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Research Article

BIANCHI TYPE-V COSMOLOGICAL MODELS WITH TIME VARYING G AND Λ

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ABSTRACT
Einstein field equations with variable gravitational and cosmological constants are considered in the presence of perfect fluid for Bianchi type -v universe by assuming the condition $\Lambda = \frac{\beta}{R^2} + H^2$ (Carvalho and Lima, 1992) where β is adjustable constant and h is Hubble parameter. We find that the model approaches isotropy at late times. Physical and kinematical properties of the model are also discussed.

Key Words: *Cosmology, Bianchi Type -V Universe, Variable Cosmological Ter*

INTRODUCTION
The cosmological constant problem is one of the most salient and unsettled problem in cosmology. To resolve the problem of huge difference between the effective cosmological constant observed today and the vacuum energy density predicted by quantum field theory, several mechanisms have been proposed (Weinberg,1989). A possible way is to consider a varying cosmological term. Due to the coupling of dynamic degree of freedom with the matter fields of the universe, Λ relaxes to its present small value through the expansion of the universe and creation of particles. From this point of view, the constant is small because the universe is old. Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by several author Vishwakarma, (1996a, 1996b, 1999, 2000, 2001, 2005); Arbab,(1998); Berman,(1991a,1991b). Some authors have argued for the dependence $\Lambda \sim t^{-2}$. Keeping in mind the dimensional considerations in the spirit of quantum cosmology, (Chen and Wu, 1990) considered Λ varying as R^{-2} (Carvalho and Lima, 1992). generalized it by taking $\Lambda = \alpha R^{-2} + \beta H^2$, where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α and β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved, expanding background, Schützhold (2002a,2002b) recently proposed a vacuum density proportional to the Hubble parameter this leads to a vacuum energy density decaying as $\Lambda \approx m^3 H$, where m ≈ 150 MeV is the energy scale of the chiral phase transmission of QCD. In a recent paper, Borges and Carnerio, (2005) have considered anisotropic and homogenous flat space filled with matter and cosmological term proportional to H, obeying the equation of state of the vacuum.
Dirac,(1937) first introduce the idea of a variable G in what he called his Large Number Hypothesis (LNH) and since various work has been done for a modified general relativity theory with this variation in G. In recent years, an appealing idea (Kalligas, 1992; Abdel-Rahaman, 1990; Pradhan, 2005; Singh, 2007) has been developed to consider joint variation of G and Λ with in the frame work of general relativity (Arbab, 1997, 1998) has considered cosmological models with viscous fluid considering G and Λ. A lot of work has been done by

Saha, (2005, 2006a, 2006b) in studying the anisotropic Bianchi type-I cosmological model in general relativity with varying G and Λ.

In recent years, the solutions of Einstein's field equations (EFEs) for homogeneous and anisotropic Bianchi type models have been studied by several authors such as Hajj-Boutros, (1985,1986); Shri Ram, (1989,1990); Mazumder, (1994); Camci *et al*., (2001); Pradhan and Kumar, (2001); Pradhan *et al.*,(2004,2005); Tiwari, (2008, 2009) by using different generating techniques. Solutions of the field equations may also be generated by applying the law of variation for Hubble's parameter, which was initially proposed by Berman,(1983) for FRW models. The main feature of this law is that it yields constant value of deceleration parameter. The theory of constant deceleration parameter in Brans-Dicke field equations were further developed by Berman and Gomide, (1988). It should be remarked that the law independs on the particular gravitational theory being considered. It is a property valid for FRW metric, and it is approximately valid also for slowly time varying deceleration parameter.

In this paper, we study Bianchi type-V cosmological model with variable gravitational and cosmological constants are considered in the presence of perfect fluid. To obtain an explicit solution we assume that the condition $\Lambda = \frac{\beta}{R^2} + H^2$ (Carvalho and Lima, 1992) where β is adjustable constant, H is the Hubble parameter and R is the scale factor of Robertson-Walker metric.

