$ (a is the scale factor of the Robertson-Walker metric).

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Holye *et al.*, (1997) considered $\Lambda \propto a^{-3}$ whereas $\Lambda \propto a^{-m}$ (a is scale factor and m is constant) considered by Olson *et al.*, (1987); Pavon, (1991); Maia et al., (1994); Silveria et al., (1994,1997); Torres *et al.*, (1996).
Thus we take the decaying vacuum energy density

$$\Lambda = \frac{a}{R^m} \qquad .....(20)$$

where a and m are positive constants. Using eq. (12) and (20) in eq. (10), we get

$$G = \frac{am}{8\pi k}\frac{R^{3\omega+3-m}}{(3\omega+3-m)} \qquad .....(21)$$

from equations (14), (15), (20) and (21) we get

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{am(1-\omega)}{2(3\omega+3-m)R^m} - \frac{a}{R^m} = 0 \qquad .....(22)$$

Now we analyze for different values of ω :

Vacuum solution (cosmology for ω=0)
For ω =0, eq. (22) becomes

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{a(m-6)}{2(m-3)}\frac{1}{R^m} = 0 \qquad .....(23)$$

Determining the time evolution of Hubble parameter, integrating (23), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{a}{3-m}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ t + t_0\right]^{-1} \qquad .....(24)$$

where t_o is a constant of integration, The integration constant is related to the choice of origin of time.
From eq. (24), we obtain the scale factor

$$R = \left(\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ \mathrm{t} + t_0\right)^{2/m} \qquad .....(25)$$

By using eq (25) in (13), the metric (1), assumes the form

$$ds^2 = -dt^2 + \left(\frac{m}{2}\sqrt{\frac{a}{3-m}}t + t_0\right)^{\frac{4}{m}} \times$$

$$\left[m_1^2\ \exp\left\{\frac{8k_1}{3}\sqrt{\frac{3-m}{a}}\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\frac{a}{3-m}}t + t_0\right)^{\frac{m-6}{m}}\right\}dx^2\right. \qquad .....(26)$$

$$\left. + m_2^2\ \exp\left\{\frac{-4k_1}{3}\sqrt{\frac{3-m}{a}}\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\frac{a}{3-m}}t + t_0\right)^{\frac{m-6}{m}}\right\}(dy^2 + dz^2)\right]$$

where m_1 and m_2 are constants.

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For the model (26), the spatial volume V, matter density ρ, pressure p gravitational constant G, cosmological constant Λ are given by

$$V = \left(\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ \mathrm{t} + t_0 \right)^{6/m} \quad .....(27)$$

$$\rho = \frac{k}{\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ t + t \right]^{\frac{6}{m}}} \quad .....(28)$$

p =0 ......(29)

$$G = \frac{am}{8\pi k(3-m)}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ t + t_0 \right]^{\frac{2}{m}(3-m)} \quad .....(30)$$

$$\Lambda = a\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\ t + t_0 \right]^{-2} \quad .....(31)$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{a}{3-m}}\ \left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\, t + to \right]^{-1} \quad .....(32)$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\, t + to \right]^{-6/m} \quad .....(33)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{3} \quad ....(34)$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1 \quad ....(35)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(3-m)}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\, t + t_0 \right]^{-6/m} \quad .....(36)$$

$$\rho_c = \frac{3k}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-M}\right)}\ t + t_0 \right]^{-6/m} \quad .....(37)$$

Thus for the model (26), we observe that for 0<m<3 the spatial volume V is zero at t=t' where $t' = \dfrac{-t_0}{\frac{m}{2}\sqrt{\frac{a}{3-m}}}$ and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at t=t' with an infinite rate of expansion. The scale factors also vanish at t=t' and hence the space-time exhibits point type singularity at initial epoch. The energy density,

shear scalar diverges at the initial singularity. As t increases the scale factors and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also ρ, σ, ρ_v, ρ_c, Λ decrease as t increases. As $t \to \infty$ scale factors and volume become infinite whereas ρ, σ, ρ_v, ρ_c, and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant G(t) is zero at t =t' and as t increases G(t) also increases.
A partial list of cosmological models in which the gravitational constant G is increasing with time are contained in Abdussattar, (1997); Abdel-Rahman, (1990); Chow, (1981); Levitt, (1980); Milne, (1935). The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$ provided m<3. So the model approaches isotropy for large value of t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.
Further, it is observed that when 2<m<3, q>0; q=0 for m=2 and for 0<m<2, q<0. Therefore, the universe begins with a decelerating expansion changes and the expansion changes from decelerating phase to accelerating one.

