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Research Article

SELF-SIMILAR FLOW UNDER THE ACTION OF MONOCHROMATIC RADIATION BEHIND A STRONG CYLINDRICAL SHOCK WAVE IN A NON-IDEAL GAS

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ABSTRACT
Similarity solutions are obtained for one-dimensional flow under the action of monochromatic radiation behind a strong cylindrical shock wave propagating in a non-ideal gas. The initial density of the medium is assumed to be constant. It is inferred that the effects of the non-idealness of the gas and of the monochromatic radiation on the shock propagation become more significant when the ratio of specific heats is increased.

Key Word: *Shock waves, Non-ideal gas, Monochromatic radiation, Self-similar solutions.*

INTRODUCTION
The influence of radiation on a strong shock wave and on the flow-field behind the shock front has always been of great interest, for instance, in the field of nuclear power and space research. Consequently, similarity models [Sedov (1959)] for classical blast wave problems have been extended, taking radiation into account [Elliott (1960), Wang (1964), Helliwell (1969), Ray and Bhowmick (1976), Nicastro (1970), Ghoniem et al. (1982), Gretler and Steiner (1993)]. Elliott (1960) discussed the conditions leading to self-similarity with a specified functional form of the mean free-path of radiation and obtained a solution for self-similar explosions. Wang (1964), Helliwell (1969) and Nicastro (1970) treated the problems of radiating walls, either stationary or moving, generating shock at the head of self-similar flow-fields. Assuming the shock to be isothermal and transparent the self-similar solution of the central explosions in stars has been obtained by Ray and Bhowmick (1976) including the effects of radiation.

The self-similar solutions have been used by Khudyakov (1983) to discuss the problem of the motion of a gas under the action of monochromatic radiation. Khudyakov (1983) has considered that a homogeneous gas at rest occupies a half-space bounded by a fixed plane wall and assumed that a radiation flux moves through the gas in the direction of the wall with a constant intensity j_0 per unit area. From the instant of arrival of the radiation at the wall a shock wave is assumed to propagate out from the wall in the direction opposite to the radiation flux. The radiation flux is absorbed in the zone between the shock wave and the wall, and it is not absorbed in the undisturbed medium. It is also assumed that the gas itself does not radiate. Zheltukhin (1968) has developed a family of exact solutions of one dimensional motion (plane, cylindrical or spherical symmetry) of a gas taking into account of the absorption of monochromatic radiation. Nath and Takhar (1990) and Nath (1998) have studied the propagation of cylindrical shock waves in a gas under the action of monochromatic radiation when the medium is rotating or non-rotating. In all of these works, the medium is assumed to be a gas obeying the equation of state of an ideal gas.

Because of the extreme conditions that generally arise behind a shock wave, produced by an explosion, the assumption that the gas is ideal is no more valid. The popular alternative to the ideal gas is a simplified van der Waals model. Roberts and Wu (1996) and Wu and Roberts (2003) adopted this model to discuss the shock wave theory of sonoluminescence. In the present work, we too adopt this as our model of a non-ideal gas to obtain the self-similar solutions for the flow under the action of monochromatic radiation behind a strong cylindrical shock wave propagating in a non-ideal gas. The initial density of the medium is assumed to be constant.

Effects of a change in the parameter of non-idealness of the gas, in the ratio of specific heats and in the parameter of the flux of monochromatic radiation on the shock propagation are investigated. It is observed that the effects of non-idealness of the gas on the flow variables in the flow-field behind the shock are significant when the ratio of specific heats are higher. The present work may be considered as an extension of the work of Nath and Takhar (1990) by taking the medium a non-ideal gas in place of an ideal gas.

Basic Equations and Boundary Conditions

The fundamental equations for cylindrically symmetric motion of a non-ideal gas under the action of monochromatic radiation neglecting heat-conduction, viscosity, radiation of the medium, may be written as [Khudyakov (1983), Nath (1998), Zedan (2002)]

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\frac{\partial v}{\partial r} + \frac{\rho v}{r} = 0, \tag{2.1}$$

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial r} + \frac{\partial p}{\partial r} = 0, \tag{2.2}$$

$$\frac{\partial e}{\partial t} + v\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r}\right) = \frac{1}{\rho r}\frac{\partial}{\partial r}(jr), \tag{2.3}$$

$$\frac{\partial j}{\partial r} = Kj, \tag{2.4}$$

where ρ, v, p, e and j are the density, radial velocity, pressure, internal energy per unit mass and the flux of monochromatic radiation per unit area at radial distance r from the axis at time t, and K is the absorption coefficient.

