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Research Article

INFLUENCE OF POROSITY AND RADIATION IN A VISCO ELASTIC FLUID OF SECOND ORDER FLUID WITHIN A CHANNEL WITH PERMEABLE WALLS

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ABSTRACT
As the Darcy's parameter increases the velocity also increases. The no slip condition is perfectly satisfied on the boundary. For a similar values of Darcy's parameter and when K = 0.10 the no slip condition is also satisfied. However, a backward flow is observed as we move away towards the other bounding surface. When the radiation parameter is held constant, it is observed that as the Darcy's number increases the no slip condition is satisfied. But a backward flow is observed. For such similar values of Darcy's number and when R=7, the velocity profiles are seen to be of same type. An interesting point in this case is that the velocity profiles are merged till 40% of the channel width. Thereafter, the dispersion in the velocity profiles is observed. In this case also, the flow appears to be more of backward and subsequently in the forward direction.

Nomenclature:

S	:	Cauchy stress tensor
p	:	Scalar pressure
μ	:	Coefficient of viscosity
α_1	:	Coefficient of elasticity
α_2	:	Coefficient of cross-viscosity
ρ	:	Density
β	:	Coefficient of thermal expansion
g	:	Acceleration due to gravity
k_0	:	Permeability of the porous medium
υ_0	:	Transpiration cross flow
k	:	Viscoelastic parameter
R	:	Cross flow Reynolds number
Re	:	Reynold's number
Gr	:	Grashof number
Pr	:	Prandtl number
Da	:	Darcy number
γ_T	:	Wall temperature parameter
ϕ	:	Angle of inclination
A	:	Constant pressure gradient

INTRODUCTION
Non-Newtonian fluid mechanics has had to be point of concern with the development of general constitutive equations for visco elastic fluids. These constitutive equations should in principle

lead to the definition of flow properties that need to be measured to define the visco elastic fluid (rheometry) and to the development of the equivalent Navier-Stokes equations for the solution of all possible boundary value along with initial value problems that arises in several situations wherein heat and mass transfer takes place. The solution presented need to be quite specific about the experimental conditions pertaining to the relevant phenomena.

Therefore, now the question that arises is to address the situation "How do elastic liquids behave in complex flows?" and it is immediately apparent that the answer must involve a consideration of how the same liquids behave in simple flows, so that obtaining rheometrical data on the test liquids is an essential part of the exercise. Such data, when available, serve more than one useful purpose they certainly provide a foundation set of data, which must be accommodated in the associated mathematical model for the test liquids. That is to define a perfect constitutive equation, which is an essential ingredient in any theoretical resolution of the experimental dilemmas, has to be consistent with the rheometrical data. Indeed, if the model cannot simulate behaviour in simple flows, what chance does it have in complex flows?! Clearly, the choice of constitutive equation is central to the whole operation and this choice is far from trivial or obvious. Indeed, a constitutive model which satisfies the dual constraints of tractability and quantitative (or even semi quantitative) prediction may not exist! But that shouldn't and doesn't prevent a search for this missing link, but it is wise to be aware of the possibility of disappointment.

The model that has been considered here is of second order fluid whose constitutive relation has been proposed by Noll. The relation involves visco elasticity and also covers the concept of cross viscosity. In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus formed inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing.

A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries such as a porous rock. In all such cases, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistance force generated by internal friction between the fluid and the pore structure. This resistance force is characterized by Darcy's semi-empirical law established by Darcy (1856). The simplest model for flow through a porous medium is the one dimensional model derived by Darcy (1856). Obtained from empirical evidence, Darcy's law indicates that for an incompressible fluid flowing through a channel filled with a fixed, uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Subsequently, Dupuit and Frochheimer presented empirical evidence that the Darcy law or the linearity between speed and pressure variation, breaks down for large enough flow speed (a compilation of several experimental results) is presented by Mac Donald *et al.* (1979). This was emphasized later by Joseph *et al.* (1982) who stressed force modeled by the Frochheimer acts in a direction opposite to the velocity vector. It follows that, in multidimensional flow, the momentum equations for each velocity component derived using the Frochheimer extended Darcy equation is at least speculative. Later, Knupp and Lage (1995) analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Frochheimer extended Darcy unidirectional flow model.

