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Research Article

BULK VISCOUS BIANCHI TYPE-V COSMOLOGICAL MODELS WITH STIFF FLUID AND TIME DEPENDENT COSMOLOGICAL TERM Λ

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ABSTRACT

We present bulk viscous Bianchi type-V cosmological models with time-dependent cosmological term Λ. Exact solutions of Einstein field equations have been obtained by assuming W = 1, $\Lambda = \frac{\beta \ddot{R}}{R} + \frac{1}{R^2}$ (Overduin and Cooperstock, 1998) and $\zeta \propto \theta$. Where w is the equation of state parameter, Λ is the cosmological constant, ζ is the bulk viscous coefficient and θ is the expansion scalar. It is found that models obtained are expanding, shearing and non-rotating. The model approach isotropy for large values of time t. Physical and kinematical parameters of the model have also been discussed.

***Key Words**: Bianchi Type-V, Expansion Scalar, Bulk Viscosity*

INTRODUCTION

Present cosmology is based on the Friedmon Robertson-Walker cosmological model which is completely homogeneous and isotropic. The study of Bianchi type V cosmological models create more interest as these models contain isotopic special cases and permit arbitrary small anisotropy levels at some instant of cosmic time. This property makes them suitable as model of our universe. This stimulated the research for obtaining exact anisotropic solution for Einstein's field equations as a cosmologically accepted model of our universe. Bianchi V cosmological models have been studied by other researchers Farnsworth,(1967); Collins, (1974).Maartens and Nel, (1978) in different context, Saha, (2006); and Singh and Chaubey, (2006) have obtained the quadratic term of metric function for Bianchi type-V model with perfect fluid and viscous fluid.

The distribution of matter can be satisfactory described by a perfect fluid blue to the range scale distribution of galaxies in our universe. Bulk viscosity is associated with the GUT phase transition and string creation. Misner, (1967, 1968) has studied the effect of viscosity on the evaluation of cosmological models. The role of viscosity in cosmology has been investigated by Weinberg, (1971); Heller and Klimek, (1975) and have obtained a viscous universes. Recently, Bianchi type V bulk viscous cosmological models have also been studied by Bali and Singh, (2005); Pradhan and Yadav, (2002); Singh and Chaubey, (2007); Tiwari, (2009); Singh and Baghel, (2009); Singh *et al.*, (2008).

In this paper Bianchi type V cosmological model with bulk viscosity has been investigated by assuming the condition w = 1, $\Lambda = \frac{\beta \ddot{R}}{R} + \frac{1}{R^2}$ and $\zeta \propto \theta$.

The outline of the paper is as follows. In section 2, the metric and field equations are described section 3 deals with the solutions of the field equations. In section 4 some physical and geometric properties of the model are described. Finally, conclusions are summarized in the last section.

Metric and Field Equations

We consider the Bianchi type V space-time given by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2x}\{B^2(t)dy^2 + C^2(t)dz^2\} \qquad \text{... (1)}$$

We assume the cosmic matter consisting of bulk viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \overline{p}) v_i v_j + \overline{p} g_{ij} \qquad \text{... (2)}$$

where $\overline{p}$ is the effective pressure given by

$$p = p - \zeta \ vi,_i , \qquad \text{... (3)}$$

$$p = w\rho , \qquad 0 \le w \le 1 \qquad \text{... (4)}$$

The Einstein field equations (is gravitational units 8πG = C = 1) with time varying cosmological term Λ (t) are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} + \Lambda g_{ij} \qquad \text{... (5)}$$

For the line-element (1), the field equations (5) in comoving system of coordinates lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1^2}{A^2} = \Lambda - \overline{p} \qquad \text{... (6)}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \Lambda - \overline{p} \qquad \text{... (7)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \Lambda - \overline{p} \qquad \text{... (8)}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} = \Lambda + \rho \qquad \text{... (9)}$$

$$\frac{2\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \qquad \text{... (10)}$$