Most of the phenomena associated with shock wave arise in extreme conditions under which the ideal gas is not a sufficiently accurate description. To discover how deviations from the ideal gas can affect the flow behind a shock wave, we adopt a simple model. We assume that the gas obeys a simplified van der waals equation of state of the form [Roberts and Wu (1996), Wu and Roberts (2003)]

$$p = \frac{\bar{R}\rho T}{1 - b\rho} \quad , e = C_v T = \frac{p(1 - b\rho)}{\rho(\gamma - 1)}, \tag{2.5}$$

where $\bar{R}$ is the gas constant, $C_v = \bar{R}/(\gamma - 1)$ is the specific heat at constant volume and γ is the ratio of specific heats. The constant b is the "van der Waals excluded volume", it places a limit, $\rho_{max} = 1/b$, on the density of the gas.

The absorption coefficient K is considered to vary as [Khudyakov (1983), Nath (1998), Nath and Takhar (1990)]

$$K = K_0 \rho^n p^m j^q r^s t^l, \tag{2.6}$$

where the coefficient K_0 is a dimensional constant and the exponents n, m, q, s, l are rational numbers.

A diverging cylindrical shock is assumed to be propagating in the non-ideal gas with constant density. The jump conditions across the shock front are as

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$$\rho_1(D - v_1) = \rho_0 D, \tag{2.7}$$

$$p_1 + \rho_1(D - v_1)^2 = p_0 + \rho_0 D^2, \tag{2.8}$$

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}(D - v_1)^2 = e_0 + \frac{p_0}{\rho_0} + \frac{1}{2}D^2, \tag{2.9}$$

$$j_1 = j_0, \tag{2.10}$$

where the suffices '1' and '0' refer to conditions just behind and just ahead of the shock respectively, and D is the shock velocity.
If the shock is strong, the boundary conditions (2.7)-(2.10) take the form

$$v_1 = (1 - \beta)D, \tag{2.11}$$

$$\rho_1 = \frac{\rho_0}{\beta}, \tag{2.12}$$

$$p_1 = \rho_0(1 - \beta)D^2, \tag{2.13}$$

$$j_1 = j_0, \tag{2.14}$$

where the quantity β $(0 < \beta < 1)$ is obtained by the relation

$$\beta^2(\gamma + 1) - 2(\gamma + \bar{b})\beta + 2\bar{b} + \gamma - 1 = 0, \tag{2.15}$$

$\bar{b} = b\rho_0$ being the parameter of non-idealness.

The dimensions of the constant coefficient K_0 in equation (2.6) are given by

$$[K_0] = M^{-n-m-q} L^{3n+m-s-l} T^{2m+3q-l}. \tag{2.16}$$

Following the approach of Sedov [1], we get the conditions under with the formulated problem will have self-similar solutions. The dimensional constants in the present problem will be p_0, ρ_0, j_0, and K_0 in which p_0, ρ_0 and j_0 are dependent given by

$$j_0 = [p_0]^{3/2}[\rho_0]^{-1/2}. \tag{2.17}$$

For self-similarity the radiation absorption coefficient K_0 must be dependent on the dimensions of j_0, ρ_0 which is equivalent to $s + l = -1$. The self-similar independent dimensionless variable λ is taken in the form $\lambda = r/r_1$, where

$$r_1 = \bar{\beta}\, j_0^{1/3} \rho_0^{1/3}\, t, \tag{2.18}$$

r_1 being the radius of the shock surface. The value of the constant $\bar{\beta}$ is so chosen that $\lambda = 1$ at the shock surface.

Similarity Solutions

We introduce the following similarity transformations to reduce the equations of motion into ordinary differential equations;

$$v = DV(\lambda), \qquad \rho = \rho_0 R(\lambda),$$

$$p = \rho_0 D^2 P(\lambda), \qquad j = j_0 J(\lambda), \tag{3.1 – 3.4}$$

where V, R, P and J are functions of the non-dimensional variable (similarity variable) $\lambda = \dfrac{r}{r_1}$.