Field Equations

The spatially homogeneous and anisotropic Bianchi type-V space-time is described by the line element

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2x}\left[B^2(t)dy^2 + C^2(t)dz^2\right] \tag{1}$$

Where A(t), B(t) and C(t) are the cosmic scale factors.

The spatial volume of this model is given by

$$V = ABC \tag{2}$$

We define R = (ABC)$^{1/3}$ as average scale factor so that the Hubble parameters is anisotropic models may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{3}$$

Here and elsewhere dot stands for ordinary time derivative of the concerned quantity.

Also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{4}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors in the directions of x, y and z respectively.

We assume that the cosmic matter is represented by the energy-momentum tensor of a perfect fluid.

$$T_{ij} = (\rho + p)v_i v_j + p\, g_{ij} \tag{5}$$

where ρ, p are energy density, thermodynamical pressure and v_i is the four-velocity vector of the fluid satisfying the relation.

$$v_i v^i = -1$$

The expansion scalar θ, the average anisotropy parameter $\overline{A}$ and the shear scalar σ^2 are defined as follows

$$\theta = v^i_{;i} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6} \tag{7}$$

$$= \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right)\right] \tag{8}$$

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{9}$$

where $\Delta H_i = H_i - H (i = 1,2,3)$.

We define the deceleration parameter q as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{10}$$

We assume that the matter content obeys an equation of state

$$p = \omega\rho \quad , \qquad 0 \le \omega \le 1 \tag{11}$$

The Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G(t)\, T_{ij} + \Lambda(t) \tag{12}$$

For the metric (1) and energy momentum tensor (5) in co-moving system of coordinates the field equation (7) yields

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = -8\pi Gp + \Lambda \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = -8\pi Gp + \Lambda \tag{14}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{A^2} = -8\pi Gp + \Lambda \tag{15}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3}{A^2} = 8\pi G\rho + \Lambda \tag{16}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{17}$$

Taking into account of the conservation equation $div(T_i^{\,j}) = 0$, we have

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{18}$$

Field equations (13) – (16) and (18) can be written in terms of Hubble parameter H, shear scalar σ and deceleration parameter q as:

$$H^2(2q-1)-\sigma^2+\frac{1}{A^2}=8\pi Gp-\Lambda \quad (19)$$

$$3H^2-\sigma^2-\frac{3}{A^2}=8\pi G\rho+\Lambda \quad (20)$$

The summation of (13) – (15) and three times of (16) gives the equation for R as

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}=4\pi G(\rho-p)+\frac{2}{A^2} \quad (21)$$

Integrating (17), we get

$$A^2=BC \quad (22)$$

Therefore, from R = $(ABC)^{1/3}$ and (22), we get

$$A = R \quad (23)$$

Using equation (11), (23) in (21), we have

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}-\frac{2}{R^2}-4\pi G(1-\omega)\,\rho-\Lambda=0 \quad (24)$$

Solutions of the Field Equations

The system of equations (11), (13) – (17) supply only six equations in seven unknowns (A, B, C, ρ, p, G and Λ). One extra equation is needed to solve the system completely. For this purpose we take a relation

$$\Lambda=\frac{\beta}{R^2}+H^2\ , \quad (25)$$

Equation (24) becomes

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}-\frac{2}{R^2}-4\pi\,(1-\omega)\,G\rho-\left(\frac{\beta}{R^2}+H^2\right)=0 \quad (26)$$

For ω = 1, we get

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}-\frac{2}{R^2}=\frac{\beta}{R^2}+H^2 \quad (27)$$

Substituting $H=\frac{\dot{R}}{R}$ in equation (27) we have

$$\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}-\frac{(2+\beta)}{R^2}=0 \quad (28)$$

