International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Online International Journal Available at http://www.cibtech.org/jpms.htm
2011 Vol. 1 (1) October-December, pp.80-85/Dangwal et al.
Research Article

A VAGUE SET THEORETIC TO GOAL PROGRAMMING

***Rajesh Dangwal[1], M.K. Sharma[2] and Padmendra Singh[1]**
[1]Department of Mathematics H.N.B.Garhwal University, Campus Pauri, Uttarakhand
[2]Department of Mathematics R.S.S.(P.G.) College, Pilkhuwa, Ghaziabad
**Author for Correspondence*

ABSTRACT
An additive Fuzzy goal programming model was given by Tiwari et al., (1987). In their model they dealt with the achievement of goals by using membership function for each goal. This approach of solving the problem considers only the favourable aspect of belongingness. It would be more realistic if the achievement of goals be viewed by considering all arguments, that may favour or disfavour the achievement of goal. This leads to changing the fuzzy set theoretic approach to vague set theory. Present paper provides a goal programming model using vague set theory. The process of solution is illustrated using the example of Tiwari et al., (1987). It has been observed that the achievements of goals by our method are more close to the aspiration level.

INTRODUCTION
Goal programming (GP) is a multi-criteria decision making technique which applies in many real world problems in a precise manner. Goal programming is an extension of linear programming to include multiple objectives. In FGP, each objective function should be substantially less than or equal to same value, called aspiration level. Often, in real world problems, aspiration levels and/or priority factors of the DM, some time even the weights to be assigned to the goals are not assigned in precise manner. To overcome this ambiguity, Fuzzy set theory plays an important role.
Narasimhan (1980, 1981) , Hannan, (1981, 1981,1982) Narasomhan, (1981), Ignizio (1982), Rubin and Narasimhan (1984) and Tiwari et. al. (1986, 1987) are those persons which use fuzzy set theory in Goal programming and investigated various aspects of decision problems using FGP.
In FGP, for each of the objective function assume that the DM has a fuzzy goal such as "objective function should be substantially less than or equal to aspiration level". So, DM takes a linear membership function for each fuzzy goal.
In this present paper, we investigate a particular modelling in which for each goal, we investigate non-membership function as mellas membership function and show that sum of non-membership function and membership function for each goal is less than or equal to 1.
In conventional GP the simple additive model for m goal S G_i (X) with deviational variability's d_i^+, d_i^- is defined as

$$\text{minimize } \sum_{i=1}^{m} (d_i^+ + d_i^-)$$

subject to $\sum G_i(X) + d_i^- - d_i^+ = g_i$

$$d_i^+, d_i^- = 0$$

$$d_i^+, d_i^-,\ x \geq 0,\ i = 1,2....m$$

where g_i is the aspiration level of the goal. Here we develop a similar model using membership function and non-membership function. Instead of deviational variables. So far, we had an additive model in the research paper Tiwari et. al., in which we use membership functions only.
Consider the FGP problem:
Find X
to satisfy $G_i(X) \geq g_i$, $i = 1,2,3,..........m$

subject to $AX \le B,\ X \ge 0$

Where x is an n-vector with components x_1, $x_2 \ldots x_n$ and $AX \le B,$ are system constraints in vector notation.

A linear membership function μ_i for the i[th] fuzzy goal $G_i(X) \ge g_i$ can be expressed as

$$\mu_i = \begin{cases} 1 \ \ if \ \ G_i(X) \ge g_i\,, \\ \dfrac{G_i\left(X\right) - L_i}{g_i - L_i} \quad if \quad L_i \le G_i(X) \le g_i \\ 0 \qquad\qquad if \quad G_i\left(X\right) \le L_i \end{cases}$$

where L_i is the lower tolerance limit for the fuzzy goal $G_i(x)$.

