International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2012 Vol. 2 (1) January-March, pp.148-156/ Dubey et al.

Research Article

AN QUALITATIVE STUDY OF ANISOTROPIC BIANCHI TYPE-II COSMOLOGICAL MODELS WITH VARYING COSMOLOGICAL TERM

***R. K. Dubey[1], Bijendra Kumar Singh[2] and Abhijeet Mitra[3]**
[1]*Department of Mathematics, Govt. Science P.G.College, Rewa, Madhya Pradesh, India*
[2]*Department of Mathematics, Govt. P G College, Satna, Madhya Pradesh, India*
[3]*Department of Mathematics, Govt. P G College, Satna, Madhya Pradesh, India*
**Author for Correspondence*

ABSTRACT
Einstein's field equations are considered for a Bianchi type-II space time with time- decaying cosmological term Λ) . Applying a special law of variation for Hubble's parameter exact solutions of field equations are obtained. An anisotropic cosmological model with a negative constant deceleration parameter is presented for accelerated phase of universe. Kinematic properties and behavior of the models has been discussed.

INTRODUCTION
Evolution of universe has been investigated in general relativistic cosmological models. Our present cosmology is based on Friedmann-Robertson-Walker (FRW) model. For simplification and description of the large scale behavior of the actual universe, locally rotationally symmetric Bianchi type-II space time has been studied. In FRW cosmological model, the universe is completely homogeneous and isotropic which is agreed by the observational data about the large scale structure of the universe. Experimental studies of the isotropy done by cosmic microwave background radiation and speculation about the amount of the helium formed at the early stage of the evolution of universe have stimulated theoretical interest in an anisotropic cosmological model. Recent experimental data and critical arguments support the existence of an anisotropic phase of the cosmic expansion that approaches an isotropic one. This results for exact anisotropic solution of Einstein's field equations for cosmological models for universe. Bianchi type-II cosmological models are important for constructing models with geometrical and physical structure for describing the early stages of evolution of universe. Asseo and Sol (1987) describes the importance of Bianchi type-II cosmological models. Lorenz (1980) has presented exact solutions for rotationally symmetric Bianchi type-II space time stiff matter models in an electromagnetic field theory. Exact solutions to Einstein's field equations for locally rotationally symmetric Bianchi type-II cosmological models was obtained by Hajj-Boutors (1986). For perfect fluid of matter, Shanthi and Rao (1991), presented Bianchi type II model in Barber's self creation theory of gravitation. Cosmological solution of Bianchi type II stiff fluid models using electromagnetic field was obtained by Venkateswarlu and Reddy (1991). Bianchi type II inflationary model with constant deceleration parameter was derived by Singh and Kumar (2007).
One of the most important and outstanding problems in cosmology is the cosmological constant and gravitational constant. Recent observations by Perlmutter et al. (1998,1999) and Riess et al. (1998) strongly favour a significant and positive value of Λ . Al.Rawaf and Taha and Al-Rawaf (1998) and Overdin and Cooperstock (1998) proposed a cosmological model with a cosmological constant of the form $\Lambda = \beta \frac{\ddot{a}_1}{a_1}$, where a_1 is scale factor of universe and β is a constant. Recently, Pradhan et al. (2008) have shown that spatially homogeneous Bianchi type II cosmological model with stiff matter and time decaying Λ term is compatible with recent observations on accelerated universe. One of the

Research Article

motivations for introducing Λ terms to reconcile the age parameter and density parameter of universe with recent observational data.

In this paper we consider Einstein's field equations for anisotropic LRS Bianchi type-II space in the presence of stiff matter with a time decaying cosmological term Λ. An attempt has been made to formulate a law of variation for Hubble's parameter in anisotropic Bianchi type-II space time that yields a constant value of deceleration parameter, the law together with the Einstein's field equations leads to a number of new solutions of Bianchi space time. The most significant study done by Chen and Wu (1990) has been further modified by several authors as Berman & Mom.(1990).

Motivated by the stimulations discussed above, in this paper we have studied Bianchi type-II cosmological models in the presence of the bulk viscous fluid with varying Λ. Some physical and geometrical features of the models are discussed.

