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***Research Article***

# BULK VISCOUS BIANCHI-I UNIVERSE WITH DECAYING VACUUM ENERGY

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**ABSTRACT**
In the present article, we have investigated Bianchi type-I cosmological model with varying cosmological constant in the presence of bulk viscous fluid by taking the condition $\zeta = \zeta_0 \theta$. Einstein's field equations have been solved by assuming the decay law $\Lambda = \beta \frac{\ddot{R}}{R} + \frac{1}{R^2}$ (Overduin and Cooperstock, 1998). Physical and geometrical properties of the model have also been discussed.

***Key Words**: Bianchi I Space-Time, Bulk Viscous Fluid, Cosmological Constan*

## INTRODUCTION
The universe at large scale is homogeneous and isotropic and the accelerating phase of universe (Gasperini, 2003). It is well known that the exact solutions of general theory of relativity for homogeneous space times belongs to Bianchi types. Taking into account dissipative process due to viscosity, the nature of cosmological solutions for homogeneous Bianchi type-I model was investigated by Belinski and Khalatnikov, (1975). They showed that the viscosity can not remove the cosmological singularity but results in a qualitatively new behaviour of the solutions near singularity. They found the remarkable property that during the time of the big bang matter is created by the gravitational field. Bianchi type-I model with bulk viscosity a power function of energy density when the universe is filled with stiff matter were studied by Banerjee,(1985); Huang, (1990). The effect of bulk viscosity with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models in the context of open thermodynamics system was studied by Desikan, (1997).
Models with dynamic cosmological term $\Lambda$ (t) are becoming popular as they solve the cosmological constant problem in a natural way. There is a significant observational evidence for the detection of Einstein's cosmological constant $\Lambda$ or a component of material content of the universe that varies slowly with the time and space and so acts like $\Lambda$. Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant have been studied by Dolgov, (1993,1997); Sahani and Starobinsky,(2000); Padmnabhan,(2003); Vishwakarma,(1999,2000); Pradhan *et al.*, (2001,2002).
Bulk viscosity is associated with GUT phase transition and string creation. The model studied by Murphy,(1973) has an interesting feature that the big bang type of singularity of infinite space-time curvature does not possess a finite past. However, the relationship assumed by Murphy between the viscocity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on cosmological evolution has been investigated by number of authors in the frame work of general relativity (Johri and Sudershan,1988; Maartens, 1995; Zimdahl, 1996). This motivates the study of cosmological bulk viscous fluid model.
In this paper, we have investigated Bianchi type-I cosmological model with varying cosmological constant in the presence of bulk viscous fluid by taking the condition $\Lambda = \beta \frac{\ddot{R}}{R} + \frac{1}{R^2}$ model and field

equation are discussed in section 2. The solution of field equation is given in section 3. Conclusion are summarised in last section.

**METRIC AND FIELD EQUATIONS**

We consider the Bianchi type-I space-time represented by the line-element

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad \text{... (1)}$$

where A, B and C are functions of t only.

We assume the cosmic matter consisting of viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p}) v_i v_j + \bar{p} \; g_{ij} \quad \text{... (2)}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta \; v^i_{;i} \quad \text{... (3)}$$

We consider the linear equation of state

$$p = w\rho \,, \qquad 0 \le w \le 1 \quad \text{... (4)}$$

where ρ and p are energy density and isotropic pressure respectively and $v^i$ the four velocity vector of the fluid satisfying $v_i\, v^i = -1$. The Einstein's field equations (in gravitational units 8πG = c = 1) with time-dependent cosmological term Λ (t) are

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda \, g_i^j \quad \text{... (5)}$$

For the line-element (1), the field equations (5) in comoving system of coordinates lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \Lambda - \bar{p} \quad \text{... (6)}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = \Lambda - \bar{p} \quad \text{... (7)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \Lambda - \bar{p} \quad \text{... (8)}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \Lambda + p \quad \text{... (9)}$$

The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + (\rho + \bar{p}) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \dot{\Lambda} = 0 \quad \text{... (10)}$$

We define average scale factor R for Bianchi I universe as

$$R^3 = ABC \quad \text{... (11)}$$

In analogy with FRW universe, we define generalized Hubble parameter H and deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \quad \text{... (12)}$$

and
$$q = -\frac{\ddot{R}}{RH^2} \qquad \text{... (13)}$$

where $H_1 = \dot{A}/A, H_2 = \dot{B}/B, H_3 = \dot{C}/C$ are directional Hubble factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ for the Bianchi I metric as

$$\theta = v^i_{ji} \qquad \text{... (14)}$$

and
$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} \qquad \text{... (15)}$$

For the metric (1), we have

$$\theta = 3\dot{R}/R \qquad \text{... (16)}$$

and
$$\sigma = K/R^3 \qquad \text{... (17)}$$

where $3k^2 = k_1^2 + k_2^2 + k_1 k_2$ and $k_1$, $k_2$ are integration constant.