Heat transfer in porous medium is gaining utmost importance due to its applicability in geothermal energy extraction, nuclear waste disposal, fossil fuels detection, regenerator bed etc. Understanding the development of hydro dynamic and thermal boundary layers along with the heat transfer characteristics is the basic requirement to further investigate the problem. Cheng and Minkowycz (1977) had analyzed the steady free convection about a vertical plate embedded in porous dynamics in the form of dissipative inequality (Clausius – Duhem) and commonly accepted the idea that the specific Helmholtz free energy should be a minimum in equilibrium. From the point of medium applied to heat transfer from dike. Murthy and Singh (1977) using method of similarity solution studied the influence of lateral mass flux and thermal dispersion on non - Darcy natural convection over a vertical plate in porous medium. They have discussed the combined effect of thermal dispersion and fluid injection on heat stratification on non - Darcy mixed transfer. Hassanien *et al.* (1998) had studied the effects of thermal dispersion and dissipation effects on non – Darcy mixed convection problems and established the trend of heat transfer rate convection from a vertical plate in porous medium and investigated the flow and temperature fields. Subsequently, Murthy (1998) had examined the dispersion while, Kuznetsov (2000) investigated the effect of transverse thermal dispersion on forced convection in porous media and identified the situations favorable to heat transfer under dispersion effects.

Mathematical Formulation

We consider the laminar mixed convection flow of a visco elastic fluid through a porous medium in an inclined permeable channel, the space between the plates is h, as shown

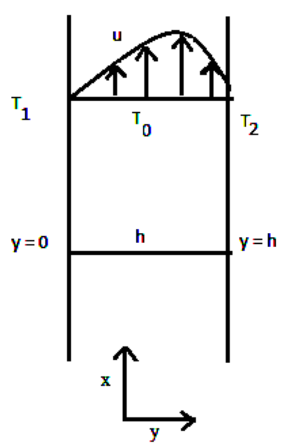

Figure 1: Geometry of the flow field when the channel is vertical

It is assumed that the rate of injection at one wall is equal to the rate of suction at the other wall. A rectangular coordinate system (x, y) is chosen such that the x axis is parallel to the gravitational acceleration vector g, but with opposite direction and the y - axis is transverse to the channel walls. The left wall (i.e. at $y = 0$) is maintained at constant temperature T_1 and the right wall (i.e. at $y = h$) is maintained at constant temperature T_2, where $T_1 > T_2$. The flow is assumed to be laminar, steady and is fully developed, i.e. the transverse velocity is zero. Then, the continuity equation drops to $\frac{\partial u}{\partial x} = 0$.

The fluid under consideration is assumed to be of Rivlin-Ericksen type whose constitutive equation is proposed as

$$S = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

The material constants μ, α_1 and α_2 can be determined from viscometric flows for any real fluid. A_1 and A_2 are Rivlin-Ericksen tensors and they denote respectively the rate of strain and acceleration. A_1 and A_2 are defined by

$$A_1 = \nabla V + (\nabla V)^T \tag{2}$$

$$A_2 = \frac{dA_1}{dt} + A_1 \nabla V + (\nabla V)^T A_1 \tag{3}$$

where $\frac{d}{dt}$ is the material time derivative and ∇ gradient operator and $(\ )^T$ transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second grade fluids. A detailed account of the characteristics of second – grade fluids is well documented by Dunn and Rajagopal (1995). Later, Rajagopal and Gupta [11] had studied the thermodynamics consideration and it is assumed that:

$$\mu \geq 0 \ , \alpha_1 > 0 \text{ and } \alpha_1 + \alpha_2 = 0 \tag{4}$$

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \alpha_1 v_0 \frac{d^3u}{dy^3} - \frac{\mu}{k_0} u + \rho g \beta (T - T_0) \tag{5}$$

$$v_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \tag{6}$$

here $\frac{dp}{dx}$ is a constant.