2. The Metric and Field Equations

We consider the locally rotationally symmetric metric for anisotropic Bianchi type-II in the form

$$ds^2 = - \quad + \quad + \quad + \quad + \tag{1}$$

Where a & b are functions of cosmic time t. The energy-momentum tensor for a perfect fluid is,

$$T_{\alpha\beta} = \quad + \quad + \tag{2}$$

Where ρ is mass density and p is pressure, u^α is the fluid four velocity vector such that $u^\alpha u_\alpha =$

For energy momentum tensor in (2) and the LRS Bianchi type-II space time (1), Einstein's field equations

$$R_{\alpha\beta} - \frac{1}{2} \quad + \Lambda \quad = \tag{3}$$

In commoving coordinates leads to the following set of three independent and non-linear differential equations

$$2\frac{\ddot{}}{b} + \frac{}{b} - \frac{}{4\,b^{}} = - \quad - \Lambda \tag{4}$$

$$\frac{\ddot{}}{a} + \frac{\ddot{}}{b} + \frac{\dot{}\,\dot{}}{b\,a} + \frac{}{4b^{}} = - \quad - \Lambda \tag{5}$$

$$2\frac{\dot{}}{b}\frac{\dot{}}{a} + \frac{}{b} - \frac{}{4b^{}} = \quad - \Lambda \tag{6}$$

Where $\Lambda \quad)$ is the cosmological constant.

We consider the equation of state

$$p = \quad \leq \quad \leq \tag{7}$$

The average scale factor $a_1(t)$ is defined as

$$a_1(t) = \tag{8}$$

A volume scale factor V is given by $V = \quad =$ (9)

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3} \quad + \quad + \tag{10}$$

Where $Hx = \frac{\dot{}}{a} = \quad = \frac{\dot{}}{b}$ are the directional Hubble's parameters in the direction of x, y, z respectively.

From equation (9) &(10), the average Hubble's parameter may be generalized in anisotropic cosmological model as $H = \frac{1}{3}\frac{\dot{}}{V} = \frac{\dot{}}{a_1} = \frac{}{3} \quad \frac{\dot{}}{b} + \frac{\dot{}}{a}$ (11)

The energy conservation equation $T^{\beta}_{\alpha\,\beta} =$ leads to

$$\dot{} \quad \frac{\dot{}}{b} + \frac{\dot{}}{a} = \frac{}{8\pi} \tag{12}$$

Einstein's field equations (4)-(6) are a coupled system of highly non linear equations. Kramer et al. (1994) have pointed out that most of the authors solve the Einstein's field equations with a stress energy tensor of perfect fluid type by assuming an equation of state linking the pressure p and energy density ρ in order to build analytical methods near the singularity. Davidson (1962) and later many others (e.g.; Coley and Tupper (1986)) have considered models with variable equation of state, which essentially deals with the (FRW) metric.

In order to solve the Einstein's field equations, we normally assume a form for the matter content or suppose that the space-time admits killing vector symmetries. Solution to the field equations may also be generated by applying a law of variation for Hubble's parameter, which was first proposed by Berman (1983) in FRW models and that yields constant value of deceleration parameter. Recently the present authors (Singh and Kumar; 2006) have proposed a similar law of variation for Hubble's parameter in locally rotationally symmetric Bianchi type II space-time that also yields a constant value of deceleration parameter. The variation for Hubble's parameter as assumed is not inconsistent with the observations.

3. Solution of Field Equation

Constant deceleration parameter q defined by $q = -\frac{\ddot{}}{\dot{}} = \quad t$ (13)

Where $a_1(t)$ is the average scale factor

Solving (13), $q = -\frac{\ddot{}}{\dot{}} \Rightarrow \quad \dot{} \quad \ddot{}$ (14)

Here a_1 is function of time t. Let $\frac{da_1}{dt} =$,

Equation (14) reduces to, $qz^2 + \dot{} \quad \frac{}{q\ z} = -$

Integrating, $\frac{a_1}{q}\frac{1}{z} = \; + \; \Rightarrow \; \frac{}{t+} - \quad \because \quad \frac{}{dt}$, again integrating,

$$q \log a_1 = \quad + \quad + \quad \Rightarrow \quad = \quad + \quad \Rightarrow \quad = \quad + \tag{15}$$

Where c_1, c_2, c_3 are integration constants.

