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Research Article

HYDROMAGNETIC OSCILLATORY FLOW OF A VISCOUS LIQUID

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ABSTRACT
Unsteady oscillatory flow of an incompressible, electrically conducting viscous liquid through a porous medium past an infinite vertical porous plate with constant suction has been studied. A transverse uniform magnetic field has been applied in a rotating frame of reference. The temperature on the vertical surface fluctuates in time about a nonzero constant mean .The analytical expressions for the velocity, temperature are presented showing the effects of pertinent parameters.

***Key Words**: Unsteady, Hydromagnetic, Oscillatory Flow*

INTRODUCTION
The study of magnetohydrodynamic(MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow has wide application in engineering and technology. Jha (1998) has studied the effects of applied magnetic field on transient free convective flow in a vertical channel. Singh *et al,*. (2001) have discussed the free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. Cookey *et al.* (2003)have studied the unsteady MHD free-convection and mass transfer flow past an infinite heated porous vertical plate with time dependant suction. Das *et al.,* (2004) have reported free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium. Panda *et al.* (2004) have analyzed the unsteady free convection MHD flow and mass transfer in a rotating porous medium. Kurtcebe *et al*., (2005) have reported the heat transfer of a visco-elastic fluid in a porous channel. Ogulu *et al.,* (2005) have analyzed the numerical study of mixed convection heat transfer from thermal sources on a vertical surface. Cheng *et al.* (2006) have analyzed the free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluid. In this paper unsteady oscillatory flow of an incompressible, electrically conducting viscous liquid through a porous medium past an infinite vertical porous plate with constant suction has been studied.

Formulation of the problem
Consider an oscillatory flow of an incompressible, electrically conducting viscous liquid through a porous medium past an infinite vertical porous plate in a rotating system.
The analysis of the problem is based on following assumptions:
The x and y axes are on two dimensional infinite vertical porous plate and z-axis is normal to it and the component of velocity in these directions are u , v , w respectively. The plate is porous with suction velocity $w = -w_o$, where w_o is real and positive.
The liquid and the plate both are in a state of rigid body rotation with uniform angular velocity Ω about z-axis. The plate is infinite in extent therefore all the physical variables depend on z and t only.
The uniform magnetic field $B_o=\mu_e H$, where $H=(o,o,H_o)$ has been applied in the z-direction i.e. normal to the flow. The buoyancy force and Hall effect are not considered and the effects due to perturbation of the field have been neglected.
The free stream velocity is $U = 1 + \varepsilon e^{\text{int}}$,where n < < 1.
Under these assumptions, the equation of continuity is

$$\frac{dw}{dz} = 0$$

The equations of motion and temperature are

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} - 2\Omega v = \frac{\partial U}{\partial t} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma}{\rho} \mu_e{}^2 H_0{}^2 (u - U) \qquad (1)$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} + 2\Omega (u - U) = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma}{\rho} \mu_e{}^2 H_0{}^2 v \qquad (2)$$

$$\frac{\partial T}{\partial t} - w_0 \frac{\partial T}{\partial z} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial z^2} + S(T - T_\infty) \qquad (3)$$

where U is the free stream velocity, H_0 is the constant magnetic field, S is the source, C_p is the specific heat and other symbols have their usual meaning.

The boundary conditions relevant to the problem are

$$u = 0, \quad v = 0, \quad T = T_\omega + \varepsilon (T_\omega - T_\infty) e^{\mathrm{i}nt} \qquad \text{at } z = 0.$$

$$u \to U(t), \quad U = U_0 (1 + \varepsilon e^{\mathrm{i}nt}), \quad T = T_\infty \qquad \text{as } z \to \infty. \qquad (4)$$