The boundary conditions are given by

$$u(0) = u(h) = 0 \ , \quad T(0) = T_1 \quad \text{and} \quad T(h) = T_2 \tag{7}$$

Introducing the following non-dimensional variables

$$\bar{y} = \frac{y}{h}, \ \bar{u} = \frac{u}{h^2} \text{ and } \theta = \frac{T - T_0}{T_2 - T_0}$$

into the Eqn. (5) and Eqn. (6), we obtain

$$kR \frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - \frac{1}{Da} u + \frac{Gr}{\mathrm{Re}} \theta + A + GSin\phi = 0 \tag{8}$$

$$\frac{d^2\theta}{dy^2} - R\Pr\frac{d\theta}{dy} = 0 \tag{9}$$

where $k = \frac{\alpha_1}{\rho h^2}$ is the visco elastic parameter, $R = \frac{\rho v_0 h}{\mu}$ is the cross flow Reynolds number, $Gr = \frac{g\beta(T_2 - T_1)h^3}{\nu^2}$ is the Grashof number, $\mathrm{Re} = \frac{\rho U_0 h}{\mu}$ is the Reynolds number, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $r_T = \frac{T_1 - T_0}{T_2 - T_0}$ is the wall temperature parameter and $A = -(\frac{dp}{dx})\frac{U_0 \nu}{h^2}$ is the constant pressure gradient.

The corresponding dimensionless boundary conditions are given by

$$u(0) = u(1) = 0,\ \theta(0) = r_T \text{ and } \theta(1) = 1 \tag{10}$$

Method of Solution

We consider the first – order perturbation solution of the boundary value problem for small k. Since the constitute Eqn. (1) has been derived up to only the first–order of smallness of k, therefore, the perturbation solution obtained by retaining the terms up to the same order of smallness of k must be quite logical and reasonable. We write

$$u = u_0 + ku_1 \tag{11}$$

and

$$\theta = \theta_0 + k\theta_1 \tag{12}$$

Substituting Eqn. (11) and Eqn. (12) into Eqn. (8) and Eqn. (9) and boundary conditions given by Eqn. (10) and then equating the like powers of k, we obtain

Zeroth-order system (k^0) **:**

$$\frac{d^2u_0}{dy^2} - R\frac{du_0}{dy} - \frac{1}{Da}u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 - A - GSin\phi \tag{13}$$

$$\frac{d^2\theta_0}{dy^2} - R\Pr\frac{d\theta_0}{dy} = 0 \tag{14}$$

together with boundary conditions

$$u_0(0) = u_0(1) = 0\ , \quad \theta_0(0) = r_T \text{ and } \theta_0(1) = 1 \tag{15}$$

First-order system (k^1) **:**

$$\frac{d^2u_1}{dy^2} - R\frac{du_1}{dy} - \frac{1}{Da}u_1 = -R\frac{d^3u_0}{dy^3} - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2} - R\Pr\frac{d\theta_1}{dy} = 0 \tag{17}$$

together with boundary conditions

$$u_1(0) = u_1(1) = 0 \text{ and } \theta_1(0) = \theta_1(1) = 0 \tag{18}$$

Zeroth-order solution (or Solution for a Newtonian fluid):

Solving Eqn. (13) and Eqn. (14) using the boundary conditions (15), we get

$$\theta_0 = \frac{(1-r_T e^{R\Pr}) + (r_T - 1)e^{R\Pr y}}{(1-e^{R\Pr})} \tag{19}$$

$$u_0 = c_1 e^{ay} + c_2 e^{by} + \frac{Gr}{\mathrm{Re}}(f_1 - f_2 e^{R\Pr y}) + ADa + GDaSin\phi \tag{20}$$

where

$$a = \frac{R+\sqrt{R^2+4/Da}}{2}, b = \frac{R-\sqrt{R^2+4/Da}}{2}, \ f_1 = \frac{(1-r_T e^{R\Pr})Da}{(1-e^{R\Pr})},$$

$$f_2 = \frac{(r_T - 1)}{(1-e^{R\Pr})(R^2\Pr^2 - R^2\Pr - 1/Da)}, \ f_3 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2) + ADa + DaGSin\phi,$$

$$f_4 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2 e^{R\Pr}) + ADa + DaGSin\phi, \ c_1 = \frac{f_4 - f_3 e^b}{e^b - e^a}, \ c_2 = \frac{f_3 e^a - f_4}{e^b - e^a}.$$

First-order solution (or Solution for a second-grade fluid):