Using the transformations (3.1)-(3.4), the equations of motion take the form

$$R'\lambda(V-\lambda) + R(V'\lambda + V) = 0, \tag{3.5}$$

$$P' - RV'(\lambda - V) = 0, \tag{3.6}$$

$$P'(V-\lambda)(1-\bar{b}R) + \frac{\gamma P}{\lambda}(\lambda V' + V) = \frac{\gamma - 1}{\lambda}[\lambda J' + J], \tag{3.7}$$

$$J' = \alpha \lambda^s R^n P^m J^{q+1} \tag{3.8}$$

where

$$\alpha = K_0 j_0^{q+1} \rho_0^{n+m-1} D^{2(m-1)+s} \tag{3.9}$$

is a dimensionless quantity and a quantity with a 'dash' represents the derivative of that quantity with respect to λ. The quantity α is taken as the parameter which characterizes the interaction between the gas and the incident radiation flux [Khudyakov (1983), Nath (1998), Nath and Takhar (1990)].

Solving equations (3.6)-(3.10) for $\dfrac{dV}{d\lambda}$, $\dfrac{dR}{d\lambda}$, $\dfrac{dP}{d\lambda}$ and $\dfrac{dJ}{d\lambda}$, we have

$$V' = \frac{\left[\gamma PV - (\gamma-1)(\alpha\lambda^{s+1}R^n P^m J^{q+1} + J)\right]}{\left[\lambda\left\{R(\lambda - V)^2(1-\bar{b}R) - \gamma P\right\}\right]}, \tag{3.10}$$

$$R' = \frac{R(V'\lambda + V)}{\lambda(\lambda - V)}, \tag{3.11}$$

$$P' = \frac{\left[\gamma P(\lambda V' + V) - (\gamma - 1)(\lambda J' + J)\right]}{(\lambda - V)(1-\bar{b}R)\lambda}, \tag{3.12}$$

$$J' = \alpha\lambda^s R^n P^m J^{q+1}. \tag{3.13}$$

The shock conditions (2.11)-(2.14) are transformed into

$$V(1) = 1 - \beta, \tag{3.14}$$

$$R(1) = \frac{1}{\beta}, \tag{3.15}$$

$$P(1) = (1-\beta), \tag{3.16}$$

$$J(1) = 1. \tag{3.17}$$

At the inner boundary surface of the flow-field behind the shock the condition is that the velocity of the surface is equal to the normal velocity of the fluid on the surface. The kinematic condition, from equation (3.1), can be written as

$$V(\lambda_p) = \lambda_p, \tag{3.18}$$

where λ_p is the value of λ at the inner surface.

For exhibiting the numerical solutions, it is convenient to write the flow variables in the non-dimensional form as

$$\frac{v}{v_1} = \frac{V(\lambda)}{V(1)}, \quad \frac{\rho}{\rho_1} = \frac{R(\lambda)}{R(1)}, \quad \frac{p}{p_1} = \frac{P(\lambda)}{P(1)}, \quad \frac{j}{j_1} = \frac{J(\lambda)}{J(1)}. \tag{3.19}$$

RESULTS AND DISCUSSION

The set of differential equations (3.10)-(3.13) are numerically integrated with the boundary conditions (3.14)-(3.17) to obtain the non-dimensional variables of the flow-field V, R, P and J against the similarity variable λ, by using the Runge-Kutta method of order four, for the values [Khudyakov (1983), Nath (1998), Nath and Takhar (1990), Ranga Rao and Purohit (1976)] $\alpha = 0.1, 0.2$; $n = -1/2$; $m = 3/2$; $q = 0$; $s = 1$; $\gamma = 7/5, 5/3$; $\bar{b} = 0, 0.05, 0.1$. The case $\bar{b} = 0$ corresponds to the perfect gas case studied by Nath and Takhar (1990).