From (15) we have that condition of expansion for $q >$. If $q >$ the model expands but always decelerate.
There can be arises many cases for physical solutions, but here we consider only one case.
Considering the case for physical solutions of equation (4)-(6)
Case Study:
For stiff matter $k =$

Adding equation (5) & (6), $\frac{}{a} + \frac{}{b} + \frac{}{ba} + \frac{}{b} = - \Lambda$ (16)

Subtracting (5) from (4), we obtain the condition for isotropy

$$\frac{}{b} - \frac{}{a} + \frac{}{b} - \frac{}{ba} + \frac{}{b} = \frac{}{b} \tag{17}$$

Equation (16) can be also written as, $\frac{d(b^2}{b^2 a} = - \Lambda$

Or $d(b^2$,

Integrating, b^2 (18)

Where c_4 is integrating constant.

Using the phenomenological decay law for Λ of the form $\Lambda = \quad = \left(\quad \right)$ (19)

Using (8), (15) & (19) in (18), we have

$$b^2 \dot{}$$

$$= - \int + \quad + \frac{1}{}$$

$$= - \frac{}{q^2} \int + \quad = - \frac{}{q} \quad 3 - \quad +$$

Dividing by $b^2 a$ by both sides,

$$\frac{\dot{}}{a} + \frac{\dot{}}{b} = - \frac{}{q(3-} + \quad +$$

$$\frac{\dot{}}{a} + \frac{\dot{}}{b} = - \frac{}{q(3-} + \quad + \quad + \quad +$$

$$\frac{\dot{}}{a} + \frac{\dot{}}{b} = - \frac{}{q(3-} + \quad + \quad +$$

$$\Rightarrow \frac{d(ab)}{ab} = - \frac{}{q(3-} + \quad + \quad +$$, integrating both sides,

$$\Rightarrow \quad = - \frac{}{q(3-} + \quad + \quad c_2 \quad + \quad - \frac{}{q} +$$

c_5 is integrating constant.

$$\Rightarrow \quad = \quad + \quad (20)$$

From (8) & (15), we have $b^2 a = \quad + \quad$ (21)

Dividing (21) by (20)

Research Article

$$b(t) = \frac{1}{c_5} + \ldots \quad (q- \qquad (22)$$

Putting value of $b(t)$ in (21), we obtain,

$$a(t) = + \quad \frac{c_4 q}{q-} \qquad (23)$$

From (15) & (19), the cosmological term Λ) is given by

$$\Lambda = \times \frac{2}{q} + + = \frac{2}{q} + \qquad (24)$$

For $q <$ Λ is always positive. $q <$ Provides the exponential expansion and later accelerates the universe. The evolution of the universe in such a scenario is shown to be consistent with the present observations predicting an accelerated expansion.

Using (17) and putting values of $a(t)$ & $b(t)$ from (23) and (22) into (6), we obtain, density parameter

$$\rho = + + + - + \qquad (25)$$

Where m_1, m_2 constants in terms of are β , m_3 is also constant.

Hubble's parameter $H = \frac{\dot{}}{a_1} = \frac{}{q} +$

The directional Hubble's factors, Hx, Hy & Hz are given by

$$Hx = \frac{\dot{}}{a} = \frac{}{c_2} + + \left(\right) \qquad (26)$$

$$Hy = \quad = \frac{\dot{}}{b} = -\frac{}{c_2} + + \left(\right) \qquad (27)$$

The scalar of expansion $= \quad = \left(\dot{} \right) +$ (28)

Using (7) and (11) into eq. (12), we have (for $k =$)

$$\Rightarrow \frac{\dot{}}{\rho} + \times = -\pi \Rightarrow \frac{\dot{}}{\rho} + = -\pi\rho$$

Research Article

$$\Rightarrow \dot{\rho} + \left(\dot{} \right) \tag{29}$$

($\because$ for continuing equation)

Integrating (29)

$$\log \rho + \quad + \quad =$$

$\Rightarrow \quad =$ is integrating constant

$$G = \quad + \quad \left[\quad \right]$$

$$= \quad + \quad + \quad + \quad -$$

$$G = \quad + \quad + \quad + \quad + \tag{30}$$

Where A, B & C are constants

For $q <$ is always decreasing. Hubble's parameter $H = \frac{\dot{}}{a_1} = \frac{}{q} + $

