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Research Article

BIANCHI TYPE –V UNIVERSE WITH VARIABLE DECELERATION PARAMETER IN GENERAL RELATIVITY

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ABSTRACT
Bianchi type – V cosmological model of the universe have been studied in the cosmological theory. A new class of exact solutions have been obtained by considering variable deceleration parameter.

Key Words: *Cosmology, Variable Deceleration Parameter, Bianchi Type – V Universe*

INTRODUCTION
Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form of the matter content or suppose that space time admits killing vector symmetry (Kramer and Schmutzer, 1980). Solution to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman and Cimento, (1983). In simplest case the Hubble law yields a constant value for the deceleration parameter. It is worth observing that most of the well known models of Einstein's theory and Brans-Diske theory with curvature parameter k = 0 including inflationary models, are models with constant deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by several like Kramer and Schmutzer,(1980); Berman and Cimento, (1983); Berman and Gomid, (1988); Maharaj and Naidoo, (1993). But red-shift magnitude test has had a chequered history. During the 1960s and the 1970s, it was used to draw very categorical conclusions. The deceleration parameter q_0 was then claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. Todays situation , we feel, is hardly different. Observations (Riess *et al.*, 2004) of type Ia supernovae (SNe) allow to probe the expansion history of the universe. The main conclusion of these observations is that the expansion of the universe is accelerating. So we can consider the cosmological models with variable deceleration parameter. The readers are advised to see the papers by Narlikar and Vishwakarma, (2005) and references the rein for a review on the determination of the deceleration parameter from super novae data.
Pradhan and Otarod, (2006) have studied the universe with time dependent deceleration parameter in presence of perfect fluid motivated by the recent results on the BOOMERANG experiment on cosmic Microwave Background Radiation (Bernardis, 2000).
In this paper we investigate Bianchi type-V model by taking the deceleration parameter to be variable. First we present the basic equations of the models and the solutions of fields equations of Sen, (1957); Sen and Dunn, (1997). Then we discuss the models and present our results.

METRIC AND FIELD EQUATION
We consider the Bianchi type – V metric
$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}\left(B^2dy^2 + C^2dz^2\right) \tag{1}$$
where A , B and C are functions of time t.
The distribution of matter in the space time consist of perfect fluid given by the energy momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \tag{2}$$

Satisfying the equation of state

$$p = \omega \rho, \qquad 0 \le \omega \le 1 \tag{3}$$

where p and ρ are pressure and energy density respectively and v_i is the unit flow vector satisfying $v_i v^i = -1$

In comoving coordinates the field equations in case of perfect fluid with variable G are

$$R_{ij} - \frac{1}{2} Rg_{ij} = -8\pi GT_{ij} \tag{4}$$

The Einstein field equations (4) for the metric (1) and matter distribution (2) give rise to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = -8\pi Gp \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -8\pi Gp \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -8\pi Gp \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{3}{A^2} = -8\pi G\rho \tag{8}$$

$$2\frac{A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{9}$$

The usual energy conservation equation $T^{j}_{i;i} = 0$ yields

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{10}$$

hear and elsewhere suffix " 4 " denotes ordinary differentiation with respect to t.

To write metric functions explicity, we introduces the average scale factor R of Bianchi type – V space time defined by $R^3 = ABC$. From equations (5) – (7) and (9), we obtain

$$\frac{A_4}{A} = \frac{R_4}{R} \tag{11}$$

$$\frac{B_4}{B} = \frac{R_4}{R} - \frac{k_1}{R^3} \tag{12}$$

$$\frac{C_4}{C} = \frac{R_4}{R} + \frac{k_1}{R^3} \tag{13}$$

where k_1 is a constant of integration. Integrating equations (11) – (13), we obtain

$$A = m_1 R \tag{14}$$

$$B = m_2 \operatorname{Re} xp\left(-k_1 \int \tfrac{dt}{R^3}\right) \tag{15}$$

$$C = m_3 \operatorname{Re} xp\left(k_1 \int \tfrac{dt}{R^3}\right) \tag{16}$$

where m_1 , m_2 and m_3 are constants of integration satisfying $m_1\, m_2\, m_3 = 1$.

The Hubble parameter H, Volume expansion θ, shear scalar σ, and deceleration parameter q are given by

$$H = \frac{R_4}{R} \tag{17}$$

$$\theta = v^i_{;i} = \frac{3R_4}{R} = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{18}$$

$$\sigma^2 = \frac{k^2}{3R^6} = \frac{1}{3}\left\{\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{CA}\right\} \tag{19}$$

the deceleration parameter q is given by

$$q = -\frac{RR_{44}}{R_4^2} \tag{20}$$

Equation (5) - (8) and (10) can be written in terms of H , σ and q as

$$H^2(2q-1) - \sigma^2 + \frac{1}{R^2} = 8\pi G\mathrm{p} \tag{21}$$

$$3H^2 - \sigma^2 - \frac{3}{R^2} = 8\pi G\rho \tag{22}$$

$$\rho_4 + 3(\rho + p)\frac{R_4}{R} = 0 \tag{23}$$

Inserting equation (3) into equation (23) and then integrating, we obtain

$$\frac{\rho_4}{\rho} + 3(1+\omega)\frac{R_4}{R} = 0$$

on integration above equation, we get

$$\log\rho + 3(1+\omega)\log R = \log k_2$$

or

$$\rho = \frac{k_2}{R^{3(1+\omega)}} \tag{24}$$

where $k_2 > 0$ is a constant of integration.

