International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2012 Vol. 2 (1) January-March, pp.173-177/Abhimanu Singh
Research Article

EVALUATION OF MEASURES OF PERFORMANCE OF A PRODUCTION LINE WITH IMMEDIATE FEEDBACK AND SINGLE SERVER AT EACH OF THE PROCESSING UNITS

***Abhimanu Singh**
Department of Applied Mathematics, Bhagwan Parshuram Institute of Technology,
I P University, Delhi, India
**Author for Correspondence*

ABSTRACT
In this paper, we have modeled a production line consisting of an arbitrary number of processing units arranged in series, having single server, Poisson arrival and exponentially distributed service times at each of the processing units. At each of the processing units we have taken into account immediate feedback and the rejection. Taking into account the stationary behavior of queues in series, the solution for infinite queuing space have been found in the product form and hence various measures of performance such as mean number of jobs at the nodes, mean response times, mean waiting times, mean number of jobs waiting for, mean queue length, mean overall response time, etc. are obtained.

Key Words: *Queuing Network, Processing Units, Single Server Queues, Feedback, Immediate Feedback*

INTRODUCTION
A production line is an ordered set of an arbitrary number of machines, called processing units, arranged in a line. For a particular product, the number of processing units is fixed. The processing times at different processing units are independent but the processing times of different jobs at a processing unit are independent and identically distributed around some mean.
In a production line a job starts processing at the first processing unit. At the first processing unit it is processed for certain time interval and then it is transferred to the second processing unit for other type of processing, if its processing is done correctly at the first processing unit. This sequence is followed until the processing at the last processing unit is over. End of processing at each of the processing units give rise to the following three possibilities:

(a) Processing at a unit is done correctly and the job is transferred to the next processing unit for other type of processing.
(b) Processing is not done correctly but can be reprocessed at the same processing unit.
(c) Processing is neither done correctly nor it can be reprocessed at the same processing unit, then it is rejected and put into the scrap.

We have applied Queueing theoretic approach to find measures of performance of a production line by representing the production line by a serial network of queues, where the rate of flow of jobs to the first processing unit is represented by the rate of arrivals to the first queue, rate of processing at the first processing unit by the rate of service at the first node and so on.
Several researches have been considered the queues in series having infinite queuing space before each servicing unit. Specifically, Jackson had considered finite and infinite queuing space with phase type service taking two queues in series. In (Arya, 1972) it has been found the steady state distribution of queue length taking two queues in the system, where each of the two non-serial servers is separately in service with the other two non-serial servers. The stationary behavior of a finite space queuing model consisting of queues in series had been studied with multi-server service facility at each processing unit in (Sharma, 1973).
In our model we have introduced immediate-feedback representing reprocessing of a job at the same processing unit.

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2012 Vol. 2 (1) January-March, pp.173-177/Abhimanu Singh
Research Article

MODELING

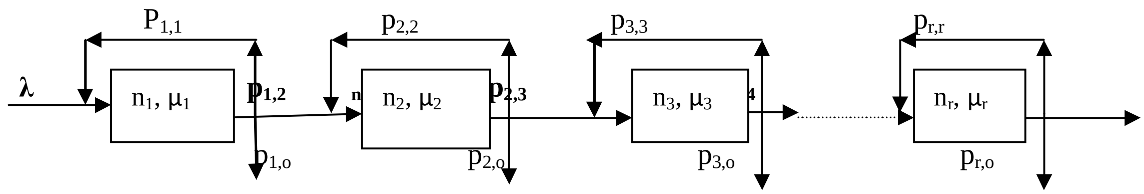

Figure 1

Let us consider a production line consisting of r (arbitrary) processing units, each having single server facility.
Let λ = Mean arrival rate to the first processing unit from an infinite source, according to Poisson's rule.

μ_i = Mean service rate at the i^{th} processing unit, having exponentially distributed service times.

n_i = Number of unprocessed items before the i^{th} processing unit, including one in service, if any.

$p_{i,i+1}$ = Probability that the processing of an item at the i^{th} processing unit is done correctly and it is transferred to the $(i+1)^{st}$ processing unit.