Zel'dovich fluid distribution (Cosmology for $\omega = 1$)
It corresponds to the equation of state $\rho=p$, This equation of state has been widely used in general relativity to obtain stellar and cosmological models for utter dense matter (Zel'dovich, 1968). In this case equation (22) becomes

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{a}{R^m} = 0 \qquad ....(38)$$

Determining the time evolution of Hubble parameter, integrating (38) we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{2a}{6-m}}\left[\frac{m}{2}\sqrt{\left(\frac{2a}{6-m}\right)}\,t + t_0\right]^{-1} \qquad ....(39)$$

where the integration constant t_o is related to the choice of origin of time.
From (38) we obtain

$$R = \left[\frac{m}{2}\sqrt{\left(\frac{2a}{6-m}\right)}\,t + t_0\right]^{\frac{2}{m}} \qquad .....(40)$$

By using (40) in (13), the metric (1) assumes the form

$$ds^2 = -dt^2 + \left(\frac{m}{2}\sqrt{\frac{2a}{6-m}}\,t + t_0\right)^{\frac{4}{m}} \times$$
$$\left[m_1^2 \exp\left\{\frac{8k_1}{3}\sqrt{\frac{6-m}{2a}}\,\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\frac{2a}{6-m}}\,t + t_0\right)^{\frac{m-6}{m}}\right\}dx^2\right. \qquad .....(41)$$
$$\left. + m_2^2 \exp\left\{\frac{-4k_1}{3}\sqrt{\frac{6-m}{2a}}\,\frac{1}{m-6}\left(\frac{m}{2}\sqrt{\frac{2a}{6-m}}\,t + t_0\right)^{\frac{m-6}{m}}\right\}(dy^2 + dz^2)\right]$$

For the model (41), spatial volume V, matter density ρ, pressure p, gravitational constant G, cosmological constant Λ are given by

$$V = \left[\frac{m}{2} \sqrt{\left(\frac{2a}{6-m} \right)} t + t_0 \right]^{\frac{6}{m}} \qquad ....(42)$$

$$\rho = p = \frac{k}{\left(\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right)^{\frac{12}{m}}} \qquad .....(43)$$

$$G = \frac{am}{8\pi k(6-m)} \left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right]^{\frac{2}{m}(6-m)} \qquad ....(44)$$

$$\Lambda = \frac{a}{\left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right]^2} \qquad .....(45)$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{2a}{6-m}} \left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right]^{-1} \qquad .....(46)$$

$$\sigma = \frac{k}{\sqrt{3}} \left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + to \right]^{-6/m} \qquad ....(47)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{6} \qquad .....(48)$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1 \qquad .....(49)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(6-m)}{m} \left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right]^{-12/m} \qquad .....(50)$$

$$\rho_c = \frac{6k}{m} \left[\frac{m}{2} \sqrt{\frac{2a}{6-m}} t + t_0 \right]^{-12/m} \qquad .....(51)$$

In the model, we observe that for m < 6, the spatial volume V is zero at $t = \dfrac{-t_0}{\frac{m}{2}\sqrt{\frac{2a}{6-m}}} = t''$ and expansion scalar θ is infinite at t= t" which shows that the universe starts evolving with zero volume and infinite rate of expansion at t= t". Initially at t = t" the energy density ρ, pressure p, cosmological constant Λ and shear scalar σ are infinite. As t increases the spatial volume increases but the expansion scalar decreases. Thus the expansion rate decreases as time increases. As t tends to ∞ the spatial volume V becomes infinitely large.

As t increases all the parameters p, ρ, Λ, θ, ρ_c, ρ_v, decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that model approaches isotropy for large values of t. The gravitational constant G(t) is zero at t =t" and as t increases, G increases and it becomes infinite large at late times, which is a similar result given by Abdussattar and Vishwakarma, (1997); Abdel-Rahman, (1990); Chow, (1981); Levitt, (1980); Milne, (1935).