We introduce the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\nu}, t^* = \frac{w_0{}^2 t}{\nu}, U^* = \frac{U}{U_0}, n^* = \frac{\nu n}{w_0{}^2}, T^* = \frac{T - T_\infty}{T_w - T_\infty}$$

$$S* = \frac{S\nu}{w_0^2}, \quad q* = \frac{u}{U_0} + i\frac{v}{U_0} = u* + iv*$$

$$R = \frac{\Omega \nu}{w_0{}^2} \qquad \text{(Rotation parameter)}$$

$$M^2 = \frac{\sigma \mu_e{}^2 H_0{}^2 \nu}{\rho w_0{}^2} \qquad \text{(Magnetic parameter)}$$

and $$\Pr = \frac{\mu C_p}{K} \qquad \text{(Prandtal number)}$$

Using the above stated non-dimensional quantities and neglecting astericks over them , equations (1) and (2) are transformed to equation (5) and equation (3) is transformed to equation(6).Thus we have

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + (M^2 + 2iR)(q - U) = \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial z^2} \qquad (5)$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial z^2} + ST \qquad (6)$$

The boundary conditions (4) become

$$q = 0, \quad T = 1 + \varepsilon e^{\mathrm{i}nt} \qquad \text{at } z = 0.$$

$$q = 1 + \varepsilon e^{\mathrm{i}nt}, \quad T = 0 \qquad \text{as } z \to \infty. \qquad (7)$$

Solutions of the problem

To solve equations (5) and (6), we assume the velocity and temperature of the liquid in the neighbourhood of the plate as

$$q = (1 - q_0) + \varepsilon (1 - q_1) e^{\mathrm{int}}$$

and $$T(z,t) = T_0(z) + \varepsilon T_1(z) e^{\mathrm{int}}$$

Using *q and U* in equation (5) and equating terms independent of ε and co-efficient of ε, we have

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - (M^2 + 2iR) q_0 = 0 \qquad (8)$$

$$\frac{d^2 q_1}{dz^2} + \frac{dq_1}{dz} - (M^2 + 2iR + in) q_1 = 0 \qquad (9)$$

The boundary conditions (7) for velocity becomes

$$q_0 = 1, \quad q_1 = 1 \qquad \text{at } z = 0.$$

$$q_0 = 0, \quad q_1 = 0 \qquad \text{as } z \to \infty. \tag{10}$$

The solutions of equations (8) and (9) in view of (10) are given by

$$q_0(z) = e^{-A_1 z} \quad and \quad q_1(z) = e^{-A_2 z}$$

where $A_1 = 0.5\left[1 + \sqrt{1 + 4(M^2 + 2iR)}\right]$

$$= \alpha_1 + i\beta_{1,}$$

$$\alpha_1 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2)^2 + 64R^2} + (1+4M^2)\right]^{\frac{1}{2}}$$

$$\beta_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2)^2 + 64R^2} - (1+4M^2)\right]^{\frac{1}{2}}$$

$$A_2 = 0.5\left[1 + \sqrt{1 + 4(M^2 + 2iR + in)}\right] = \alpha_2 + i\beta_2,$$

$$\alpha_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2)^2 + (8R+4n)^2} + (1+4M^2)\right]^{\frac{1}{2}}$$

$$\beta_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2)^2 + (8R+4n)^2} - (1+4M^2)\right]^{\frac{1}{2}}$$

Hence,

$$q(z) = (1 - q_0) + \varepsilon(1 - q_1)e^{\text{int}} \quad = u + iv,$$

where $u = 1 - e^{-\alpha_1 z}\cos\beta_1 z + \varepsilon(1 - e^{-\alpha_2 z}\cos\beta_2 z)\cos nt - \varepsilon e^{-\alpha_2 z}\sin\beta_2 z \sin nt$

$$v = e^{-\alpha_1 z}\sin\beta_1 z + \varepsilon e^{-\alpha_2 z}\sin\beta_2 z\cos nt + \varepsilon(1 - e^{-\alpha_2 z}\cos\beta_2 z)\sin nt$$