Equations (6) – (9) can be expressed in terms of H, σ and q as

$$p - \zeta\theta - \Lambda = (2q-1)H^2 - \sigma^2 \qquad \text{... (18)}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 \qquad \text{... (19)}$$

**SOLUTION OF THE FIELD EQUATIONS**

From equations (4), (18) and (19) we obtain

$$\frac{(1-w)\rho}{2} + \Lambda = \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{3}{2}\,\frac{\zeta\dot{R}}{R} \qquad \text{... (20)}$$

Thus, we have one equation with three unknowns R, ρ, Λ and ζ. We require three more conditions to close the system. We assume that p = ρ (stiff fluid) i.e. w = 1. We now consider the decay law for Λ as

$$\Lambda = \beta\frac{\ddot{R}}{R} + \frac{1}{R^2} \qquad \text{... (21)}$$

and bulk viscosity is taken as

$$\zeta = \zeta_0\theta \qquad \text{... (22)}$$

where β and $\zeta_0$ are constants.

Therefore, from (20), (21) and (22), we have

$$(1-\beta)\frac{\ddot{R}}{R} + \left(2 - \frac{3}{2}\zeta_0\right)\frac{\dot{R}^2}{R^2} = \frac{1}{R^2} \qquad \text{... (23)}$$

For β = 1, integrating (23), we get

$$R = at + b$$

where $a = 1/\sqrt{2 - 9\zeta_0/2}$ and b is an integration constant.

For this solution metric (1) assumes the following form

$$ds^2 = -dt^2 + (at+b)^2\left[e^{-\frac{2k_1+k_2}{3a(at+b)^2}}dx^2 + e^{\frac{k_1-k_2}{3a(at+b)^2}}dy^2 + e^{\frac{k_1+2k_2}{3a(at+b)^2}}dz^2\right]$$

**DISCUSSION**

Matter density ρ and cosmological term Λ are given by

$$\rho = \frac{3a^2 - 1}{(at+b)^2} - \frac{k^2}{(at+b)^6}$$

$$\Lambda = \frac{1}{(at+b)^2}$$

Expressions for average scale factor R expansion scalar θ shear scalar σ, deceleration parameters q, Hubble parameter H and bulk viscosity ζ are given by

$$R = at + b$$

$$\theta = \frac{3a}{at+b}$$

$$\sigma = \frac{k}{(at+b)^3}$$

$$q = 0$$

$$H = \frac{a}{at+b}$$

$$\zeta = \frac{3a\,\zeta_0}{at+b}$$

We observe that model has initial singularity at $t = -b/a$. The model starts with a bing bang from its initial singularity. At the initial singularity, ρ, Λ, θ, σ, ζ, H all are infinite and at late times they becomes zero. In this model we have q = 0 this indicates that expansion rate of our model is constant. We also observe that $\sigma / \theta \to 0$ as $t \to \infty$ therefore model approach isotropy at late times.

**REFERENCES**

**Banerjee A, Duttachoudhary SB and Sanyal AK (1985).** Bianchi type I cosmological model with a viscous fluid *Journal of Mathmatical Physics* **26**(11) 3010.

**Belinski VA, Khalatnikov, IM (1975).** *Influence of viscosity on the character of cosmological evolution. Soviet physics, JETP* **69**(2) 401-413.

**Desikan K (1997).** Cosmological Models with Bulk Viscosity in the Presence of Particle Creation *General Relativity and Gravitation* **29** 435.

**Dolgov AD (1993).** *Breaking of conformal invariance and electromagnetic field generation in the Universe Physical Review* D **48** 2499D.

**Dolgov AD (1997).** Higher spin fields and the problem of the cosmological constant *Physical Review* D **55** 5881.

**Gasperini M and Veneziano G (2003).**The pre –big bang scenario in string cosmology. *Physics Reports* **373(**1-2**)** 1-212.

**Huang W (1990).** Anisotropic cosmological models with energy density dependent bulk viscosity. *Journal of Mathmatical Physics,* **31** 1456

**Johri VB, Sudershan R (1988).** Friedmann universes with bulk viscosity *Physics Letter A.* **132** 316.

**Maartens R (1995).** Dissipative cosmology *Classical andQuantum Gravity* **12** 1455.

**Murphy GL (1973).** Big-Bang Model Without Singularities. *Physical Review* D **8**, 4231.

**Overduin JM and Cooperstock FI (1998).** Evolution of the with scale factor a variable cosmological term *Physical Review* D **58** 043506.

**Padmnabhan T (2003).** Cosmological Constant - the Weight of the Vacuum *Physics Reports* . **380** 235-320.
**Pradhan A, Yadav VK, Chakrabarty I (2001).** Bulk Viscous FRW Cosmology in Lyra Geometry *International Journal of Modern Physics* **D10** 339.
**Pradhan A, Yadav VK, Saste NN (2002).** Plane-Symmetric Inhomogeneous Viscous Fluid Cosmological Models with Electromagnetic Field *International Journal of Modern Physics* **D11** 857.
**Sahni V, Starobinsky A (2000).** The Case for a Positive Cosmological Λ-Term *International Journal of Modern Physics*.**D9** 373.
**Vishwakarma RG (2000).** A study of angular size-redshift relation for models in which Λ decays as the energy density *Classical andQuantum Gravity*, **17** 3833.
**Vishwakarma RG, Abdussattar Beesham A (1999).** LRS Bianchi type-I models with a time-dependent cosmological ``constant" *Physical Review* D **60** 063507.
**Zimdahl W (1996).** Bulk viscous cosmology *Physical Review* D **53** 5483.