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A NEW APPROACH TO SOLVE AN UNBALANCED ASSIGNMENT PROBLEM

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ABSTRACT

In this paper I have proposed a new approach to solve an unbalanced assignment problem (UBAP). This approach includes two parts. First is to obtain an initial basic feasible solution (IBFS) and second part is to test optimality of an IBFS. I have proposed two new methods Row Penalty Assignment Method (RPAM) and Column Penalty Assignment Method (CPAM) to obtain an IBFS of an UBAP. Also I have proposed a new method Non-basic Smallest Effectiveness Method (NBSEM) to test optimality of an IBFS. We can solve an assignment problem of maximization type using this new approach in opposite sense. By this new approach, we achieve the goal with less number of computations and steps. Further we illustrate the new approach by suitable examples.

Key Words: *BAP, UBAP, IBFS, RPAM, C PAM, NBSEM*

INTRODUCTION

The assignment problem is a special case of the transportation problem where the resources are being allocated to the activities on a one-to-one basis. Thus, each resource (e.g. an employee, machine or time slot) is to be assigned uniquely to a particular activity (e.g. a task, site or event). In assignment problems, supply in each row represents the availability of a resource such as a man, machine, vehicle, product, salesman, etc. and demand in each column represents different activities to be performed such as jobs, routes, factories, areas, etc. for each of which only one man or vehicle or product or salesman respectively is required. Entries in the square being costs, times or distances. The assignment method is a special linear programming technique for solving problems like choosing the right man for the right job when more than one choice is possible and when each man can perform all of the jobs. The ultimate objective is to assign a number of tasks to an equal number of facilities at minimum cost (or maximum profit) or some other specific goal.

Let there be 'm' resources and 'n' activities. Let c_{ij} be the effectiveness (in terms of cost, profit, time, etc.) of assigning resource i to activity j (i = 1, 2, …., m; j = 1, 2,…., n). Let $x_{ij} = 0$, if resource i is not assigned to activity j and $x_{ij} = 1$, if resource i is assigned to activity j. Then the objective is to determine x_{ij}'s that will optimize the total effectiveness (Z) satisfying all the resource constraints and activity constraints.

1. Mathematical Formulation

Let number of rows = m and number of columns = n. If m = n then an AP is said to be BAP otherwise it is said to be UBAP.

A) Case 1: If m < n then mathematically the UBAP can be stated as follows:

Minimize $$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij} \quad (1.1)$$

Subject to $$\sum_{j=1}^{n} X_{ij} = 1$$, i = 1, 2, ….., m (availability constraints), (1.2)

$$\sum_{i=1}^{n} X_{ij}$$ = 0 or 1, j = 1, 2,…, n (requirement constraints), (1.3)

And x_{ij}= 0 or 1, for all i and j. (1.4)

B) Case 2: If m > n then mathematically the UBAP can be stated as follows:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \tag{1.5}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} X_{ij} = 0 \text{ or } 1, \; i = 1, 2, \ldots, m \text{ (availability constraints)}, \tag{1.6}$$

$$\sum_{i=1}^{n} X_{ij} = 1, \; j = 1, 2, \ldots, n \quad \text{(requirement constraints)}, \tag{1.7}$$

$$\text{And} \quad x_{ij} = 0 \text{ or } 1, \text{ for all i and j.} \tag{1.8}$$

Presently an AP can be solved by using one of the four methods, (i) Enumeration method, (ii) Simplex method (iii) Transportation method and (iv) Hungarian method**.** Among these four methods Hungarian method can be used as an efficient method for finding an optimal solution of an AP. But this method also requires more number of computations and steps. For using Hungarian method to solve UBAP it is require to convert it into BAP, Hadley (1997), Taha (2008), Kanti Swarup *et al.*, (2008),Gupta and Hira (2010), Sharma (2010). This leads to consider an assignment table of higher order than the initial assignment table, to do more number of computations, iterations and steps to get an optimal solution. Also, to solve an AP of maximization type, it is require to convert it into minimization type. This leads to do more number of computations and steps to get an optimal solution.

Kore (2008) made an attempt to solve unbalanced transportation problem without balancing it. As AP is a particular case of TP in this paper, I have proposed a new approach to solve an UBAP, which overtakes the problem of degeneracy of transportation method. Using our new approach we get, optimal solution of an UBAP without balancing it, with less number of computations, steps and considering initial assignment table, without changing its order. We can illustrate the comparison between our new approach and Hungarian Method by solving various types of UBAPs.

2. A) Algorithms of the new methods to obtain an IBFS

2.1: Row Penalty Assignment Method (RPAM)

Step1: For each row determine row penalty by taking difference between smallest and next smallest effectiveness.