In figures 1 to 4, we have plotted the radial velocity v/v_1, the density ρ/ρ_1, the pressure p/p_1 and the radiation flux j/j_1 against the radial distance r/r_1 in the flow-field between the inner expanding surface and the shock surface. Tables 1 and 2 show, respectively, the density ratio across the shock and the position of inner expanding surface for various values of the parameters $\gamma, \bar{b}, \alpha$.

From tables 1 and 2 and figures 1-4 it is observed that the effects of an increase in the value of γ (ratio of specific heats) are

(i) to decrease x_p, i.e. to increase the distance of inner expanding surface from the shock front. Physically, it means that the gas behind the shock is less compressed, i.e. the shock strength is reduced;

(ii) to decrease the value of $1/\beta$, i.e. to decrease the shock strength, which is the same as given in (i) above;

(iii) to decrease the value of v/v_1 and p/p_1; at any point in the flow-field behind the shock; and

(iv) to enhance the effect of non-idealness of the gas on the profiles of v/v_1, p/p_1 and j/j_1.

Thus an increase in the ratio of specific heats decays the shock wave and enhances the effect of non-idealness of the gas on the profiles of the flow variables behind the shock.

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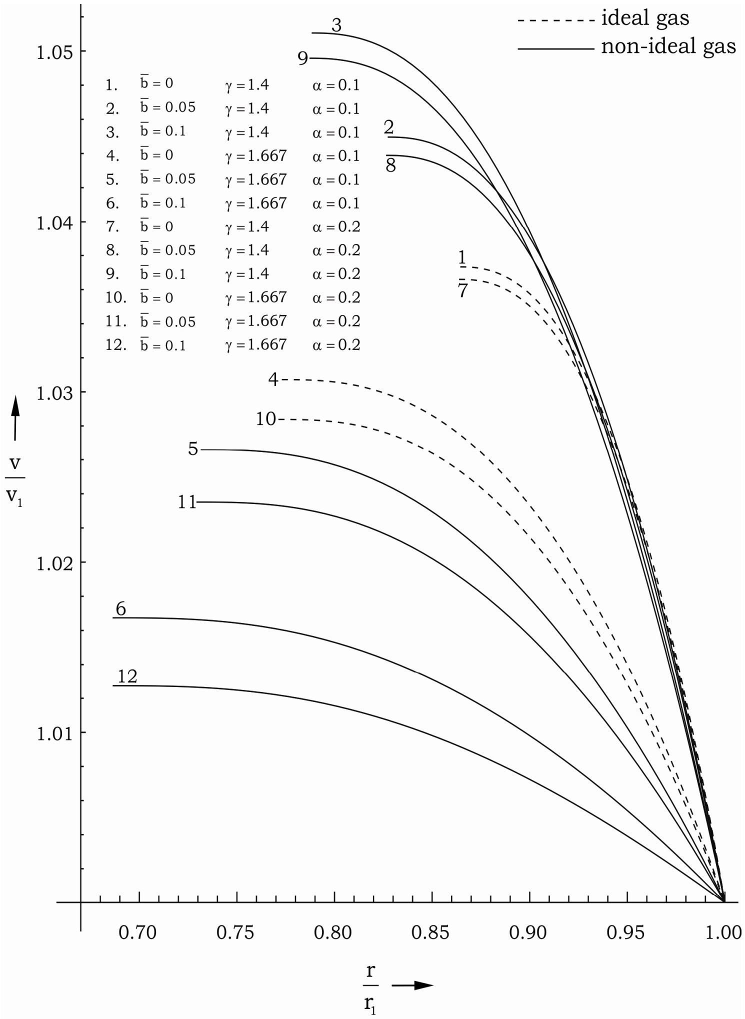

Figure1: Variation of radial fluid velocity v/v_1 with radial distance r/r_1 in the flow-field behind the shock

The effects of an increase in the value of the parameter of the non-idealness of the gas $\bar{b}$ are

(i) to increase the distance of the inner expanding surface from the shock front (see table 2);

(ii) to decrease the value of $1/\beta$ (table 1), i.e. to decrease the shock strength. Therefore the non-idealness of the gas has decaying effect on the shock wave;

(iii) to increase the value of ρ/ρ_1, j/j_1 and to decrease the value of p/p_1 at any point in the flow-field behind the shock (see figures 2, 3, 4). These effects are significant when γ is equal to 1.667 instead of 1.4.