Solving Eqn. (17) with corresponding boundary conditions, we obtain

$$\theta_1 = 0 \tag{21}$$

Substituting the Eqn. (20) and Eqn. (2)1 into the Eqn. (16) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = c_3 e^{ay} + c_4 e^{by} - f_6 y e^{ay} - f_7 y e^{by} + f_5 e^{R\Pr y} \tag{22}$$

where $f_5 = \dfrac{Gr}{\mathrm{Re}} \dfrac{f_2 R^4 \Pr^3}{(R^2\Pr^2 - R^2\Pr - 1/Da)}$, $\quad f_6 = \dfrac{Rc_1 a^3}{2a - R}$, $\quad f_7 = \dfrac{Rc_2 b^3}{2b - R}$,

$$f_8 = f_5 e^{R\Pr} - f_6 e^a - f_7 e^b, \qquad c_3 = \frac{f_8 - f_5 e^b}{e^b - e^a}, \qquad c_4 = \frac{f_5 e^a - f_8}{e^b - e^a}.$$

It can be verified that when $k = 0$, $R = 0$ and $Da \to \infty$ our results reduces to those given by Aung and Worku (1986)

Finally, temperature is given by

$$\theta = \frac{(1-r_T e^{R\Pr}) + (r_T - 1)e^{R\Pr y}}{(1-e^{R\Pr})} \tag{23}$$

and velocity is given by

$$\begin{aligned} u = {} & (c_1 + kc_3)e^{ay} + (c_2 + kc_4)e^{by} + Gr(f_1 - f_2 e^{R\Pr y})/\mathrm{Re} \\ & + ADa + GDaSin\,\phi - kyf_6 e^{ay} - kyf_7 e^{by} + kf_5 e^{R\Pr y} \end{aligned} \tag{24}$$