Step2: Observe the maximum row penalty, select smallest effectiveness corresponding to that row, and encircle it, cross out corresponding row and column. If there is a tie in maximum row penalty then select the largest effectiveness of the smallest effectiveness corresponding to them. If there is a tie in the largest effectiveness of the smallest effectiveness then select that largest effectiveness corresponding to which next to next smallest effectiveness in the row is maximum. If there is again tie then select one of them randomly,

Step3: Repeat *step1* and *step2* until only one row is remained uncrossed. Select smallest effectiveness in the last row, encircle it and cross out corresponding row and column. If there is a tie in smallest effectiveness then select that smallest effectiveness corresponding to which next smallest effectiveness in the column is minimum.

2.2: Column Penalty Assignment Method (CPAM)

*Step1***:** For each column determine column penalty by taking difference between smallest and next smallest effectiveness.

*Step2***:** Observe the maximum column penalty, select smallest effectiveness corresponding to that column, encircle it, cross out corresponding row and column. If there is a tie in maximum column penalty then select the largest effectiveness of the smallest effectiveness corresponding to them. If there is a tie in the largest effectiveness of the smallest effectiveness then select that largest effectiveness corresponding to which next to next smallest effectiveness in the column is maximum. If there is again tie then select one of them randomly.

*Step3***:** Repeat *step1* and *step2* until only one column is remained uncrossed. Select smallest effectiveness in the last column, encircle it and cross out corresponding row and column. If there is a tie in smallest effectiveness then select that smallest effectiveness corresponding to which next smallest effectiveness in the row is minimum.

B) **Algorithm of the Method to obtain an Optimal Solution**

2.3: Non-basic Smallest Effectiveness Method (NBSEM)

Step1: Select non-basic cell having smallest effectiveness.

Step 2 : a) Form a loop which starts and ends at selected non-basic cell considering two basic cells and two non-basic cells such that, the non-basic cells and basic cells are alternate in the loop, no more than two cells in the loop are in the same row or column.

b) Make the total of effectiveness in the non-basic cells (T) and the total of effectiveness in the basic cells (T').

c) If $T = T'$ then it indicates that there exists an alternative solution to the given AP.

d) If $T < T'$ then it indicates that improvement in the present IBFS is possible. If there is a tie in smallest effectiveness in the non-basic cells then select that smallest effectiveness which provides maximum improvement. Interchanging non-basic cells and basic cells in the row. Select again smallest effectiveness in non-basic cells and go to *step 2.* If $T > T'$ then go to (e)

e) Increase the number of basic cells and non-basic cells one by one up to min (m, n), form all possible loops one by one satisfying the conditions of forming a loop as stated in (a), go to (b).

Note: 1) If an UBAP is of maximization type then use RPAM to obtain an IBFS of the UBAP of *case1* and CPAM, to obtain an IBFS of the UBAP of *case 2*, in opposite sense.

2) For using the NBSEM to test an optimality of IBFS of the UBAP of *case 1 or case 2,* select non-basic smallest effectiveness corresponding to which row and column have a basic cell.

3) For an UBAP of maximization type, to test an optimality of an IBFS we can use the NBSEM in opposite sense.

C) Algorithm of the New Approach to solve an UBAP

Step 1: Express the given AP in tabular form.

Step 2: Check whether the AP is BAP or UBAP.

Step 3: If an AP is UBAP of case 1 then obtain an IBFS using RPAM. If an AP is UBAP of case 2 then obtain an IBFS by using CPAM.

Step 4: Optimize an IBFS of UBAP by using NBSEM to get an optimal solution of given UBAP.

Step 5: Write optimal solution and the optimum value of objective function (Z).

3. Applications of the Approach method

We illustrate the effectiveness of the new approach by solving various types of APs.

Example 3.1: Consider the following AP of minimization type, Sharma (2010)**:**

	1	2	3	4	5	6
1	80	140	80	100	56	98
2	48	64	94	126	170	100
3	56	80	120	100	70	64
4	99	100	100	104	80	90
5	64	90	90	60	60	70

Here, m = 5 and n = 6 i.e. m < n, the problem is UBAP of **case 1**. Using the RPAM to obtain an IBFS and testing its optimality by using the NBSEM we get,

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Table 3.1: IBFS

	1	2	3	4	5	6	R.P.
1	80	140	80	100	56	98	(24)
2	48	64	94	126	170	100	(16) 16)
3	56	80	120	100	70	64	(8) (8) (16)
4	99	100	100	104	80	90	(10) (9) (10) (0)
5	64	80	90	60	60	70	(0) (4) (10) (20)

Here, T= 200 and T' = 212 i.e. T < T', improvement in the present solution is possible by introducing the non-basic smallest effectiveness 56 in the basis. The improved solution is,