Thus the non-idealness of the gas decays the shock wave, and affects the variables in the flow-field behind the shock significantly, when the value of ratio of specific heats is higher.

Effects of an increase in the radiation parameter α are

(i) to decrease x_p (table 2), i.e. to decrease the shock strength; and to decrease v/v_1, p/p_1, and j/j_1 at any point in the flow-field behind the shock. This decrease in these flow-variables is somewhat significant when the value of the ratio of specific heats is larger.

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2012 Vol. 2 (1) January-March, pp.27-37/Vishwakarma and Pandey
Research Article

Table 1. The density ratio β across the shock front for different values of $\bar{b}$ and γ

$\bar{b}$	$\frac{\rho_0}{\rho_1} = \beta$	
	$\gamma = 1.4$	$\gamma = 1.667$
0	0.1667	0.2500
0.05	0.2083	0.2875
0.1	0.2500	0.3250

Table 2. Position of inner expanding surface x_p for various values of $\bar{b}$, γ and α

γ	α	$\bar{b}$	x_p
1.4	0.1	0	0.86442
		0.05	0.827261
		0.1	0.78829
	0.2	0	0.863831
		0.05	0.826415
		0.1	0.787183
1.667	0.1	0	0.773039
		0.05	0.731446
		0.1	0.686300
	0.2	0	0.771287
		0.05	0.729253
		0.1	0.683598