Vanishing divergence of Ei

nstein tensor $R_{ij} - \frac{1}{2} R g_{ij}$ gives rise to

$$\dot{\rho} + (\rho + \overline{p}) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \dot{\Lambda} = 0 \qquad \text{... (11)}$$

We define average scale factor R for Bianchi V universe as

$$R^3 = ABC \qquad \text{... (12)}$$

In analogy with FRW universe, we define generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \qquad \text{... (13)}$$

and

$$q = -\frac{\ddot{R}}{RH^2} \qquad \text{... (14)}$$

where $H_1 = \dot{A}/A, H_2 = \dot{B}/B, H_3 = \dot{C}/C$ are directional Hubble's factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ for the Bianchi V metric as

$$\theta = v^i_{ji} \qquad \text{... (15)}$$

and $$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} \qquad \text{... (16)}$$

For the metric (1), we have

$$\theta = 3\dot{R}/R \qquad \text{... (17)}$$

and $$\sigma = K/R^3 \qquad \text{... (18)}$$

where K is an integration constant.

Equations (6) – (9) can be expressed in terms of H, σ and q as

$$p - \zeta\theta - \Lambda = (2q-1)H^2 - \sigma^2 + 1/R^2 \qquad \text{... (19)}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 - 3/R^2 \qquad \text{... (20)}$$

Solution of the Field Equations

From equations (19) and (20) with (4), (13), (14) and (18), we obtain

$$\frac{1}{2}(1-w)\,\rho + \Lambda = \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{3\zeta\dot{R}}{2R}\frac{2}{R^2} \qquad \text{... (21)}$$

Thus, we have one equation with three unknowns R, ρ, Λ and ζ. We require three more conditions to close the system. We assume that p = ρ (stiff fluid) i.e. w = 1. Assuming the decay law for Λ as

$$\Lambda = \beta\frac{\ddot{R}}{R} + \frac{1}{R^2} \qquad \text{... (22)}$$

where β is a constant and bulk viscosity is taken as

$$\zeta = \zeta_0\theta \qquad \text{... (23)}$$

Thus (21) becomes

$$(1-\beta)\frac{\ddot{R}}{R} + \left(2 - \frac{3z_0}{2}\right)\frac{\dot{R}^2}{R^2} = \frac{3}{R^2} \qquad \text{... (24)}$$

For β = 1, integrating (24), we obtain

$$R = a\,t - t_0 \qquad \text{... (25)}$$

where $a = \sqrt{\dfrac{6}{4-9\zeta_0}}$ and to is an integration constant.

For this solution metric (1) assumes the following form

$$ds^2 = -dt^2 + (at-t_0)^2\,dx^2 + e^{2x - \frac{k}{a(at-to)^2}}(at-to)^2\left\{\frac{2k}{e^{a(at-to)^2}}\,dy^2 + dz^2\right\} \qquad \text{... (26)}$$

DISCUSSION

Matter energy ρ and cosmological term Λ are given by

$$\rho = \frac{3a^2-4}{(at-to)^2} - \frac{k^2}{(at-to)^6} \qquad \text{... (27)}$$

$$\Lambda = \frac{1}{(at-to)^2} \qquad \text{... (28)}$$

Expressions for average scale factor R, expansion scalar θ shear scalar σ, deceleration parameter q, bulk viscosity ζ are given by

$$R = at - to$$

$$\theta = \frac{3a}{at - to} \qquad \text{... (29)}$$

$$\sigma = \frac{k}{(at - to)^3} \qquad \text{... (30)}$$

$$q = 0 \qquad \text{... (31)}$$

$$\zeta = \frac{3\zeta_0 a}{at - to} \qquad \text{... (32)}$$

We observe that model has initial singularity at t = t0 / a. The model starts with big bang from its initial singularity. At t = to / a, ρ, Λ, θ, σ, ζ all are infinite and at late times they become zero. We find $\lim_{t \to \theta} \sigma / \theta = 0$, therefore, model approach isotropy at late times : q = 0 indicates expansion rate of the model is constant.

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