$p_{i,i}$ = Probability that the processing of an item at the i^{th} processing unit is not done correctly and it is transferred to the same processing unit for reprocessing.

$p_{i,o}$ = Probability that the processing of an item at the i^{th} processing unit is neither done correctly nor suitable for reprocessing, then it is rejected and put out.

$P(n_1, n_2, ...n_r, t) =$ Probability that there are n_1 jobs for processing before the first processing unit, n_2 jobs before the second processing unit, and so on,…. n_r jobs before the r^{th} unit at time t, with $n_i \geq 0 (1 \leq i \leq r)$ and $P(n_1, n_2, ...n_r, t)$=0, if some n_i <0 .

EQUATIONS GOVERNING QUEUING SYSTEM

In the above serial network of queues, each queue has immediate feedback. To analyze this serial network of queues firstly we remove the immediate feedback. After the removal of immediate feedback the above serial network of queues is replaced by one as follows:

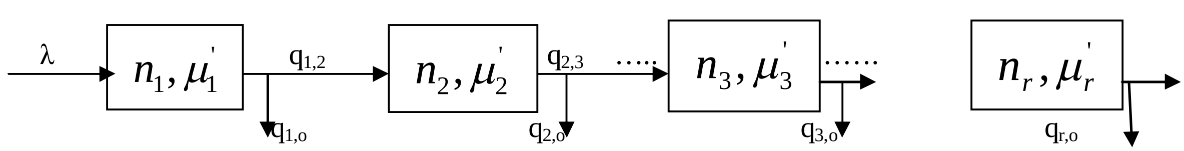

Figure 2

Here $\mu_i' = \mu_i(1 - p_{i,i})$, where μ_i' is the effective service at the i^{th} processing unit after the removal of the immediate feedback (Walrand, 1988).

We define the routing probabilities as follows

$$q_{i,o} = \frac{p_{i,o}}{(1 - p_{i,i})} \quad , \quad q_{i,i+1} = \frac{p_{i,i+1}}{(1 - p_{i,i})} \tag{1}$$

Then $\frac{\partial}{\partial t} p\ (n_1, n_2, ...n_r, t) = -\left[\lambda + \sum_{i=1}^{r}(1 - \delta_{n_i,0})\mu_i'\right] . \mathrm{P}(n_1, n_2......n_r, t)$

$$+ \lambda\ .\ \mathrm{P}(n_1 - 1, n_2, n_3, ...n_r, t)$$
$$+ \mu_1'.q_{1,2}\ .\ \mathrm{P}(n_1 + 1, n_2 - 1, n_3, ...n_r, t)$$
$$+ \mu_2'\ .\ q_{2,3}\ .\mathrm{P}(n_1, n_2 + 1, n_3 - 1, n_4..........n_r, t)$$
$$+ ...... + \mu_{r-1}'\ .\ q_{r-1,r}\ .\mathrm{P}(n_1, n_2, ...n_{r-1} + 1, n_r - 1, t)$$
$$+ \mu_r'\ .\mathrm{P}(n_1, n_2, ...n_r + 1, t)$$
$$+ \mu_1'\ .\ q_{1,o}\ .\mathrm{P}(n_1 + 1, n_2......n_r, t)$$
$$+ \mu_2'\ .\ q_{2,o}\ .\mathrm{P}(n_1, n_2 + 1, n_3, ...n_r, t) + ...$$
$$+ \mu_{r-1}'\ .\ q_{r-1,o}\ \mathrm{P}(n_1, n_2, n_3, ...n_{r-1} + 1, n_r, t) \tag{2}$$

such that $n_i \geq 0$ (1<i<r), and $\mathrm{P}(n_1, n_2, n_3, ...n_r, t)$=0, if any $n_i < 0$.