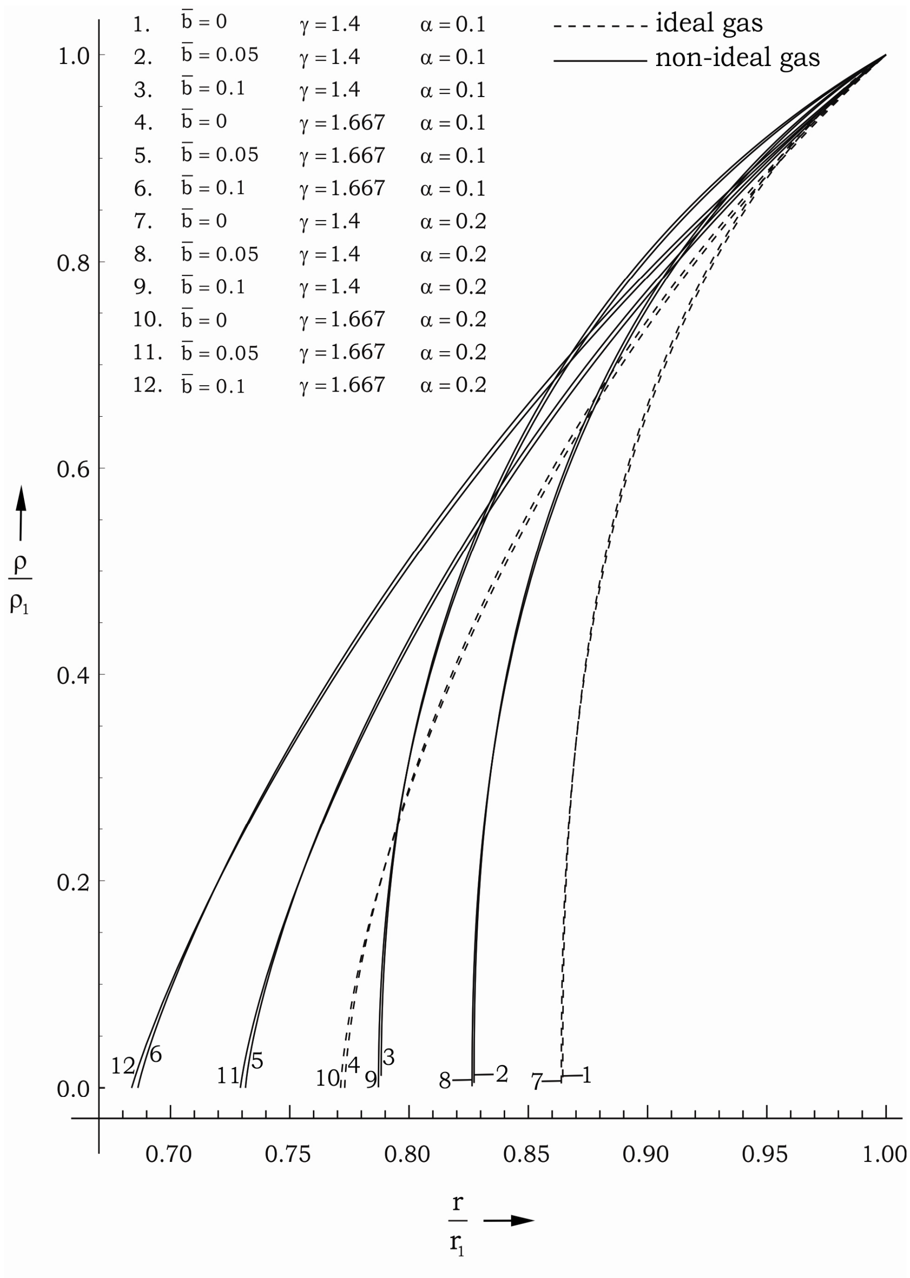

Figure 2: Variation of density ρ/ρ_1 with radial distance r/r_1 in the flow-field behind the shock

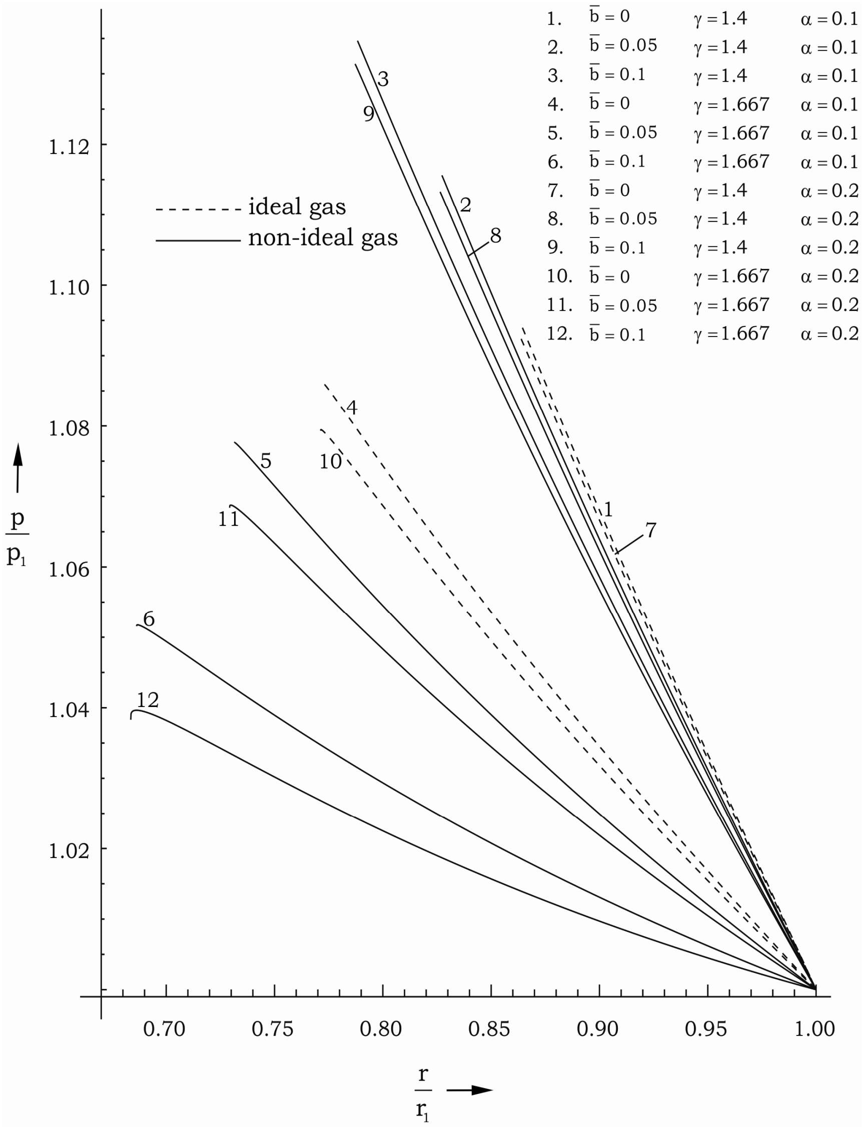

Figure 3: Variation of pressure p/p_1 with radial distance r/r_1 in the flow-field behind the shock