Putting T and its corresponding derivatives in equation (6) and equating terms independent of ε and coefficient of ε, we have

$$\frac{d^2T_0}{dz^2} + \Pr\frac{dT_0}{dz} + Pr\ S\ T_0 = 0 \tag{11}$$

$$\frac{d^2T_1}{dz^2} + \Pr\frac{dT_1}{dz} + (S - in)\Pr T_1 = 0 \tag{12}$$

The boundary conditions (7) for temperature becomes

$$T_0 = 1,\ T_1 = 1 \qquad \text{at } z = 0$$

$$T_0 = 0,\ T_1 = 0 \qquad \text{as } z \to \infty \tag{13}$$

The solutions of equations (11) and (12) in view of (13) are given by

$$T_0(z) = e^{-P_1 z} \text{ and } T_1(z) = e^{-P_2 z}$$

where $P_1 = 0.5\left[\Pr + \sqrt{\Pr^2 - 4\Pr S}\right],$ (14)

$$P_2 = 0.5\left[\Pr + \sqrt{\Pr^2 - 4(S - in)\Pr}\right] = \alpha_3 + i\beta_3,$$

So, $T(z,t) = e^{-P_1 z} + \varepsilon e^{-P_2 z + \text{int}}$

RESULTS AND DISCUSSION

Unsteady oscillatory flow of an electrically conducting viscous liquid through a porous medium under the influence of transient velocity and temperature field in a rotating system has been characterized by Magnetic parameter (M) and Rotating pameter(R) with a fluctuating free stream velocity. The main

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objective of the discussion is to bring out effect of pertinent parameters governing the flow and heat transfer phenomenon. During computation the value of ε=0.05 and n=0.01.

Fig.1 shows parabolic velocity distribution . Due to resistance of porous matrix, velocity decreases. Magnetic field as well as rotating system contributes to enhance the primary velocity. When Magnetic field and Rotating force are withdrawn(i.e.M=0 and R=0), the velocity assumes the lowest value (curves VI and VII).

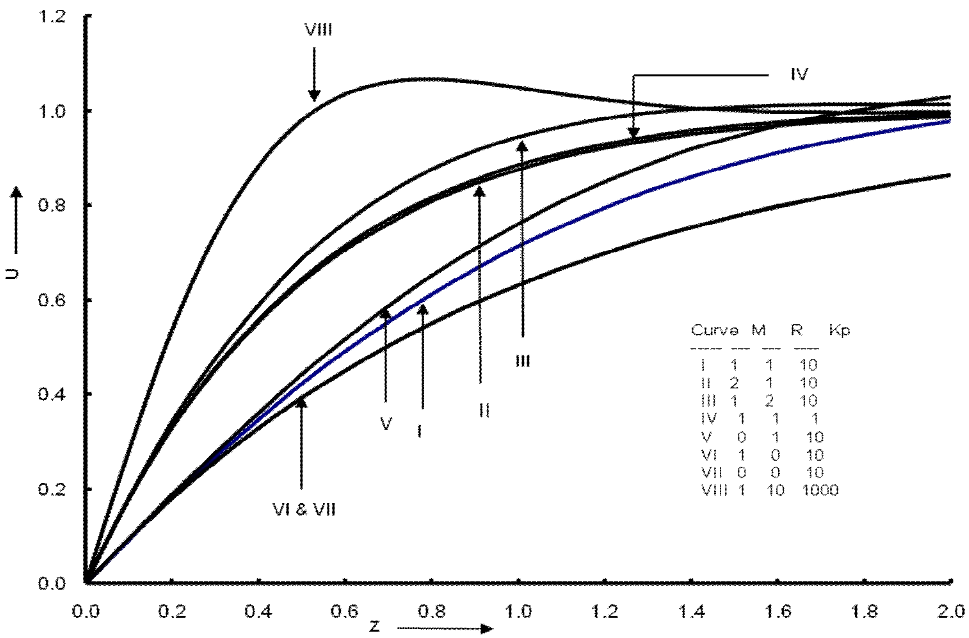

Fig - 1 Effect of M, R, Kp on Primary Velocity profiles when n=0.01, ε=0.05

CONCLUSION

The above study brings out the following conclusions:

The effects of magnetic field and rotating parameter are to enhance the primary velocity.

The contribution of rotation parameter R is independent of the magnetic field.

In the absence of rotation,the magnetic field has no significant contribution.

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