Now anisotropic Bianchi type-II model is given by

$$ds^2 = - \quad + \quad + \quad + \quad + \quad +$$

$$e^{c_2(q-} (dy^2 + \tag{31}$$

Matter energy density ; $\Omega_{m} = \frac{}{3H^2}$

$$\Omega_{m} = \frac{2}{3c_2^2} \left[\quad \right] \tag{32}$$

Dark energy matter; $$\Omega = \frac{\Lambda}{3H^2} = \tag{33}$$

In absence of any curvature, matter energy density Ω_{m} and dark energy Ω are related by the equation

$$\Omega_{m} + \Omega =$$

From (32) and (33)

$$\Omega_{...} + \Omega_{.} = \frac{\ ^2}{3c_2^2}\left[\quad \right]$$

Taking $m_1 = \ = \ = $, we have

$$\Omega_{...} + \Omega_{.} = \frac{\Omega_{...}}{3c_2^2} + - + \frac{\rho}{3} = \frac{\rho}{c_2^2} + \frac{\rho}{3}$$

Taking $m_1 = \ = $, we have $\Omega_{...} + \Omega_{.} = $

i.e all simple inflationary models predict that the universe is flat. There is no known physical reason for a non-zero cosmological constant.

4. *Conclusion*

One of the most important and outstanding problem in cosmology is the cosmological constant problem. In this paper, we have presented Einstein's field equation for LRS. Bianchi type II stiff fluid space-time with varying cosmological constant of the form $\Lambda =$. In early phases of dominated era q was positive and becomes negative during the later stage of the evolution. To obtain exact solutions to the field equation which would corresponds to an accelerated universe; we have applied a special law of variation for Hubble's parameter initially proposed by Bermann. The expressions for some important cosmological parameters have been obtained and their physical behavior is discussed. The evolution of universe starts with a singularity at $t = -\frac{}{c_2}$. The corresponding universe expands indefinitely with acceleration while all the physical parameters diverge at $t = -\frac{}{c_2}$. As t increases $t \to \infty$ $\Lambda \to \ \to$, also scalar of expansion tends to zero. For $t \to \infty$, the universe becomes essentially empty. The matter energy $\Omega_{...}$ tends to zero as $t \to \infty$. It is also observed that universe remains anisotropic throughout the evolution. In this paper we have obtained that for $q <$ Λ is positive but G is negative sometimes $q <$ provides the exponential expansion and later accelerates the universe.

From (30) the model expands indefinitely with acceleration while all the physical parameters diverge at $t = -\frac{}{c_2}$. The cosmological constant Λ in all cases is a decreasing function of time, while gravitational constant $G(t)$ is increasing function in some cases. The law of variation for Hubble's parameter explicitly determines the scale factors. It is possible that law of Hubble's parameter variation and cosmological constant may be useful in studying new solution to Einstein's field equation in other alternative theories. It is also observed that this LRS Bianchi type II universe filled with stiff fluid is compatible with the recent observations.

As $t \to \infty$, the scale factor divege and ρ Λ tend to zero.

REFERENCES

Asseo E., Sol. H (1987) Extragalatic magnetic fields . Physics Review **148**(6) 307-435.

Lorenz D (1980) Physics Letter A **79**, 19.

Hajj-Boutors J (1986) .A method for generating exact Bianchi type II cosmological models. Journal of Mathematical Physics **27**(6) 1592.

Shanthi K, Rao V.V.M. (1991). Astrophysics and Space Science **79**, 147.

Venkateswarlu R. , Reddy D.R.K. (1991). Exact Bianchi type-II,VIII & IX cosmological models with matter and electromagnetic fields in Lyra's manifold. Astrophysics and Space Science **182**(1) 97-103.

Singh C.P. and Kumar S. (2007). Bianchi type-II inflationary models with constant deceleration parameter in general relativity. Pramana-Journal of Physics **68**(5) 707-720.

Perlmutter S. et al.: Measurements of Omega and Lambda from 42 High-Redshift Supernovae/Astrophysics Journal 483,565.(1997); astro-ph/9608192; Nature, 391,**51**(1998); astroph/9712212; Astrophys.J., 517,565 (1999); astro-ph/9608192.

Riess A.G. et al. (1998): Observational evidence from supernovae for an accelerating Universe and a cosmological constant. Astronomical Journal **116**, 1009; astro-ph/9805201.

Al.Rawaf A.S. (1998). Modern Physics Letter A, **13**,429.

Overdin J.M., Cooperstock F.I. (1998). Phys.Rev.D, **58**, 043506.

Pradhan A., Srivastava D., Khadekar G.S. (2008). Can Bianchi type-II cosmological models with a decay law for lambda term be compatible with recent observations? Romanian report in physics **60**(1) 3-12.

Chen W. and Wu Y.S. (1990). Physics Review D **41**,695.

Berman M., Som M.M. (1990).International Journal of Theoretical Physics **29**,1411.