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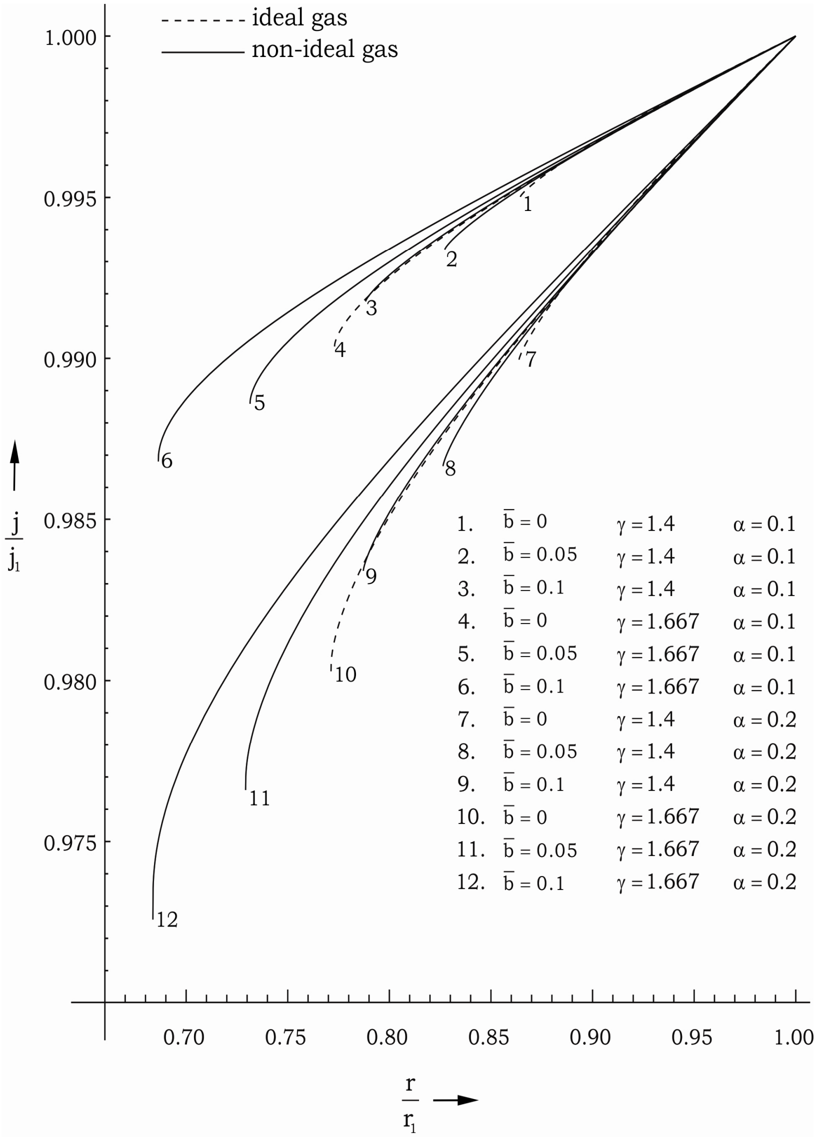

Figure 4: Variation of radiation flux j/j_1 with radial distance r/r_1 in the flow-field behind the shock

Conclusion

The present work investigates the self-similar flow of a non-ideal gas under the action of monochromatic radiation behind a strong cylindrical shock wave. The density of the ambient medium is uniform. On the basis of the present work, one may draw the following conclusions:

(i) An increase in the ratio of specific heats decays the shock wave and enhances the effect of non-idealness of the gas on the profiles of the flow-variables behind the shock.

(ii) The non-idealness of the gas decays the shock wave, and affects the flow behind the shock significantly when the values of the ratio of specific heats is higher.

(iii) An increase in α affects the profiles of the flow variables somewhat significantly when the ratio of specific heats is higher.

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