Table 3.2: Optimal Solution

	1	2	4	5	6
1	80	140	100	56	98
2	48	64	126	170	100
3	56	80	100	70	64
4	99	100	104	80	90
5	64	80	60	60	70

Here, Improvement in the present solution is not possible by introducing the non-basic smallest effectiveness 48 in the basis. The present solution is optimal solution to the given UBAP.
The optimal solution is assign, 1→5, 2→2, 3→1, 4→6 and 5→4, job 3 remained unassigned.
The optimum value of Z is, Z_{min} = 56 + 64 + 56 + 90 + 60 = 326.
Now, we solve the above UBAP by using Hungarian method, converting the given UBAP into BAP by introducing dummy row having zero effectiveness in each cell we get,

Table 3.3: BAP

	1	2	3	4	5	6
1	80	140	80	100	56	98
2	48	64	94	126	170	100
3	56	80	120	100	70	64
4	99	100	100	104	80	90
5	64	80	90	60	60	70
6	0	0	0	0	0	0

Subtracting the smallest effectiveness in each row from each effectiveness of that row and drawing he minimum number of vertical and horizontal lines necessary to cover all zeros in the reduced matrix obtained, we get,

Table 3.4

	1	2	3	4	5	6
1	24	84	24	44	0	42
2	0	16	46	78	122	52
3	0	24	64	44	14	8
4	19	20	20	24	0	10
5	4	20	30	0	0	10
6	0	0	0	0	0	0

Here, the number of lines drawn ≠ the number of rows or columns, the optimal solution cannot be obtained. Subtracting the smallest uncovered effectiveness from each uncovered effectiveness and adding into the effectiveness which lies at the intersection of two lines, and drawing the minimum number of vertical and horizontal lines necessary to cover all zeros in the new reduced matrix obtained we get,

Table 3.5

	1	2	3	4	5	6
1	24	76	16	36	0	34
2	0	8	38	70	122	44
3	0	16	56	36	14	0
4	19	12	12	16	0	2
5	12	20	30	0	8	10
6	8	0	0	0	8	0

Here, the number of lines drawn ≠ the number of rows or columns, the optimal solution cannot be obtained. Subtracting the smallest uncovered effectiveness from each uncovered effectiveness and adding into the effectiveness which lies at the intersection of two lines, and drawing the minimum number of vertical and horizontal lines necessary to cover all zeros in the new reduced matrix
obtained we get,

Table 3.6

	1	2	3	4	5	6
1	24	74	14	34	0	32
2	0	6	36	68	122	42
3	2	16	56	36	16	0
4	19	10	10	14	0	0
5	14	20	30	0	10	10
6	10	0	0	2	10	0

Here, the number of lines drawn ≠ the number of rows or columns, the optimal solution cannot be obtained. Subtracting the smallest uncovered effectiveness from each uncovered effectiveness and adding into the effectiveness which lies at the intersection of two lines, and drawing the minimum number of vertical and horizontal lines necessary to cover all zeros in the new reduced matrix obtained we get,

Table 3.7

	1	2	3	4	5	6
1	22	72	12	32	0	32
2	0	6	36	68	124	44
3	0	14	54	34	16	0
4	17	8	8	12	0	0
5	14	18	28	0	12	12
6	10	0	0	2	12	2

Here, the number of lines drawn ≠ the number of rows or columns, the optimal solution cannot be obtained. Subtracting the smallest uncovered effectiveness from each uncovered effectiveness and adding into the effectiveness which lies at the intersection of two lines, and drawing the minimum number of vertical and horizontal lines necessary to cover all zeros in the new reduced matrix obtained we get,

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Table 3.8

	1	2	3	4	5	6
1	22	66	6	28	0	32
2	0	0	30	62	124	44
3	0	8	48	28	16	0
4	17	2	2	6	0	0
5	20	18	28	0	18	18
6	16	0	0	2	18	8

Here, the number of lines drawn = the number of rows or columns. The optimal solution can be obtained.

Table 3.9: Optimal Solution

	1	2	3	4	5	6
1	22	66	6	28	0	32
2	~~0~~	0	30	62	124	44
3	0	8	48	28	16	~~0~~
4	17	2	2	6	~~0~~	0
5	20	18	28	0	18	18
6	16	~~0~~	0	2	18	8

Here, the optimal solution is assign, 1→5, 2→2, 3→1, 4→6 and 5→4.
The optimum value of Z is, $Z_{min} = 56 + 64 + 56 + 90 + 60 = 326$.
Note: For above example we get an optimal solution by using our new approach in 2 steps, and considering assignment table of order 5×6 and 5×5 in each step respectively.
By using Hungarian method we get an optimal solution in 7 steps, and considering assignment table of order 6×6 from first step to last step.
Since steps are more, computations are more, and order of assignment table is higher, the time required to solve the above UBAP by using Hungarian method is more than the time required by using our new approach.