The non-membership function v_i for that goal is

$$v_i = \begin{cases} 0 \ \ if \ \ G_i(X) \ge g_i\,, \\ \dfrac{g_i - G_i\left(X\right)}{g_i - L_i^{'}} \quad if \quad L_i^{'} \le G_i(X) \le g_i \\ 1 \qquad\qquad if \quad G_i\left(X\right) \le L_i^{'} \end{cases}$$

Where $L_i^{'} \le L_i$

Now,

$$\frac{G_i(x) - L_i}{g_i - L_i} + \frac{g_i - G_i(x)}{g_i - L_i^{'}}$$

$$= \frac{(g_i - L_i^{'})(G_i(x) - L_i) + (g_i - G_i(x))(g_i - L_i)}{(g_i - L_i)(g_i - L_i^{'})}$$

$$= \frac{g_i G_i - g_i L_i - L_i^{'} G_i + L_i L_i^{'} + g_i^2 - L_i g_i - G_i g_i + G_i L_i)}{(g_i - L_i)(g_i - L_i^{'})}$$

$$= \frac{g_i^2 - 2L_i g_i + (L_i - L_i^{'}) G_i + L_i L_i^{'})}{(g_i - L_i)(g_i - L_i^{'})}$$

$$= \frac{g_i(g_i - L_i) - L_i(g_i - L_i^{'}) + G_i(L_i - L_i^{'})}{(g_i - L_i)(g_i - L_i^{'})}$$

Taking $L_i = L_i^{'} + a, \quad where \quad a \ge 0,\ then$

$$= \frac{g_i(g_i - L_i) - L_i(g_i - L_i + a) + G_i(L_i - L_i + a)}{(g_i - L_i)(g_i - L_i^{'})}$$

$$= \frac{g_i(g_i - L_i) - L_i(g_i - L_i) - L_i a + a G_i}{(g_i - L_i)(g_i - L_i^{'})}$$

$$= \frac{(g_i - L_i)(g_i - L_i) + a(G_i - L_i)}{(g_i - L_i)(g_i - L_i^{'})}$$

Since $(G_i - L_i) \le (g_i - L_i),\ So, we \ \ have$

$$\le \frac{(g_i - L_i)(g_i - L_i) + a(g_i - L_i)}{(g_i - L_i)(g_i - L_i + a)}$$

$$= \frac{(g_i - L_i)(g_i - L_i + a)}{(g_i - L_i)(g_i - L_i + a)}$$

= 1

For goal $G_i(x) \le g_i$, *we have*

$$\mu_i = \begin{cases} 1 & if \quad G_i(x) \le g_i \\ \dfrac{U_i - G_i}{U_i - g_i} & if \quad g_i \le G_i(x) \le U_i \\ 0 & if \quad G_i(x) \ge U_i \end{cases}$$

The non-membership function is

$$\upsilon_i = \begin{cases} 0 & if \quad G_i(x) \le g_i \\ \dfrac{G_i - g_i}{U_i^{'} - g_i} & if \quad g_i \le G_i(x) \le U_i^{'} \\ 1 & if \quad G_i(x) \ge U_i^{'} \end{cases}$$

Where $U_i \le U^{'}_i$, *take* $U^{'}_i = U_i + b,\ b \ge 0$

Now,

$$\frac{U_i - G_i}{U_i - g_i} + \frac{G_i - g_i}{U_i^{'} - g_i}$$

$$= \frac{(U_i - G_i)(U_i^{'} - g_i) + (G_i - g_i)(U_i - g_i)}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{U_i U_i^{'} - U_i g_i - G_i U_i^{'} + G_i g_i + G_i U_i - G_i g_i - g_i U_i + g^2{}_i}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{g_i(g_i - U_i) - U_i(g_i - U_i^{'}) + G_i(U_i - U_i^{'})}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{g_i(g_i - U_i) - U_i(g_i - (U_i + b)) + G_i(U_i - (U_i - b))}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{g_i(g_i - U_i) - U_i(g_i - U_i - b) + G_i(U_i - U_i - b))}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{(U_i - g_i)^2 + U_i b - G_i b}{(U_i - g_i)(U_i^{'} - g_i)}$$

$$= \frac{(U_i - g_i)^2 + (U_i - G_i)b}{(U_i - g_i)(U_i^{'} - g_i)}$$

Since $U_i - G_i \le U_i - g_i$, *So, we have*

$$\Rightarrow G_i - U_i \ge g_i - U_i$$

$$\le \frac{(U_i - g_i)^2 + (U_i - g_i)b}{(U_i - g_i)(U_i - g_i + b)}$$

$$= \frac{(U_i - g_i)(U_i - g_i + b)}{(U_i - g_i)(U_i - g_i + b)}$$

= 1.