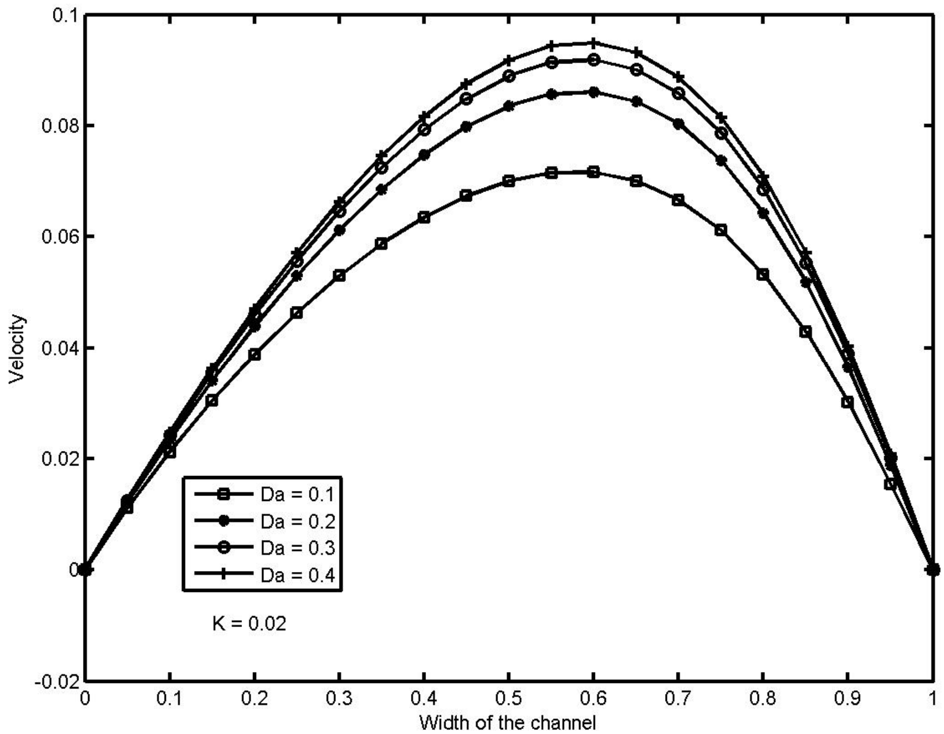

Figure 1: Velocity Profiles with respect to Darcy's Parameter

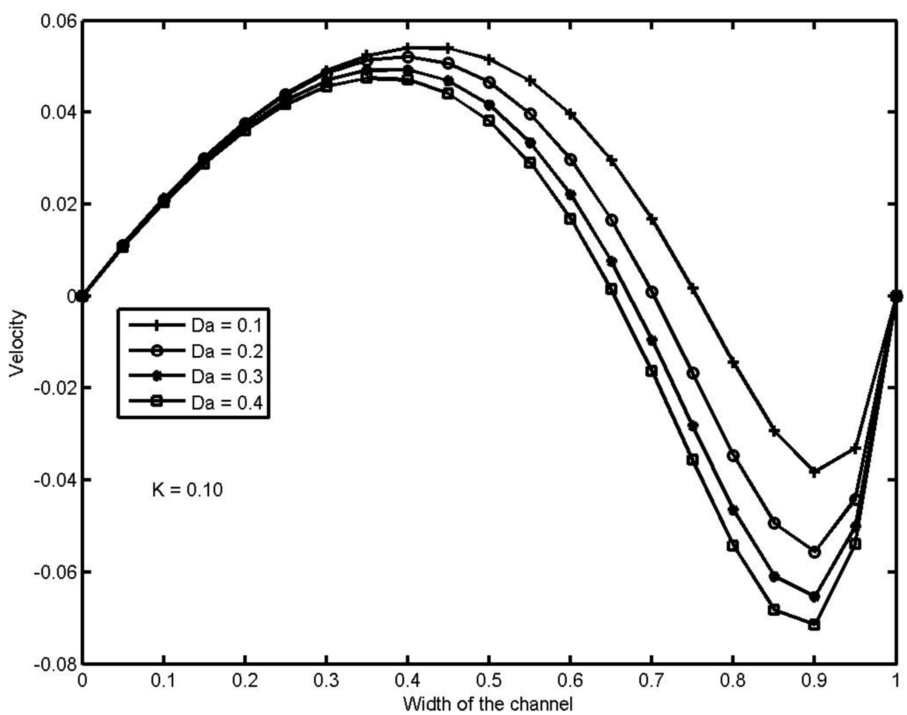

Figure 2: Influence of Darcy's Parameter on Velocity Profiles

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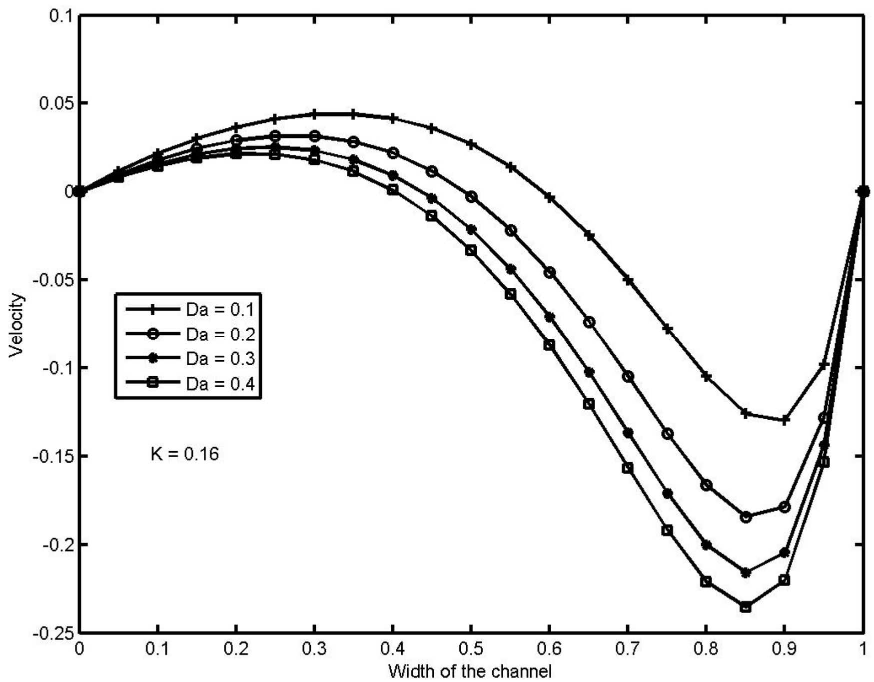