Further, we observe that $\Lambda \alpha \frac{1}{t^2}$ which follows from the model of Kalligas *et al.*, (1992); Berman, (1990); Berman and Som, (1990); Berman *et al.*, (1989); Bertolami, (1986a,1986b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

3.3 Radiation dominated solution (ρ =3p) [Cosmology for ω =1/3]

In this case eq. (22) becomes

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 + \frac{2(m-6)a}{3(4-m)R^m} = 0 \qquad .....(52)$$

Determining the time evolution of Hubble's parameter, integrating eq. (52), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{4a}{3(4-m)}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{-1} \qquad ....(53)$$

where t_0 is constant of integration and the integration constant t_0 is related to the choice of origin of time

From eq. (53) we obtain

$$R = \left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{2/m} \qquad .....(54)$$

By using eq. (54) in (13), the metric (1) assumes the form

$$ds^2 = -dt^2 + \left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m}{4}} \times$$

$$\left[m_1^2 \exp\left\{\frac{4k_1}{3}\frac{1}{(m-6)}\sqrt{\frac{3(4-m)}{a}}\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m-6}{m}}\right\}dx^2 \right. \qquad .....(55)$$

$$\left. + m_2^2 \exp\left\{\frac{-2k_1}{3}\frac{1}{(m-6)}\sqrt{\frac{3(4-m)}{a}}\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m-6}{m}}\right\}(dy^2 + dz^2)\right]$$

For the model (55), matter density ρ, pressure p, gravitational constant G, cosmological constant Λ are given by

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$$\rho = \frac{k}{\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right)^{\frac{8}{m}}} \qquad .....(56)$$

$$p = \frac{k}{3\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right)^{\frac{8}{m}}} \qquad .....(57)$$

$$G = \frac{am}{8\pi k(4-m)}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right]^{\frac{2}{m}(4-m)} \qquad ....(58)$$

$$\Lambda = \frac{a}{\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right]^{2}} \qquad .....(59)$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{4a}{3(4-m)}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right]^{-1} \qquad .....(60)$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + to\right]^{-6/m} \qquad ....(61)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{4} \qquad .....(62)$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1 \qquad .....(63)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(4-m)}{m\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right)^{\frac{8}{m}}} \qquad ......(64)$$

$$\rho_c = \frac{4k}{m\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}\,t + t_0\right)^{\frac{8}{m}}} \qquad .....(65)$$

Thus for the model (55), we observe that for 0<m<4 the spatial volume V is zero at t=t''' where t''' = $\dfrac{-t_0}{\dfrac{m}{2}\sqrt{\dfrac{4a}{3(4-m)}}}$ and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at t=t''' with an infinite rate of expansion. The scale factors also vanish at t=t''' and hence the space-time exhibits point type singularity at initial epoch. The energy density, shear scalar diverges at the initial singularity. As t increases the scale factors and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also ρ, σ, ρ_v, ρ_c, Λ decrease as t increases. As t $\to\infty$ scale factors and volume become infinite whereas ρ, σ, ρ_v, ρ_c, and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant G(t) is zero at t = t''' and as t increases G(t) also increases, which is a similar result obtained by Abdussattar, (1997); Abdel-Rahman, (1990); Chow, (1998); Levitt, (1981); Milne, (1980).The ratio $\dfrac{\sigma}{\theta} \to 0$ as $t \to \infty$ provided m < 4. So the model approaches isotropy for large value of t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.

CONCLUSIONS

In this paper we have studied a spatially homogeneous and isotropic LRS Bianchi type-I space-time with variable gravitational constant G(t) and cosmological constant Λ(t). The field equations have been solved exactly by using a law of variation of scale factor with a variable cosmological term i.e. a cosmological term that scales as $\Lambda \propto R^{-m}$ (where R is scale factor). Three exact LRS Bianchi type-I models have been obtained in sec 3.1, 3.2 and 3.3. Expressions for some important cosmological parameters have been obtained for all the models and physical behavior of the models is discussed in detail. All the cases, the models represent shearing, non-rotating and expanding model of the universe with a big-bang start approaching to isotropy at late times. It is interesting that, the proposed variation law provides an alternative approach to obtain exact solutions of Einstein's filed equations. It presents a unified description of the evolution of the universe which starts with a decelerating expansion and expands with acceleration at late times. Also, gravitational constant G(t) is zero at initial singularity and it is increasing with time increase which corresponds the results already given by Abdussattar, (1997) and others (vishwakarma,1990; chow1981; Levitt,1980). The cosmological constant $\Lambda(t) \propto 1/t^2$ which follows from the model of Kalligas *et al.*, (1992); Berman, (1990); Berman and Som, (1990); Berman *et al.*, (1989); Bertolami, (1986a,1986b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. Finally, the solutions presented in the paper are new and useful or better understanding of the evolution of the universe in LRS Bianchi type-I space-time with variable G and Λ.

REFERENCES

Abdel-Rahman A-M M (1990). A Critical Density Cosmological Model with Varying Gravitational and Cosmological "Constants" *General Relativity and Gravitation* **22**(6) 655.