Without loss of generality we take β = -2 and then in this case scale factors takes the form

$$A(t)=(2kt+c)^{1/2} \quad (29)$$

$$B(t)=k_1(2kt+c)^{1/2}\exp\left(\frac{-k_2}{3k(2kt+c)^{1/2}}\right) \quad (30)$$

$$C(t)=k_1^{-1}(2kt+c)^{1/2}\exp\left(\frac{k_2}{3k(2kt+c)^{1/2}}\right) \quad (31)$$

where k, k_1, k_2 and c are constants. This is similar result obtained by Singh *et al.*, (2008).
Using (29), (30) and (31) we get the directional Hubble's factors as

$$H_1=k(2kt+c)^{-1} \quad (32)$$

$$H_2 = k(2kt+c)^{-1} + \frac{k_2}{3}(2kt+c)^{-3/2} \quad (33)$$

$$H_3 = k(2kt+c)^{-1} - \frac{k_2}{3}(2kt+c)^{-3/2} \quad (34)$$

The average generalized Hubble's parameter is given by

$$H = k(2kt+c)^{-1} \quad (35)$$

The physical quantities θ, $\bar{A}$, σ and Λ are respectively given by

$$\theta = 3k(2kt+c)^{-1} \quad (36)$$

$$\bar{A} = \frac{2}{27}\frac{k_2^2}{k^2}(2kt+c)^{-1} \quad (37)$$

$$\sigma^2 = \frac{k_2^2}{9}(2kt+c)^{-3} \quad (38)$$

$$\Lambda = (2kt+c)^{-1}[-2 + k^2(2kt+c)^{-1}] \quad (39)$$

Using (29) into (10) we get

$$q = 1 \quad (40)$$

Using (29), (30), (31) into (18), we get

$$p = \rho = (2kt+c)^{-3} \quad (41)$$

$$G = \frac{k^2}{4\pi}(2kt+c) - \frac{k_2^2}{72\pi} - \frac{(2kt+c)^2}{8\pi} \quad (42)$$

From the above results, it is observed that the volume V is zero at $t = t_0$ where $t_0 = \frac{-c}{2k}$ and expansion scalar θ, Hubble constant H, cosmological constant Λ, shear scalar σ, density ρ are infinite, which shows that the universe starts evolving with zero volume at $t = t_0$ with an infinite rate of expansion. The model has a point type singularity at t_0. As t increase H, θ, σ, Λ, ρ decrease whereas V and G increase. Thus, the rate of expansion slows down with increase in time. Thus, the cosmic scenario starts from a Big Bang at $t = t_0$ and continuous till $t = \infty$. Since $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$ so the model approaches isotropy for large value of t. As $t \to \infty$, the scale factors A (t), B (t) and C (t) tend to infinity. The scalar curvature R tends to infinity as $t \to \infty$. As $t \to \infty$, volume V becomes infinite whereas expansion scalar θ, mean anisotropy parameter $\bar{A}$, shear scalar σ, cosmological constant Λ and energy density ρ tend to zero, which indicate that universe is expanding with the increase of cosmic time but the rate of expansion decrease to zero as cosmic time t very large. Therefore, the model would essentially give an empty universe for large time t. As $t \to \infty$, the directional Hubble's parameter H_1, H_2 and H_3 tend to zero and therefore Hubble parameter H tend to zero. This indicate that the rate of expansion of the universe decrease as t increase and as time tends to infinite the rate of expansion of the universe tend to zero.

CONCLUSIONS

We investigated the Bianchi type -V cosmological model with variable gravitational and cosmological constants in the presence of perfect fluid by assuming the cosmological constant

$\Lambda = \frac{\beta}{R^2} + H^2$ suggested by Carvalho and Lima, (1992). We have found that the model has "point type singularity" at initial stage, as $t \to \infty$ all the cosmological parameters decrease. the cosmic scenario starts from a Big-Bang at initial singularity and continuous asymptotically. Also the model approaches isotropy at late times. The cosmological term Λ being very large at initial stage relaxes to constant at the infinity. Gravitational constant G is decreasing with time increasing which is similar result of Abdusattar and Vishwakarma,(1997); Vishwakarma (2005).It is interesting to note that we have found the results same as that of Singh *et al.*, (2008) which was taken to be of different approach of Λ for solving the Einstein field equations.

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