Figure 3: Effect of Darcy's Parameter on Velocity Profiles

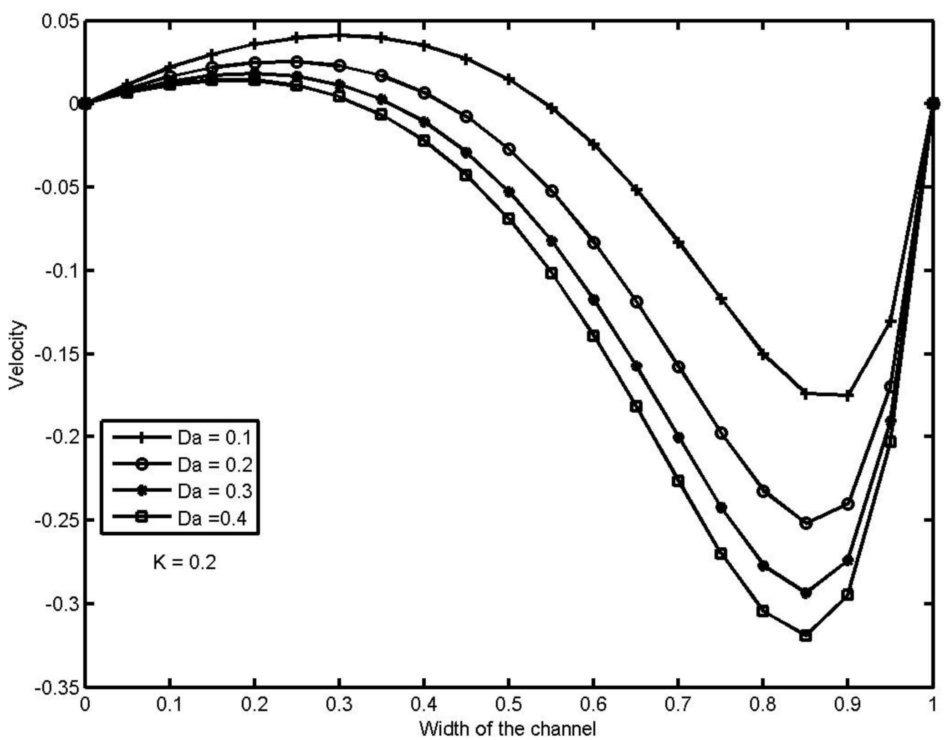

Figure 4: Variation in Velocity Profiles with respect to Darcy's Parameter

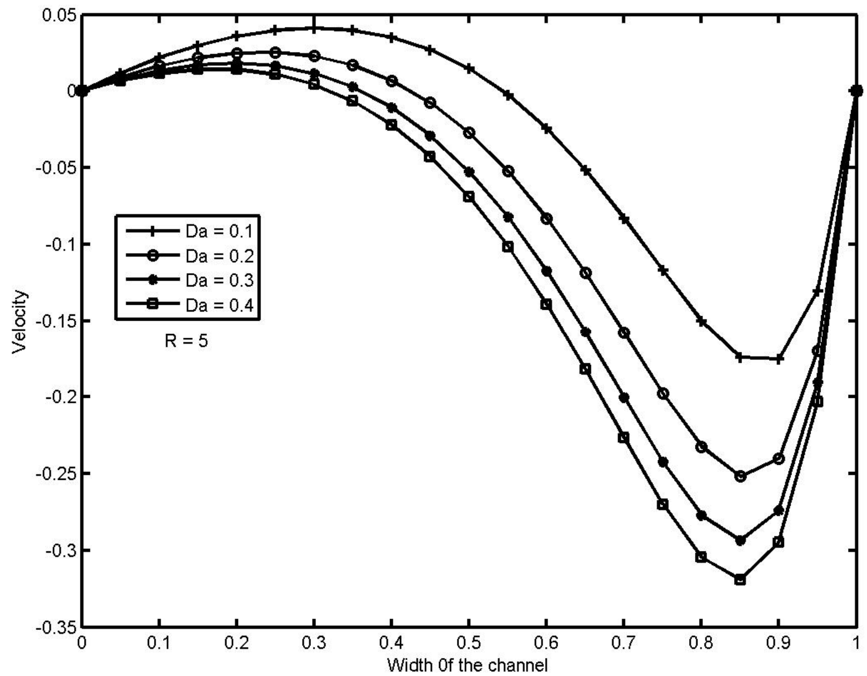

Figure 5: Velocity Profiles with respect to Radiation Parameter