Abdussattar, Vishwakarma RG (1997). Some FRW models with variable <B> G</B> and ? *Classical and Quantum Gravity* **14** 945.

Berman MS (1991). Cosmological Models with Variable Cosmological Term, *Physical Review D* ***43*** *1075-8.*

Berman MS(1990). Static universe in a modified brans-dicke cosmology. *International Journal of Theoretical Physics* **29** 567.

Berman MS, Som MM and Gomide FM (1989). Brans-Dicke Static Universes. *General Relativity and Gravitation* **21** 287.
Bertolami O (1986b). Brans-Dicke cosmology with a scalar field dependent cosmological term. Fortschr. Physics **34** 829.
Bertolami O (1986a). Time-dependence cosmological term. Nuovo Cimento B **93** 36.
Borges, HA and Carnerio S (2005).Friedmann cosmology with decaying vacuum density *General Relativity and Gravitation* **37(8)** *1385*-1394.
Carvalho JC, Lima JAS and Waga I (1992). Cosmological consequences of a time-dependent Λ term: *PhysicalReview D* **46(6)** 2404.
Chen W and Wu YS (1990). Implications of a cosmological constant varying as R^{-2} *Physical Review D*.**41** 695.
Chow TL (1981). The variability of the gravitational constant.Nuovo Cimento Lettere **31** 119.
Dirac PAM (1937). The Cosmological Constants *Nature*. **139**(3512) 323.
Holye F, Burbidge G, Narlikar J V (1997). On the Hubble constant and The Cosmological Constant. *Monthly Notices of the Royal Astronomical Society* **286** 173.
Kalligas D, Wesson P, Everitt CWF (1992). Flat FRW models with variable G and Λ *General Relativity and Gravitation* **24** 351.
Kramer D, Stephani H, MacCallum M and Herlt E (1980). Exact Solutions of EFE, Cambridge University Press, Cambridge .
Lau YK (1985). The large number hypothesis and Einstein's theory of gravitation *Journal of Phys*ics **38** 547.
Levitt LS (1980). The gravitational constant at time zero Nuovo Cimento, Lettere, Serie 2 **29** 23.
MD and Silva GS (1994). Geometrical constraints on the cosmological constant. *Physical Review D* **50** 7233 .
Milne EA (1935). Relativity, Gravitation and world structure, Oxford University Press, Oxford .
Olson TS and Jordan TF (1987). Ages of the Universe for decreasing cosmological constants *Physical Review D*. **35** 3258 .
Overduin JM and Cooperstock FI (1998). Evolution of the scale factor with a variable cosmological term *Physical Review D* **58** 043506 .
Pavon D (1991). Nonequilibrium fluctuations in cosmic vacuum decay. *Physical Review D* **43** 375.
Pradhan A and Otarod S (2006). Universe with Time Dependent Deceleration Parameter and Λ Term in General Relativity *Astrophysics and Space Science* **306** 11.
Silveira V and Waga I (1994). Decaying Λ cosmologies and power spectrum *Physical Review D*. **50** 4890 .
Silveria V and Waga J (1997). Cosmological properties of a class of Λ decaying cosmologies *Physical Review D*. **56** 4625.
Singh CP, Kumar S and Pradhan A (2007). Early viscous universe with variable gravitational and cosmological 'constants' *Classical and Quantum Gravity* **24** 455.
Singh JP and Tiwai RK (2008). Perfect fluid Bianchi Type-I cosmological models with time varying G and L. *Pramana Journal of Physics* **70** 565.
Singh JP and Tiwai RK I (2007). An LRS Bianchi Type-I Cosmological Model with Time-Dependent Λ Term. *International Journal of Modern Physics* D**16** 745.
Sistero RF (1991). Cosmology withG and Λ coupling scalars *General Relativity and Gravitation* **23** 1265.
Tiwari RK (2008). Bianchi type-I cosmological models with time dependent G and Λ *Astrophysics and Space Science* **318** 243.
Tiwari RK (2009). Some Robertson-Walker models with time dependent G and Λ *Astrophysics and Space Science* **321** 147.

Torres LFB and Waga I (1996). Decaying Lambda cosmologies and statistical properties of gravitational lenses. *Monthly Notices of the Royal Astronomical Society* **279** 712 .
Vishwakarma RG (2005). A model to explain varying Λ, G and σ^2 simultaneously *General Relativity and Gravitation* **37** 1305.
Vishwakarma RG (2000). A study of angular size-redshift relation for models in which Λ decays as the energy density *Class Quantum Gravity* **17** 3833.
Weinberg S (1972). Gravitational and Cosmology, Wiley, New York.