Example 3.2: Consider the following AP of maximization type, Kanti Swarup et al., (2008):

	1	2	3	4
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	3
6	5	7	6	4

Here, m = 6 and n = 4 i.e. m > n, the problem is UBAP of **case 2**.Using the CPAM in opposite sense to obtain an IBFS and using the NBSEM in opposite sense to test its optimality we get,

Table 3.10: IBFS

	1	2	3	4
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	3
6	5	7	6	4
C. P.	(1)	(1)	(1)	(1)
	(1)	(2)		(1)
	(1)			(1)

Here, T= 19 and T' = 18 i.e. T > T', improvement in the present solution is possible by introducing the non-basic largest effectiveness 8 in the basis. The improved solution is,

Table 3.11

	1	2	3	4
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
6	5	7	6	2

Here, T= 15 and T' = 12 i.e. T > T', improvement in the present solution is possible by introducing the non-basic largest effectiveness 8 in the basis. The improved solution is,

Table 3.12

	1	2	3	4
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
6	5	7	6	2

Here, improvement in the present solution is not possible by introducing the non-basic largest effectiveness 8 in the basis. The present solution is an optimal solution to the given UBAP.
The optimal solution is assign, 2→1, 3→2, 4→4, and 6→3, job 1and 5 remained unassigned.
The optimum value of Z is, $Z_{max} = 7 + 8 + 7 + 6 = 28$.
Now, we solve the above UBAP by using Hungarian method converting the given UBAP of maximization type into minimization type by subtracting all the effectiveness from largest effectiveness we get,

Table 3.13: Minimization Problem

	1	2	3	4
1	5	2	6	2
2	1	7	4	4
3	5	0	3	0
4	2	4	5	1
5	3	6	4	5
6	3	1	2	4

Converting the given UBAP into BAP by introducing two dummy columns having zero effectiveness in each cell we get,

Table 3.14: BAP

	1	2	3	4	5	6
1	5	2	6	2	0	0
2	1	7	4	4	0	0
3	5	0	3	0	0	0
4	2	4	5	1	0	0
5	3	6	4	5	0	0
6	3	1	2	4	0	0

Subtracting the smallest effectiveness in each column from each effectiveness of that column and drawing the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained we get,

Table 3.15

	1	2	3	4	5	6
1	4	2	4	2	0	0
2	0	7	2	4	0	0
3	4	0	1	0	0	0
4	1	4	3	1	0	0
5	2	6	2	5	0	0
6	2	1	0	4	0	0

Here, the number of lines drawn ≠ the number of row or columns the optimal solution cannot be obtained. Subtracting the smallest uncovered effectiveness from each uncovered effectiveness and adding into the effectiveness which lies at the intersection of two lines, and drawing the minimum number of vertical and horizontal lines necessary to cover all zeros in the new reduced matrix obtained we get,

Table 3.16

	1	2	3	4	5	6
1	3	1	3	1	0	0
2	0	7	2	4	1	1
3	4	0	1	0	1	1
4	0	3	2	0	0	0
5	1	5	1	4	0	0
6	2	1	0	4	1	1

Here, the number of lines drawn = the number of rows or columns. The optimal solution can be obtained.

Table 3.17: Optimal Solution

	1	2	3	4	5	6
1	3	1	3	1	0	0
2	0	7	2	4	1	1
3	4	0	1	0	1	1
4	0	3	2	0	0	0
5	1	5	1	4	0	0
6	2	1	0	4	1	1

The optimal solution is assign, 2→1, 3→2, 4→4, and 6→3.
The optimum value of Z is, $Z_{max} = 7 + 8 + 7 + 6 = 28$.
Note: For above example we get an optimal solution by using our new approach in 3 steps and considering assignment table of order 6×4, 4×4 and4×4 respectively.
By using Hungarian method we get an optimal solution in 5 steps and considering assignment table of order 6×6 from first step to last step.
Since steps are more, computations are more and order of assignment table is higher the time required to solve the above UBAP by using Hungarian method is more than the time required by using our new approach.

CONCLUSIONS

1) It is not required to convert **UBAP** in the form of **BAP** to get an optimal solution.
2) If an **IBFS** of the **UBAP** of **case 1**and **case 2**is obtained by using **RPAM** and **CPAM** respectively, without balancing it and it is optimized by using **NBSEM** method then the least possible optimum value of **Z** is achieved.
3) Using our new approach to solve the **UBAP** we get, optimal solution fastly, without changing the order of assignment table, with less number of steps, iterations and computations.

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