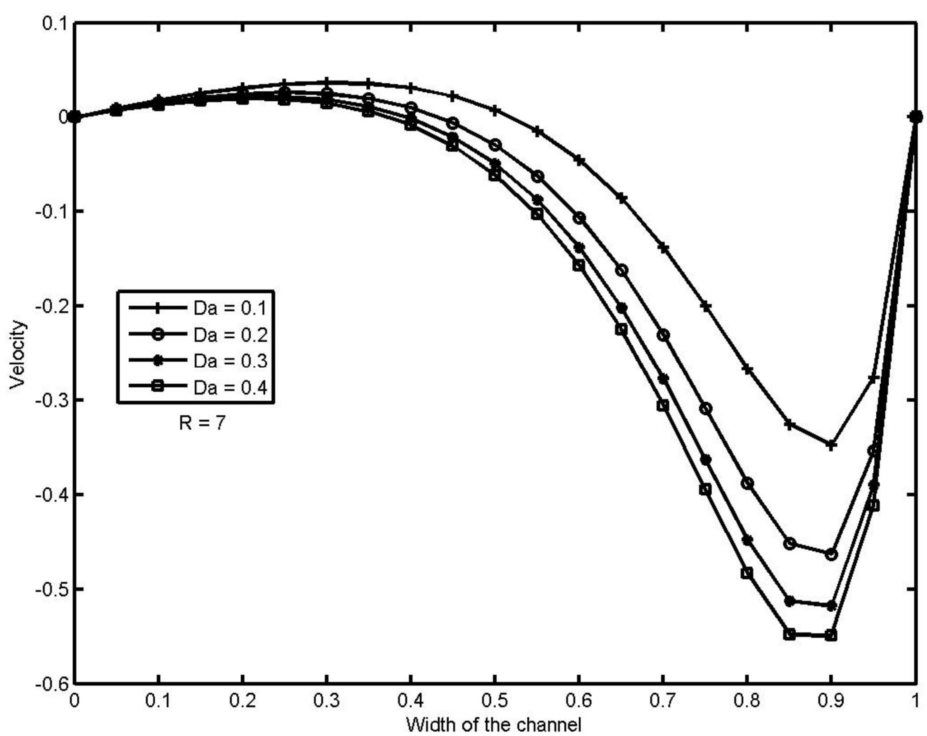

Figure 6: Variation of Velocity with respect to Darcy's parameter and Radiation Parameter

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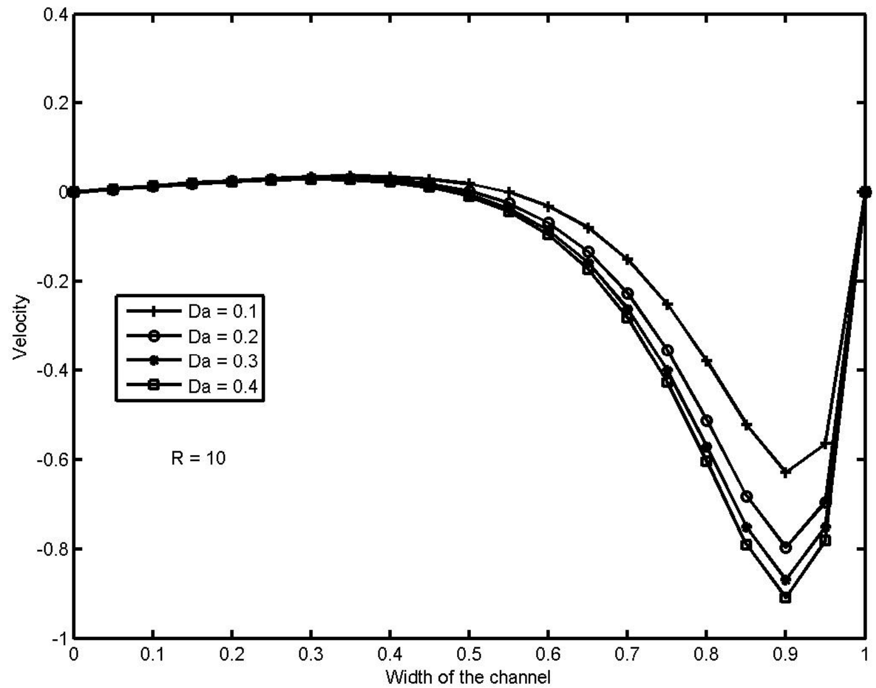