SOLUTION TO THE FIELD EQUATION

The system of equations (3) and (5) – (8) supply only five equations in six unknowns A , B, C, ρ, p and G. One extra equation is needed to solve the system completely. We consider the deceleration parameter to be variable

$$q = \frac{-RR_{44}}{R_4^2} = b \quad \text{(variable)}$$

above equation may be rewritten as

$$\frac{R_{44}}{R} + b\left(\frac{R_4}{R}\right)^2 = 0 \tag{25}$$

The general solution of equation (25) is given by

$$\int\left(e^{\int \frac{b}{R}dR}\right)dR = t + n \tag{26}$$

where n is integrating constant. In order to solve the problem completely, we have to choose in $\int \frac{b}{R} dR$ such a manner that equation (26) be integrable. Without any loss of generality we consider.

$$\int \frac{b}{R} dR = I_n L(R) \tag{27}$$

Which does not effect the nature of generality of solution. Hence from equation (26) and (27) we obtain

$$\int L(R) dR = t + n \tag{28}$$

Let us consider $L(R) = \dfrac{1}{2k_3\sqrt{R + k_4}}$, where k_3 and k_4 are constants.

Inserting the value of L (R) into (28) then integrating we obtain

$$R(t) = \alpha_1 t^2 + \alpha_2 t + \alpha_3$$

we take $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 0$

$$R(t) = (t^2 + t) \tag{29}$$

Putting the value of R from equation (29) in equations (14), (15) and (16) we have

$$A = m_1(t^2 + t)$$

$$B = m_2(t^2 + t)\exp(-k_1 f(t))$$

$$C = m_3(t^2 + t)\exp(k_1 f(t))$$

For this solutions metric (1) assumes the form

$$ds^2 = -dt^2 + (t^2 + t)^2\left[m_1^2 dx^2 + e^{2x}\left\{m_2^2 e^{-k_1 f(t)} dy^2 + m_3^2 e^{k_1 f(t)} dz^2\right\}\right] \tag{30}$$

Where $f(t) = \int \dfrac{dt}{(t^2 + t)^3}$

From equation (17), (18), and (19) Hubble parameter H, expansion scalar θ, shear scalar σ for the model (30) are

$$H = \frac{2}{(t+1)} + \frac{1}{(t+1)t} \tag{31}$$

$$\theta = \frac{6}{(t+1)} + \frac{3}{(t+1)t} \tag{32}$$

$$\sigma^2 = \frac{k^2}{3(t^2 + t)^6} \tag{33}$$

From equation (20), (3), and (24) deceleration parameter q , the spatial volume V, cosmological energy density ρ, and pressure p are given by

$$q = \frac{-2(t^2 + t)}{(4t^2 + 4t + 1)} \tag{34}$$

$$V = (t^2 + t)^3 \tag{35}$$

$$p = \omega\rho = \frac{\omega k_2}{(t^2 + t)^{3(1+\omega)}} \tag{36}$$

Using equation (29), (31) and (33) in (22) we obtain

$$8\pi G k_2 = \frac{3(2t+1)}{\left(t^2+t\right)^{-(2+3\omega)}} - \frac{k^2}{3\left(t^2+t\right)^{3-3\omega}} \tag{37}$$

DISCUSSION

In the model, we observe that the spatial volume $V \to 0$ as $t \to 0$, and expansion scalar $\theta \to \infty$ as $t \to 0$, which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factor also vanish at t = 0 and hence the model has a point type singularity at the initial epoch. The cosmological energy density ρ, pressure p and shear scalar σ are approach to infinite as $t \to 0$, .

With t increases the expansion scalar and shear scalar decrease but spatial volume increases. As t increases all the parameters ρ, p, and θ decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large value of t .The gravitational term G is increasing function of cosmic time t provided, $\omega > 1$ .The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for the large value of t.

CONCLUSIONS

In this paper we have studied a spatially homogeneous and isotropic Bianchi type-V space time with the variable deceleration parameter. Einstein's field equations have been solved. Expressions for some important cosmological parameters have been obtained and physical behaviour of the models are discussed in detail, clearly the model represent shearing, non-rotating and expanding models with a big-bang start. The models have point type singularity at the initial epoch and approach isotropy at late times. Finally the solutions presented here are new and useful for a better understanding of the evolution of the universe in the Bianchi type-V universe with variable deceleration parameter.

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