NUMERICAL EXAMPLE

Let us consider a mathematical model of the problem having 5 fuzzy goals with 4 variables and 4 system constraints as follows:

Find X satisfying the following fuzzy goals:

$$\begin{aligned} &4x_1 + 2x_2 + 8x_3 + x_4 \le 35 \\ &4x_1 + 7x_2 + 6x_3 + 2x_4 \ge 100 \\ &x_1 - 6x_2 + 5x_3 + 10x_4 \ge 120 \\ &5x_1 + 3x_2 + 2x_4 \ge 70 \\ &4x_1 + 4x_2 + 4x_3 \ge 40 \end{aligned} \qquad \text{.......} \quad (3)$$

S.t.,

$$\begin{aligned} &7x_1 + 5x_2 + 3x_3 + 2x_4 \le 98 \\ &7x_1 + x_2 + 6x_3 + 6x_4 \le 117 \\ &x_1 + x_2 + 2x_3 + 6x_4 \le 130 \\ &9x_1 + x_2 + 6x_4 \le 105 \\ &x_1,\ x_2,\ x_3,\ x_4 \ge 0 \end{aligned} \qquad \text{.......}(4)$$

Let the tolerance limits of the 5 goals be (55, 40, 70, 30, 10) respectively. Now, the fuzzy goals are converted into crisp ones by using membership functions μ_i as defined in (2.1, 2.3). Thus this problem reduces to

$$\max V(\mu) = \sum_{i=1}^{5} \mu_i \qquad \text{......}(5a)$$

S. t. $\mu_1 = \dfrac{55 - (4x_1 + 2x_2 + 8x_3 + x_4)}{20}$

$$\Rightarrow \quad 4x_1 + 2x_2 + 8x_3 + x_4 + 20\mu_1 = 55$$

$$\mu_2 = \frac{4x_1 + 7x_2 + 6x_3 + 2x_4 - 40}{60}$$

$$\Rightarrow 4x_1 + 7x_2 + 6x_3 + 2x_4 - 60\mu_2 = 40$$

$$\mu_3 = \frac{x_1 - 6x_2 + 5x_3 + 10x_4 - 70}{50}$$

$$\Rightarrow x_1 - 6x_2 + 5x_3 + 10x_4 - 50\mu_3 = 70$$

$$\mu_4 = \frac{5x_1 + 3x_2 + 2x_4 - 30}{40}$$

$$\Rightarrow 5x_1 + 3x_2 + 2x_4 - 40\mu_4 = 30$$

$$\mu_5 = \frac{4x_1 + 4x_2 + 4x_3 - 10}{30}$$

$$\Rightarrow 4x_1 + 4x_2 + 4x_3 - 30\mu_5 = 10,$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 \le 98$$