Under the steady state conditions $\mathrm{P}(n_1, n_2, ...n_r, t)$ $=\mathrm{P}(n_1, n_2, ...n_r)$

$$\therefore \qquad \frac{\partial}{\partial t}\mathrm{P}(n_1, n_2, ...n_r, t) = 0$$

Under the steady state conditions, the above set of equations is reduced to

$$[\lambda + \mu_1 + \mu_2 + ... + \mu_r].\mathrm{P}(n_1, n_2, ...n_r)$$
$$= \lambda\ \mathrm{P}(n_1 - 1, n_2 n_3, ...n_r) + \sum_{i=1}^{r} \mu_i'.q_{i,i+1}.P(n_1, n_2, ...n_i + 1, n_{i+1} - 1, ...n_r) +$$
$$\sum_{i=1}^{r} \mu_i'.q_{i,o}.P(n_1, n_2, ...n_i + 1.n_{i+1}, ...n_r) \tag{3}$$

Dividing the above steady state equation by the factor $[\lambda + \mu_1 + \mu_2 + ... + \mu_r]$, the above equation is reduced to P.Q=P, where P is the row vector of the steady state probability matrix and Q is the stochastic transition matrix of an irreducible and aperiodic Markov chain.

SOLUTION FOR INFINITE QUEUING SYSTEM

Under the steady state conditions all the queues behave independently and the solution of steady state equation in product form is given by

$$\mathrm{P}(n_1, n_2, ...n_r) = \prod_{i=1}^{r}(1 - \rho_i)\rho_i^{n_i}, \text{ where } n_i \geq 0(1 \leq i \leq r) \text{ and } \rho_i < 1\ (1 \leq i \leq r) \tag{4}$$

If any $\rho_i \ (1 \le i \le r)$>1, then the steady state is not reached and the solution will not remain valid.

We have

$$\rho_i = \frac{\lambda_i}{\mu_i}, \quad \text{where } \lambda_i = \lambda \prod_{k=1}^{i} \frac{p_{k-1,k}}{(1 - p_{k-1,k-1})}, \ p_{0,0} = 0$$

or

$$\rho_i = \frac{\lambda}{\mu_i} \prod_{k=1}^{i} \frac{p_{k-1,k}}{(1 - p_{k,k})}, \quad \text{with } p_{0,1} = 1 \qquad (5)$$

It can be seen that $\sum_{i=1}^{r} \lambda_i . q_{i,o} + \lambda_r . q_{r,f} = \lambda$ (6)

MEASURES OF PERFORMANCE

Mean number of jobs in the system $E(X) = \sum_{i=1}^{r} E(X_i)$

$$= \sum_{i=1}^{r} \frac{\rho_i}{1 - \rho_i} \qquad (7)$$

Mean Response time E (W_i) $= \dfrac{1}{\mu_i - \lambda_i}$ (8)

Mean Queue lengths Q_i $= \dfrac{\rho_i^2}{1 - \rho_i}$ (9)

Mean waiting times $= \dfrac{\frac{\rho_i}{\mu_i}}{1 - \rho_i}$ (10)

Mean overall Response time of a job $= \dfrac{W}{\lambda} = \dfrac{\sum_{i=1}^{r} w_i}{\lambda}$ (11)

Necessary marginal probabilities can be computed by

$$p_i(n_s) = (1 - \rho_i)\rho_i^{n_s} \qquad (1 \le i \le r, n \ge 0) \qquad (12)$$

CONCLUSION

This model can be used to find the measures of performance of a production line and for further studies of complicated production lines and assembly lines.

REFERENCES

Arya KL (1972). Study of a Network of Serial and Non-serial Servers with Phase Type Service and Finite Queueing Space, *Journal of Applied Probability* ,**9**(1), 198-201.

Bhat UN (2008). An Introduction to Queueing Theory(Modeling and Analysis in Applications), Birkhauser, Boston.

Gross D and Harris CM (1985). Fundamentals of Queueing Theory, John-Wiley, NewYork.

Trivedi KS (1997). Probability &Statistics with Reliability, Queueing and Computer Science Applications. Prentice Hall of India, New Delhi.

Saaty TL (1961). Elements of Queueing Theory, McGraw-Hill, New York.

Sharma OP, (1973). A Model for Queues in Series. *Journal of Applied Probability*, **10**(3), 691-696. Printed in Israel.

Walrand J, (1988). An Introduction to Queueing Networks, Prentice Hall, Englewood Clifs, New Jersey.