Figure 7: Influence of Radiation parameter and Darcy's Radiation parameter

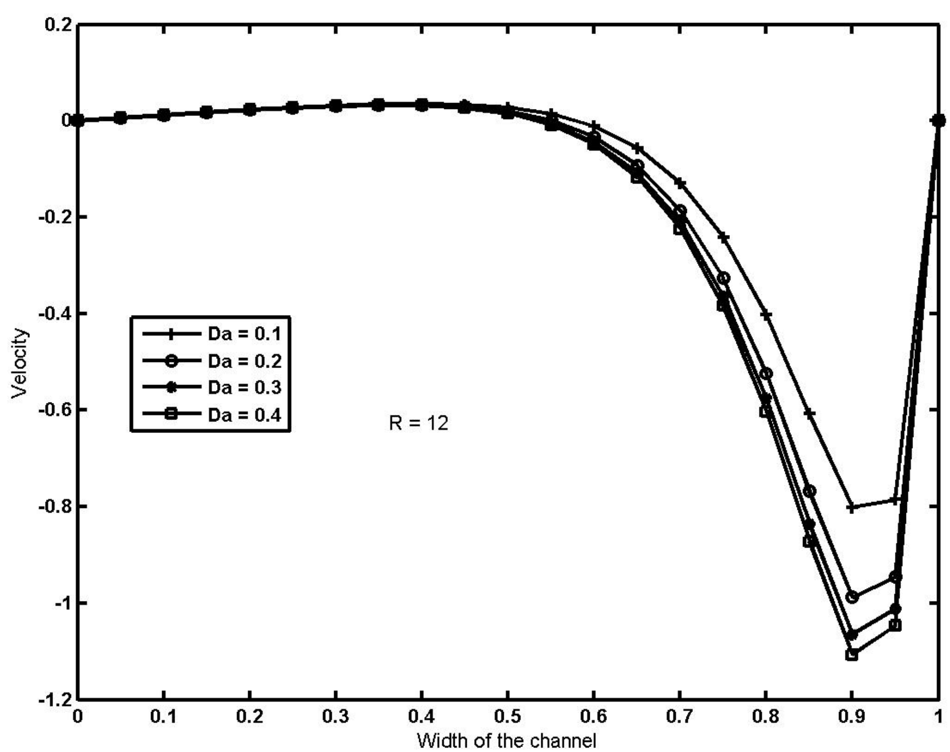

Figure 8: Effect of Darcy's parameter on Velocity profiles with respect to Radiation parameter

RESULTS AND CONCLUSIONS

1. The nature of velocity profiles with respect to the Darcy's and porosity of the fluid bed are illustrative in Fig-1, Fig-2, Fig-3, Fig-4. In Fig-1it is noticed that, for a constant value of porosity, as the Darcy's parameter increases the velocity also increases. The no slip condition is perfectly satisfied on the boundary. For a similar values of Darcy's parameter and when K = 0.10 the no slip condition is also satisfied. However, a backward flow is observed as we move away towards the other bounding surface. Similar such a trend as stated above is noticed when K = 0.16. The velocity profiles are observed to be more dispersed as we approach the other boundary. Even for, K = 0.2 not much of significant trend in the velocity profiles is observed. Moreover, there seems to be no change in the nature of velocity profiles.

2. The influence of radiation and Darcy's parameter on the nature of velocity profiles is shown in Fig-5, Fig-6, Fig-7 and Fig-8. In Fig-5, when the radiation parameter is held constant, it is observed that as the Darcy's number increases the no slip condition is satisfied. But a backward flow is observed. For such similar values of Darcy's number and when R=7, the velocity profiles are seen to be of same type. However, the magnitude of the backward flow is noticed to be slight altered. In Fig-7, when the radiation parameter is (R = 10), and for same values of Darcy's parameter, a slight change in the velocity profiles is noticed. As usual and as discussed above, more of backward flow is observed in this case. An interesting point in this case is that the velocity profiles are merged till 40% of the channel width. Thereafter, the dispersion in the velocity profiles is observed. Fig-8 depicts the variation in the velocity profiles when R=12 and the Darcy's number ranging from 0.1 to 0.4. In this case also, the flow appears to be more of backward and subsequently in the forward direction. The conditions of the no slip are satisfied in this situation also.

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