$$7x_1 + x_2 + 6x_3 + 6x_4 \le 117$$

$$x_1 + x_2 + 2x_3 + 6x_4 \le 130 \qquad \text{......(5 b)}$$

$$9x_1 + x_2 + 6x_4 \le 105$$

$$and \mu_i \le 1$$

$$\mathrm{x}_j, \mu_i \ge 0,\ i = 1,2,....5,\ j = 1,2......4.$$

Introducing slack variables, then

$$7x_1 + 5x_2 + 3x_3 + 2x_4 + x_5 = 98$$

$$7x_1 + x_2 + 6x_3 + 6x_4 + x_6 = 117$$

$$x_1 + x_2 + 2x_3 + 6x_4 + x_7 = 130$$

$$9x_1 + x_2 + 6x_4 + x_8 = 105$$

$$\mu_1 + x_9 = 1,\ \mu_2 + x_{10} = 1,\ \mu_3 + x_{11} = 1$$

$$\mu_4 + x_{12} = 1,\ \mu_5 + x_{13} = 1$$

All $x_i \ge 0,\ \mu_i \ge 0$

This problem is solved by 'TORA' software we have the results as

$$x_1 = 0,\ x_2 = 9.75,\ x_3 = 0,\ x_4 = 15.875$$

with achieved Goal values

$$G_1 = 35.375,\ G_2 = 100.0,\ G_3 = 100.25,\ G_4 = 61.0,\ G_5 = 39.0$$

and membership values

$$\mu_1 = 0.981,\ \mu_2 = 1.00,\ \mu_3 = 0.605,\ \mu_4 = 0.775,\ \mu_5 = 0.967$$

For nonmembership functions

$$\text{Max V}(v) = \sum_{i=1}^{5} (1 - vi) \qquad .....(6a)$$

Research Article

Subject to

$$4x_1 + 2x_2 + 8x_3 + x_4 - 25v_1 = 35$$
$$4x_1 + 7x_2 + 6x_3 + 2x_4 + 65v_2 = 100$$
$$x_1 - 6x_2 + 5x_3 + 10x_4 + 55v_3 = 120$$
$$5x_1 + 3x_2 + 2x_4 + 45v_4 = 70 \quad \text{......(6b)}$$
$$4x_1 + 4x_2 + 4x_3 + 35v_5 = 40$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 + x_5 = 98$$
$$7x_1 + x_2 + 6x_3 + 6x_4 + x_6 = 117$$
$$x_1 + x_2 + 2x_3 + 6x_4 + x_7 = 130$$
$$9x_1 + x_2 + 6x_4 + x_8 = 105$$
$$v_1 + x_4 = 1,\ v_2 + x_{10} = 1,$$
$$v_3 + x_{11} = 1,\ v_4 + x_{12} = 1,$$
$$v_5 + x_{13} = 1$$

All $x_i \geq 0$ and $v_i \geq 0$

The solution is

$x_1 = 0$, $x_2 = 9.75$, $x_3 = 0$, $x_4 = 15.884$

With achieved goal values

$G_1 = 35.384$, $G_2 = 100.018$, $G_3 = 100.34$

$G_4 = 61.018$, $G_5 = 39.0$

and non-membership values

$v_1 = .01$, $v_2 = 0$, $v_3 = .36$, $v_4 = .20$, $v_5 = .03$

REFERENCES

Hannan E.L. (1981). Linear programming with multiple fuzzy goals, *Fuzzy Sets and Systems.* **6** 235-248.

Hannan E.L. (1981). On fuzzy goal programming. *Decision Sciences.* **12** 522-531.

Hannan E.L.(1982). Contrasting fuzzy goal programming and fuzzy multi criteria programming. *Decision Sciences.* **13** 337-339.

Ignizio J.P. (1982) On the (re) discovery of fuzzy goal programming. *Decision Sciences.* **13** 331-336.

Narasimhan R.(1980). Goal programming in a fuzzy environment. *Decision Sciences.* **11** 325-336.

Narasimhan R. (1981). On fuzzy goal programming-some Comments. *Decision Sciences.* **12** 532-538.

Rubin P.A. and Narasimhan R. (1984). Fuzzy goal programming with nested priorities. *Fuzzy Sets and Systems* **14** 115-129.

Tiwari R.N., Dharmar S (1986). Rao J.R., Priority structure in fuzzy goal programming. *Fuzzy Sets and Systems* **19** 251-259.

Tiwari R.N., Dharmar S. and Rao J.R. (1987). Fuzzy goal programming An additive model. *Fuzzy Sets and Systems* **24** 27-34.

Zadeh L.A. (1965). Fuzzy Sets. *Information and Control.* **8** 338-353.

Zimmermann H.J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems* **1** 45-55.